A MILP Approach to the short term Hydrothermal Self-Scheduling Problem

C. K. Simoglou, *Student Member*, *IEEE*, P. N. Biskas, *Member*, *IEEE*, A. G. Bakirtzis, *Senior Member*, *IEEE*

***Abstract*— This paper addresses the hydrothermal electricity producer self-scheduling problem in day-ahead energy and reserves markets. A 0/1 mixed integer linear formulation of the producer self-scheduling problem is presented, which allows a realistic modeling of the unit's operating phases (synchronization, soak, dispatchable, desynchronization). Prohibited operating zones and daily hydro energy production constraints are also modeled. Test results address the effect of the energy and reserves market clearing prices on the producer units' day-ahead commitment status and profits.**

***Index Terms*— electric energy markets, reserves markets, generation scheduling, mixed integer linear programming (MILP)**

I. NOMENCLATURE

f ($\mathcal{F}^i$)	index (set) of steps of the marginal cost function of unit *i*
i ($\mathcal{I}$)	index (set) of generating units (thermal & hydro)
j ($\mathcal{J}$)	index (set) of hydro units (subset of $\mathcal{I}$)
k ($\mathcal{K}$)	index (set) of the permissible operating zones of hydro unit *j*
m ($\mathcal{M}$)	index (set) of reserves types $\mathcal{M}=\{1+,1-,2+,2-,3S,3NS\}$, where *m*=1+: primary-up, *m*=1-: primary-down, *m*=2+: secondary-up, *m*=2-: secondary-down, *m*=3S: tertiary spinning, *m*=3NS: tertiary non-spinning
t ($\mathcal{T}$)	index (set) of hours of the planning period
λ_t^E	forecasted price of energy in hour *t*, in €/MWh
λ_t^m	forecasted price of reserve type *m* in hour *t*, in €/MW&h
B_{if}	size of step *f* of unit *i* marginal cost function, in MW
b_{ift}	portion of step *f* of the *i*-th unit's marginal cost function loaded in hour *t*, in MW
c_{if}	marginal cost of step *f* of unit *i* marginal cost function, in €/MWh
$c_{it}(p_{it})$	total production cost of unit *i* in hour *t* at level p_{it}, in €
D_t	system load demand in hour *t*, in MW
DT_i	minimum down time of unit *i,* in h
$\bar{H}_j$	maximum energy production of hydro unit *j* during the planning period, in MWh
$\underline{H}_j$	minimum energy production of hydro unit *j* during the planning period, in MWh
NLC_i	no-load cost of unit *i* (for one hour operation), in €
$P_i^{\max}$	maximum power output of unit *i*, in MW
$P_i^{\max,AGC}$	maximum power output of unit *i* while operating under AGC, in MW
$P_i^{\max,k}$	upper bound of the *k*-th permissible operating zone of unit *i* , in MW
$P_i^{\min}$	minimum power output of unit *i,* in MW
$P_i^{\min,AGC}$	minimum power output of unit *i* while operating under AGC, in MW
$P_i^{\min,k}$	lower bound of the *k*-th permissible operating zone of unit *i* , in MW
$\bar{P}_i^{soak}$	fixed power output of unit *i* while in soak phase, in MW
p_{it}	power output of unit *i* accepted by the ISO in hour *t* in the day-ahead energy auction, in MW
p_{it}^{des}	power output of unit *i* during the desynchronization phase in hour *t*, in MW
p_{it}^{soak}	power output of unit *i* during the soak phase in hour *t*, in MW
R_i^m	maximum contribution of unit *i* in reserve type *m*, in MW
r_{it}^m	contribution of unit *i* in reserve type *m* during hour *t*, in MW
RD_i	ramp-down rate of unit *i,* in MW/min
RD_i^{AGC}	ramp-down rate of unit *i* while operating under AGC*,* in MW/min
RR_t^m	system requirement in reserve type *m* during hour *t*, in MW
RU_i	ramp-up rate of unit *i,* in MW/min
RU_i^{AGC}	ramp-up rate of unit *i* while operating under AGC*,* in MW/min
SDC_i	shut-down cost of unit *i*, in €
SUC_i	start-up cost of unit *i*, in €
T_i^{syn}	synchronization time of unit *i*, in h
T_i^{soak}	soak time of unit *i*, in h

C. K. Simoglou and A.G Bakirtzis are with the Department of Electrical & Computer Engineering, Aristotle University of Thessaloniki, 54124, Thessaloniki, GREECE (e-mail: chsimoglou@ee.auth.gr, bakiana@eng.auth.gr).

P.N. Biskas is with the Hellenic TSO (e-mail: pbiskas@desmie.gr)

978-1-4244-2235-7/09/$25.00 ©2009 IEEE

T_i^{des}	time from technical minimum power output to desynchronization of unit *i*, in h
UT_i	minimum up time of unit *i*, in h
u_{it}	binary variable which is equal to 1 if unit *i* is committed during hour *t*
u_{it}^{syn}	binary variable which is equal to 1 if unit *i* is in the synchronization phase during hour *t*
u_{it}^{soak}	binary variable which is equal to 1 if unit *i* is in the soak phase during hour *t*
u_{it}^{disp}	binary variable which is equal to 1 if unit *i* is in the dispatchable phase during hour *t*
u_{it}^{des}	binary variable which is equal to 1 if unit *i* is in the desynchronization phase during hour *t*
u_{it}^{AGC}	binary variable which is equal to 1 if unit *i* provides secondary reserve during hour *t*
u_{it}^{3NS}	binary variable which is equal to 1 if unit *i* provides tertiary non-spinning reserve during hour *t*
$\mathbf{x}_{it}$	vector of the continuous and the binary decision variables associated with the operation of unit *i* during hour *t*
y_{it}	binary variable which is equal to 1 if unit *i* is started-up during hour *t*
z_{it}	binary variable which is equal to 1 if unit *j* is shut-down during hour *t*

II. Introduction

In a competitive electricity market, an important problem that electricity producers face every day is the self-scheduling of their generating units (thermal and hydro) so as to maximize their total profits in the day-ahead energy and reserves markets.

The self-scheduling problem of a price-taker producer owning one thermal unit and participating in a day-ahead energy market is modeled and solved as a mixed-integer linear program (MILP) in [1]. A detailed formulation of a thermal unit start-up and shut-down procedures is given in [2]. The optimal response of a price-taker power generator to simultaneous energy and reserves pool-based markets is modeled in [3]. An analytical MILP method to solve the hydro producer self-scheduling problem is presented in [4]. The hydrothermal self-scheduling problem of a price-taker producer in a day-ahead energy market is modeled and solved as a MILP problem in [5]-[6].

This paper formulates and solves the hydrothermal producer (from now on "Producer") self-scheduling problem as a 0/1 mixed-integer linear program, which provides an actual and precise modeling of the unit's operating phases (synchronization, soak, dispatchable, desynchronization). The Producer self-scheduling problem is implemented in a pool framework similar to the Greek day-ahead market structure, considering simultaneously the following markets, as required by the Greek Grid and Exchange Code [7]:

1) energy market
2) primary up/down reserve markets
3) secondary up/down reserve markets
4) tertiary spinning/non-spinning reserve markets

In this paper, it is assumed that the Producer acts as a price-taker in all above markets, responding to forecasted price curves.

The solution of the Producer self-scheduling problem provides the desired energy and reserve contribution of each Producer unit (thermal and hydro), for all hours of the next day. After computing the optimal self-schedule, the Producer can design a suitable bidding strategy [8] that will result to this optimal schedule, by offering the quantities he desires to be scheduled at prices below the forecasted energy and reserves market-clearing prices.

III. The Producer Self-Scheduling Problem

The Producer self-scheduling problem is modeled as a mathematical optimization problem as follows:

$$Max \sum_{t\in\mathcal{T}}\sum_{i\in I}\left[\lambda_t^E\cdot p_{it}+\sum_{m\in\mathcal{M}}\left(\lambda_t^m\cdot r_{it}^m\right)-c_{it}\left(p_{it}\right)\right] \qquad (1)$$

Subject to:

$$\mathbf{x}_{it}=\left[p_{it},u_{it},y_{it},z_{it}....\right]\in \Pi_i \qquad \forall i\in I, t\in\mathcal{T} \qquad (2)$$

The Producer profit during the planning period (1) is the Producer revenue from the energy market and the various reserves markets minus the total Producer cost.

Constraints (2) represent the operating constraints of the Producer units, i.e. min/max power output restrictions, ramp-rate limits, minimum up/down times, start-up and shut-down procedures etc. These constraints can be expressed as linear constraints on the continuous and the binary decision variables associated with the operation of the unit, denoted by the vector decision variable $\mathbf{x}_{it}$, further described in the following.

Other constraints modeled are the prohibited operating zones constraints, total hydro energy production constraints and system demand and reserves requirement constraints, all described in detail in the following sections.

A) Unit Operating State Modeling

Fig. 1 shows the different operating states of a unit [9]. After being reserved ($u_{it}=0$) for T_i^{off} hours ($T_i^{off}\geq DT_i$) the unit starts-up at hour t_1 ($y_{it}=1$) and remains committed ($u_{it}=1$) until it is shut-down at hour t_5 ($z_{it}=1$).

Once committed, the unit follows four consecutive operation phases, a) synchronization, b) soak, c) dispatchable and d) desynchronization, denoted by binary variables $u_{it}^{syn}, u_{it}^{soak}, u_{it}^{disp}$ and u_{it}^{des} respectively (Fig. 1). The first two phases comprise the unit start-up phase.

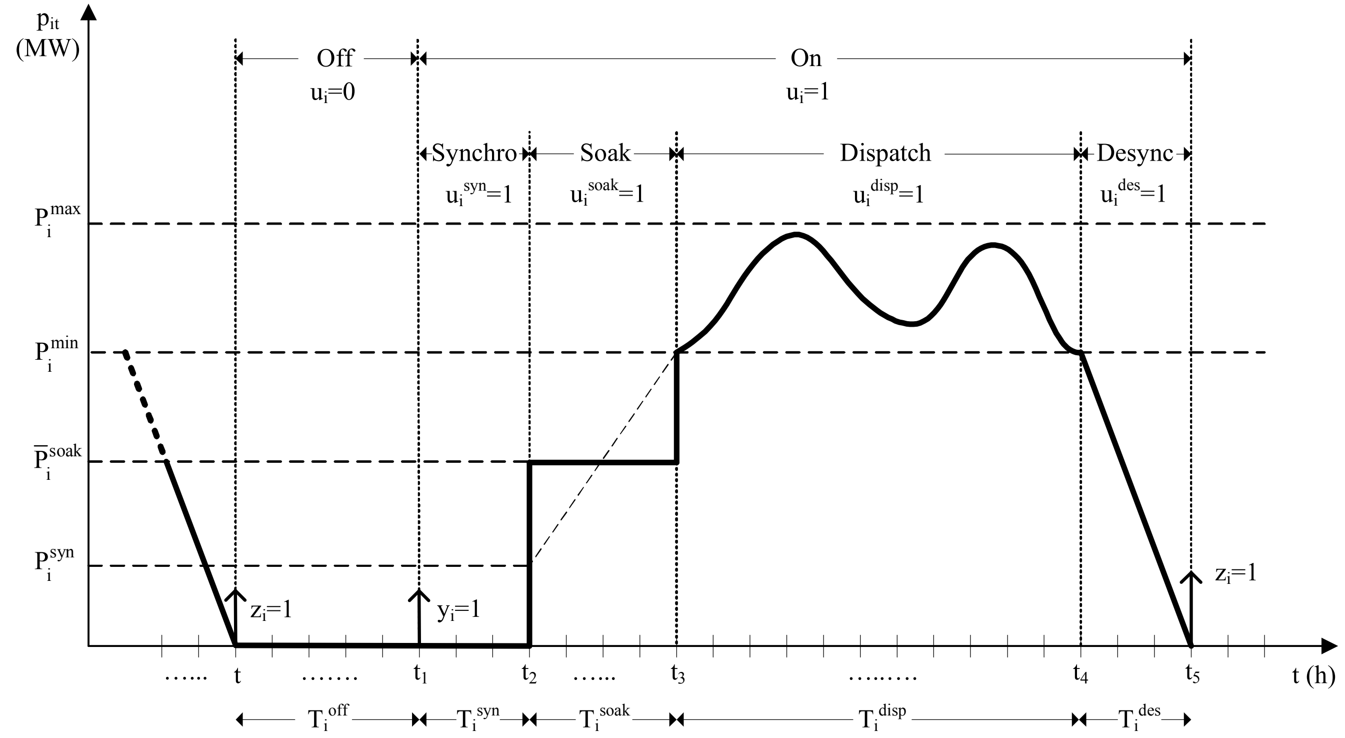

Fig. 1 Operating phases of a thermal unit

Special care has been given to the modeling of the unit start-up sequence. The unit start-up sequence consists of two phases a) synchronization phase and b) soak phase.

Once a start-up decision is taken, $y_{it}=1$, (see Fig. 1) the thermal unit enters the synchronization phase which lasts for T_i^{syn} hours and during which the power output of the unit is zero MW. Subsequently, the unit enters the soak phase which lasts for T_i^{soak} hours and during which the unit's power output increases linearly from the synchronization load P_i^{syn} to the technical minimum $P_i^{\min}$. For the sake of simplicity, in our model the unit's power output during the soak phase is fixed to the constant value $\overline{P}_i^{soak}$ MW (see Fig. 1).

It should be noted that fast start-units (open cycle gas turbines-OCGT and hydro), due to their ability to synchronize in a few minutes, do not follow the start-up sequence described above and so they enter the dispatchable phase directly after their start-up [i. e. $u_{it}^{syn}=u_{it}^{soak}=0$].

B) Linear Expression of Thermal Units Constraints

1) Start-up sequence constraints

$$u_{it}^{syn} = \sum_{\tau=t-T_i^{syn}+1}^{t} y_{i\tau} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (3)$$

$$u_{it}^{soak} = \sum_{\tau=t-T_i^{syn}-T_i^{soak}+1}^{t-T_i^{syn}} y_{i\tau} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (4)$$

$$p_{it}^{soak} = \overline{P}_i^{soak}\cdot u_{it}^{soak} \qquad \forall\ i\in I,\ t\in \mathcal{T} \qquad (5)$$

Constraints (3) ensure that thermal unit i enters the synchronization phase immediately following start-up (see Fig. 1). This is achieved by turning ON the synchronization phase binary variable, u_{it}^{syn}, whenever there is a startup of the unit in the prior T_i^{syn} hours.

Constraints (4) ensure that thermal unit i should enter a soak phase lasting T_i^{soak} hours following its synchronization, during which the soak phase binary variable, u_{it}^{soak}, is turned ON and the unit's power output is fixed to the constant value $\overline{P}_i^{soak}$ MW (5) (see Fig. 1).

2) Desynchronization phase constraints

$$u_{it}^{des} = \sum_{\tau=t+1}^{t+T_i^{des}-1} z_{i\tau} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (6)$$

$$p_{it}^{des} = \sum_{\tau=t}^{t+T_i^{des}-1} z_{i\tau}\cdot(\tau-t)\cdot\frac{P_i^{\min}}{T_i^{des}} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (7)$$

Constraints (6) ensure that thermal unit i should operate in a desynchronization phase lasting T_i^{des} hours before its shut-down (see Fig. 1). The thermal unit power output during the desynchronization process decreases linearly from its technical minimum power output to zero MW (7).

3) Minimum Up/Down Time Constraints

$$\sum_{\tau=t-UT_i+1}^{t} y_{i\tau} \le u_{it} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (8)$$

$$\sum_{\tau=t-DT_i+1}^{t} z_{i\tau} \le 1-u_{it} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (9)$$

Constraints (8)-(9) enforce the minimum up and down time constraints, respectively, i.e. unit i must remain committed (de-committed) at hour t if its start-up (shut-down) started during the previous UT_i-1 (DT_i-1) hours [10].

4) Logical status of commitment

$$u_{it} = u_{it}^{syn} + u_{it}^{soak} + u_{it}^{disp} + u_{it}^{des} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (10)$$

$$y_{it} - z_{it} = u_{it} - u_{i(t-1)} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (11)$$

$$y_{it} + z_{it} \le 1 \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (12)$$

Constraints (10) ensure that only one at most of the binary variables corresponding to the different commitment states of unit i can equal 1 in every hour. Constraints (11) model the logic of the start-up and shut-down status change. Constraints (12) require that unit i may not be started-up and shut-down simultaneously in a given hour.

5) Power output constraints

$$u_{it}^{AGC} \le u_{it}^{disp} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (13)$$

$$r_{it}^{1+} \le R_i^{1} \cdot u_{it}^{disp} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (14)$$

$$r_{it}^{1-} \le R_i^{1} \cdot u_{it}^{disp} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (15)$$

$$r_{it}^{2+} \le 15 \cdot RU_i^{AGC} \cdot u_{it}^{AGC} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (16)$$

$$r_{it}^{2-} \le 15 \cdot RD_i^{AGC} \cdot u_{it}^{AGC} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (17)$$

$$r_{it}^{3S} \le R_i^{3S} \cdot u_{it}^{disp} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (18)$$

$$r_{it}^{3NS} \le R_i^{3NS} \cdot u_{it}^{3NS} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (19)$$

$$r_{it}^{3NS} \ge P_i^{\min} \cdot u_{it}^{3NS} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (20)$$

$$u_{it}^{3NS} \le 1 - u_{it} \qquad \forall i\in I,\ t\in \mathcal{T} \qquad (21)$$

$$\begin{aligned} p_{it} - r_{it}^{2-} \ge\ & 0 \cdot u_{it}^{syn} + p_{it}^{soak} + p_{it}^{des} + P_i^{\min} \cdot \left(u_{it}^{disp} - u_{it}^{AGC}\right) \\ & + P_i^{\min,AGC} \cdot u_{it}^{AGC} \qquad \forall i\in I,\ t\in \mathcal{T} \end{aligned} \qquad (22)$$

$$\begin{aligned} p_{it} + r_{it}^{2+} \le\ & 0 \cdot u_{it}^{syn} + p_{it}^{soak} + p_{it}^{des} + P_i^{\max} \cdot \left(u_{it}^{disp} - u_{it}^{AGC}\right) \\ & + P_i^{\max,AGC} \cdot u_{it}^{AGC} \qquad \forall i\in I,\ t\in \mathcal{T} \end{aligned} \qquad (23)$$

$$\begin{aligned} p_{it} - r_{it}^{1-} - r_{it}^{2-} \ge\ & 0 \cdot u_{it}^{syn} + p_{it}^{soak} + p_{it}^{des} + P_i^{\min} \cdot u_{it}^{disp} \\ & \forall i\in I,\ t\in \mathcal{T} \end{aligned} \qquad (24)$$

$$\begin{aligned} p_{it} + r_{it}^{1} + r_{it}^{2+} + r_{it}^{3S} \le\ & 0 \cdot u_{it}^{syn} + p_{it}^{soak} + p_{it}^{des} + P_i^{\max} \cdot u_{it}^{disp} \\ & + \left(P_i^{\min} - P_i^{\max}\right) \cdot z_{i\left(t+T_i^{des}\right)} \qquad \forall i\in I,\ t\in \mathcal{T} \end{aligned} \qquad (25)$$

Constraints (13) state that unit i may provide AGC if and only if it is on dispatch. Constraints (14)-(19) set the upper limits of primary-up/down, secondary-up/down and tertiary spinning/non-spinning reserves, respectively. As shown by constraints (13)-(18), unit i may contribute in synchronized reserves if and only if it is dispatchable, while during the synchronization, soak and desynchronization phases the synchronized reserves contribution is equal to zero. Constraints (20) enforce the tertiary non-spinning reserve contribution to be greater than the minimum power output of unit i. Constraints (21) state that unit i may provide tertiary non-spinning reserve if and only if it is off-line.

Constraints (22)-(25) define the limits of the power output of thermal unit i in every commitment state. The first three terms of the right hand side of (22)-(25) constrain the output of the unit during synchronization, soak and desynchronization. If unit i is on synchronization in hour t [i.e. $u_{it}^{syn} = 1$], the power output will be equal to 0, while the terms p_{it}^{soak} and p_{it}^{des} are defined in (5) and (7), respectively. The last term of the right-hand-side of (25) ensures that the unit will operate at its technical minimum power output the hour prior to entering the desynchronization phase; this term must be omitted for fast-start thermal units.

6) Ramp-up/down constraints

$$p_{it} - p_{i(t-1)} \le RU_i \cdot 60 + N \cdot \left(u_{it}^{syn} + u_{it}^{soak}\right) \qquad \forall i\in I, t\in \mathcal{T} \qquad (26)$$

$$p_{i(t-1)} - p_{it} \le RD_i \cdot 60 + N \cdot \left(z_{it} + u_{it}^{des}\right) \qquad \forall i\in I, t\in \mathcal{T} \qquad (27)$$

Constraints (26)-(27) introduce the effect of ramp rate limits on the power output. Note that N is a large constant, so that constraints (26) and (27) are relaxed when unit i is in synchronization, soak or desynchronization phase.

7) Total production cost

$$c_{it}\left(p_{it}\right) = SUC_i \cdot y_{it} + SDC_i \cdot z_{it} + NLC_i \cdot u_{it} + \sum_{f\in\mathcal{F}} c_{if} \cdot b_{ift}$$

$$\forall i\in I, t\in \mathcal{T} \qquad (28)$$

where,

$$\sum_{f\in\mathcal{F}} b_{ift} = p_{it} \qquad \forall i\in I, t\in \mathcal{T} \qquad (29)$$

$$0 \le b_{ift} \le B_{if} \qquad \forall i\in I, f\in\mathcal{F}, t\in \mathcal{T} \qquad (30)$$

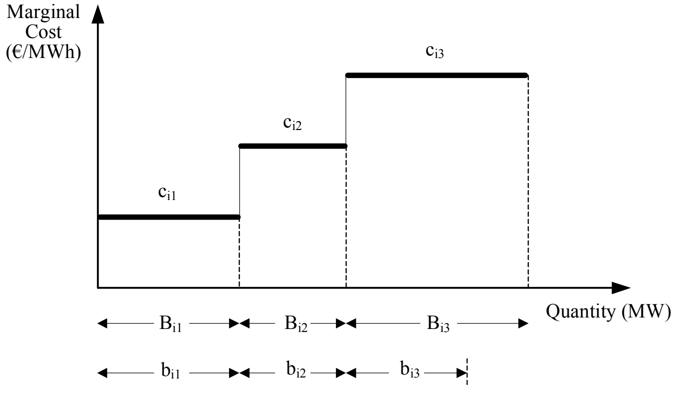

Fig. 2 Thermal generating unit step-wise marginal cost function

The total production cost of a thermal unit (28) includes the unit's start-up cost, SUC_i , shut-down cost SDC_i and the hourly operating cost defined by the unit's no-load cost, NLC_i and the step-wise marginal cost function, $\left(B_{if}, c_{if}\right)$, $f \in \mathcal{F}^i$, shown in Fig. 2.

In this paper we assume that the units' marginal cost function is non-decreasing (convex cost function).

C) Linear Expression of Hydro Units Constraints

This section refers to the constraints imposed to the operation of every hydro unit *j* over the market time horizon.

As referred in Section III-B, given that hydro units can be committed/de-committed in a few minutes, they do not follow the start-up and shut-down procedures described by constraints (3)-(7) [i.e. $u_{jt}^{syn} = u_{jt}^{soak} = u_{jt}^{des} = 0$] and so these constraints should be omitted in the case of hydro units. Additionally, the total operating cost (including start-up cost) of a hydro unit is negligible and so constraints (28)-(30) are not necessary in the case of hydro units.

1) Intertemporal Constraints

Constraints enforcing the expressions of minimum up/down time as well as the logical status of commitment and ramp rate limits are given by constraints (8)-(12) and (26)-(27) for thermal units and are also valid for hydro units.

2) Power output constraints

Constraints enforcing the reserves contribution given by constraints (13)-(21) for thermal units are also valid for hydro units, with the exception of constraints (14)-(15) that are omitted in the case of hydro units, as these units do not contribute in primary up/down reserves due to technical reasons.

As concerns the constraints enforcing the power output limitations, constraints (22)-(25) standing for thermal units are also valid for hydro units by omitting the first three terms of the right-hand side of these constraints as well as the last term of the right hand side of (25), which are referred to the synchronization, soak and desynchronization phases.

3) Total hydro energy constraints

$$\underline{H}_j \le \sum_{t \in \mathcal{T}} p_{jt} \le \bar{H}_j \qquad \forall j \in \mathcal{J} \tag{31}$$

Constraints (31) establish lower and upper bounds on the sum of the energy production of a hydro unit *j* during the planning period.

4) Prohibited operating zones constraints

$$\sum_{k \in \mathcal{K}} u_{jt}^k \cdot P_j^{\min,k} \le p_{jt} \le \sum_{k \in \mathcal{K}} u_{jt}^k \cdot P_j^{\max,k} \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{32}$$

$$\sum_{k \in \mathcal{K}} u_{jt}^k = u_{jt}^{disp} \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{33}$$

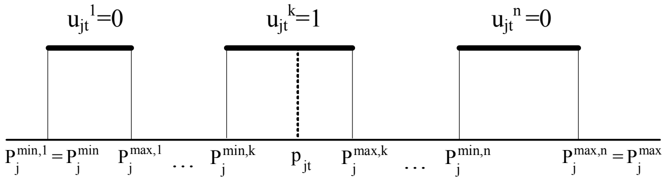

Fig. 3 Hydro unit permissible operating zones

A hydro generating unit may have prohibited operating zone(s) due to oscillations and physical limitations of unit components [11]. For an on-line hydro unit with *n-1* prohibited zone(s), the total operating region is divided into *n* discrete sub-regions, named permissible operating zones, and the hydro unit must operate in one of these sub-regions (see Fig. 3).

Constraints (32)-(33) ensure that hydro unit *j* power output will be within the bounds of only one permissible operating zone in every hour *t* that hydro unit *j* is in dispatchable phase.

D) System Constraints

$$\sum_{i \in I} p_{it} \le D_t \qquad \forall t \in \mathcal{T} \tag{34}$$

$$\sum_{i \in I} r_{it}^{1+} \le RR_t^{1+} \qquad \forall t \in \mathcal{T} \tag{35}$$

$$\sum_{i \in I} r_{it}^{1-} \le RR_t^{1-} \qquad \forall t \in \mathcal{T} \tag{36}$$

$$\sum_{i \in I} r_{it}^{2+} \le RR_t^{2+} \qquad \forall t \in \mathcal{T} \tag{37}$$

$$\sum_{i \in I} r_{it}^{2-} \le RR_t^{2-} \qquad \forall t \in \mathcal{T} \tag{38}$$

$$\sum_{i \in I} r_{it}^{3S} + \sum_{i \in I} r_{it}^{3NS} \le RR_t^3 \qquad \forall t \in \mathcal{T} \tag{39}$$

The System (Coupling) Constraints (34)-(39) reflect the fact that, no matter how high the energy and the reserves prices are, the energy and the reserves that the System Operator will purchase from the Producer will not exceed the corresponding system requirements. These coupling constraints may be relaxed if the size of the producer is small compared to the system requirements. In this case the Producer problem may be decomposed by unit.

IV. TEST RESULTS

The developed method has been tested for the scheduling of a hypothetical Producer with 5 thermal and 2 hydro units.

The main techno-economic data of the thermal and hydro units are presented in Tables I and II. Units 1-2 are base-load (lignite-fired) units; units 3-4 are intermediate-load (combined cycle gas turbine- CCGT) units and units 5-7 are peak-load (5: open cycle gas turbine- OCGT, 6-7: hydro) units. The units' technical and economic data were selected or estimated based on publicly available information (e. g. [12]-[14]).

In the time interval before the market horizon (t=0) units 1-4 were operating at their nominal power output and had been synchronized for 20 hours, while units 5-7 were decommitted and had been desynchronized for 10 hours.

TABLE I
THERMAL & HYDRO UNITS TECHNO-ECONOMIC DATA

Units	Type	$P^{\max}$ [MW]	$P^{\min}$ [MW]	$\bar{P}^{soak}$ [MW]	$P^{\max,AGC}$ [MW]	$P^{\min,AGC}$ [MW]	R^1 [MW]	R^{3NS} [MW]	RU [MW/min]	RD [MW/min]	UT [h]	DT [h]	T^{syn} [h]	T^{soak} [h]	T^{des} [h]	NL C [€/h]	SUC [€]	SDC [€]	Marginal Cost Range [€/MWh]	
																			min	max
# 1	A	274	160	90	-	-	13.7	0	2	2	8	4	2	2	3	1,894	46,600	10,000	32.0	33.5
# 2	A	342	180	100	-	-	17.1	0	2	2	8	4	2	2	3	1,644	58,165	10,000	33.0	35.0
# 3	B	378	200	110	302.4	240.0	18.9	360	24	24	4	3	1	1	2	3,367	16,012	5,000	52.0	54.5
# 4	B	476	250	135	380.8	300.0	23.8	360	24	24	4	3	1	1	2	3,839	19,766	5,000	55.0	57.5
# 5	C	152	63	0	121.6	75.6	7.6	120	8	8	1	1	0	0	0	965	2,568	500	81.0	83.5
# 6	D	100	0	0	90.0	10.0	0	100	100	100	1	1	0	0	0	0	0	0	0	0
# 7	D	150	0	0	135.0	15.0	0	150	100	100	1	1	0	0	0	0	0	0	0	0

TABLE II
HYDRO UNITS ADDITIONAL DATA

Units	$\overline{H}$ [MWh]	$\underline{H}$ [MWh]	Permissible Operating Zones			
			$P^{\min,1}$ [MW]	$P^{\max,1}$ [MW]	$P^{\min,2}$ [MW]	$P^{\max,2}$ [MW]
# 6	360	180	0	40	70	100
# 7	530	265	0	60	100	150

TABLE III
SYSTEM PRICE DATA

Hour	λ^{E} [€/MWh]	λ^{1+} [€/MW&h]	λ^{2+} [€/MW&h]	λ^{2-} [€/MW&h]	λ^{3NS} [€/MW&h]
1	64.04	0.7	0.1	0.1	0.0
2	49.40	0.7	0.1	0.1	0.0
3	35.33	0.7	0.1	0.1	0.0
4	32.73	0.7	0.1	0.1	0.0
5	30.47	0.7	0.1	0.1	0.0
6	30.39	0.7	0.1	0.1	0.0
7	30.62	0.7	2.0	2.0	0.0
8	49.94	0.7	2.0	2.0	0.0
9	64.00	0.7	2.0	0.1	0.0
10	64.85	0.7	2.0	0.1	0.0
11	92.40	0.5	2.0	0.1	0.5
12	92.41	0.5	2.0	0.1	0.5
13	92.42	0.5	2.0	0.1	0.5
14	92.41	0.5	0.1	0.1	0.5
15	92.41	0.5	0.1	0.1	0.5
16	89.00	24.3	0.1	0.1	0.5
17	89.00	24.3	0.1	0.1	0.5
18	83.05	0.5	2.0	0.1	0.5
19	89.00	24.3	2.0	0.1	0.5
20	83.14	0.5	2.0	0.1	0.5
21	83.61	0.5	2.0	0.1	0.5
22	83.05	0.5	0.1	0.1	0.5
23	64.85	0.5	0.1	0.1	0.0
24	64.79	0.5	0.1	0.1	0.0

The forecasted prices of energy and reserves (primary-up and secondary-up/down) are the ex-post market clearing prices obtained in the Greek deregulated market on January 31, 2009 [12] and are presented in Table III.

Price for tertiary non-spinning reserve is considered equal to €0.5/MW&h during peak-load hours (i.e. hours 11-22) and zero during off-peak hours.Prices for primary-down and tertiary spinning reserves are considered equal to zero, in accordance with the Greek Grid and Exchange Code [7].

All tests were performed on a 2.4 GHz Intel Core 2 Duo with 4 GB of RAM. The models were implemented in GAMS using the CPLEX solver [15].

Two sets of tests are performed. In the first set, the Producer participates in the energy market only. In the second set, the Producer participates in all energy and reserves markets, determining his degree of involvement in each market described in the introduction.

a) Day-ahead Energy Market

In this case the Producer participates in the day-ahead energy market only, in which he acts as a price-taker responding to a forecasted price curve. Table IV and Fig. 4 present a) the optimal producer generating schedule for the various unit types and b) the energy system marginal price (SMP) curve.

Lignite units 1-2 remain synchronized all day long and operate below their technical maximum from 4:00 p.m. to 7:00 p.m and 3:00 p.m. to 7:00 p.m, respectively, where the SMP is lower than their marginal cost. This is preferable to shutting-down and subsequently starting-up the lignite units owing to their high start-up cost and start-up time. Owing to the low SMP of the early morning hours, the combined cycle units (CCGTs) are desynchronized from 3:00 p.m. to 7:00 p.m, following a two hour desynchronization phase (hours 1-2). In hours 10-24, lignite and combined cycle units operate in their nominal power output as the SMP is much higher than their variable cost. Additionally, the operation of the open cycle unit is restricted in hours 11-17, where the SMP is higher than the unit's marginal cost and so it is profitable to operate this unit at its nominal power output. Finally, hydro units 6-7 operate during high SMP hours only (hours 12-15), so as to maximize profits.

Please note that each hydro unit's daily energy production identifies with the maximum available hydro energy production given in Table II.

TABLE IV
PRICE DATA AND OPTIMAL GENERATING SCHEDULE
ENERGY MARKET

Hour	Energy Price [€/MWh]	p_{1t} [MW]	p_{2t} [MW]	p_{3t} [MW]	p_{4t} [MW]	p_{5t} [MW]	p_{6t} [MW]	p_{7t} [MW]
0	65.46	274	342	378	476	0	0	0
1	64.04	274	342	200	250	0	0	0
2	49.40	274	342	100	125	0	0	0
3	35.33	274	310	0	0	0	0	0
4	32.73	180	190	0	0	0	0	0
5	30.47	160	180	0	0	0	0	0
6	30.39	160	180	0	0	0	0	0
7	30.62	160	222	0	0	0	0	0
8	49.94	274	342	0	0	0	0	0
9	64.00	274	342	110	135	0	0	0
10	64.85	274	342	378	476	0	0	0
11	92.40	274	342	378	476	152	0	0
12	92.41	274	342	378	476	152	90	130
13	92.42	274	342	378	476	152	100	150
14	92.41	274	342	378	476	152	100	150
15	92.41	274	342	378	476	152	70	100
16	89.00	274	342	378	476	152	0	0
17	89.00	274	342	378	476	152	0	0
18	83.05	274	342	378	476	0	0	0
19	89.00	274	342	378	476	0	0	0
20	83.14	274	342	378	476	0	0	0
21	83.61	274	342	378	476	0	0	0
22	83.05	274	342	378	476	0	0	0
23	64.85	274	342	378	476	0	0	0
24	64.79	274	342	378	476	0	0	0

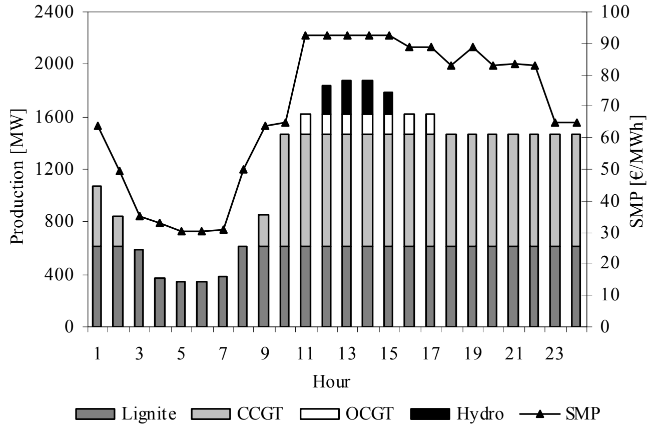

Fig. 4 Optimal generating schedule for the various unit types and system marginal price (Energy market only)

Also note that peak load units (OCGT and hydro) can be committed/de-committed in a few minutes and so they can reach their technical maximum/minimum in one hour, without following the corresponding start-up and shut-down procedures, as explained in Section III.

b) Day-ahead Energy and Reserves Markets

In this case the Producer participates as a price taker in all day-ahead markets (energy and reserves) responding to the respective forecasted price curves (see Table III).

TABLE V
PRICE DATA AND OPTIMAL GENERATING SCHEDULE
ENERGY AND RESERVES MARKETS

Hour	Energy Price [€/MWh]	p_{1t} [MW]	p_{2t} [MW]	p_{3t} [MW]	p_{4t} [MW]	p_{5t} [MW]	p_{6t} [MW]	p_{7t} [MW]
0	65.46	274	342	378	476	0	0	0
1	64.04	274	342	200	250	0	0	0
2	49.40	274	342	100	125	0	0	0
3	35.33	274	310	0	0	0	0	0
4	32.73	180	190	0	0	0	0	0
5	30.47	160	180	0	0	0	0	0
6	30.39	160	180	0	0	0	0	0
7	30.62	160	222	0	0	0	0	0
8	49.94	274	342	0	0	0	0	0
9	64.00	274	342	110	135	0	0	0
10	64.85	274	342	378	476	0	0	0
11	92.40	274	342	378	476	152	10	15
12	92.41	274	342	378	476	152	10	15
13	92.42	274	342	378	476	152	100	150
14	92.41	274	342	378	476	152	100	150
15	92.41	274	342	378	476	152	100	140
16	89.00	274	342	378	476	144.4	0	0
17	89.00	274	342	378	476	144.4	0	0
18	83.05	274	342	378	476	0	10	15
19	89.00	274	342	378	476	0	10	15
20	83.14	274	342	378	476	0	10	15
21	83.61	274	342	378	476	0	10	15
22	83.05	274	342	378	476	0	0	0
23	64.85	274	342	378	476	0	0	0
24	64.79	274	342	378	476	0	0	0

TABLE VI
PRIMARY-UP & SECONDARY-UP RESERVE CONTRIBUTION
ENERGY AND RESERVES MARKETS

Hour	RR^{1+} [MW]	r^{1+}_{1t} [MW]	r^{1+}_{2t} [MW]	r^{1+}_{5t} [MW]	RR^{2+} [MW]	r^{2+}_{6t} [MW]	r^{2+}_{7t} [MW]
1	80	0	0	0	100	0	0
2	80	0	0	0	100	0	0
3	80	0	17.1	0	100	0	0
4	80	13.7	17.1	0	100	0	0
5	80	13.7	17.1	0	100	0	0
6	80	13.7	17.1	0	100	0	0
7	80	13.7	17.1	0	250	0	0
8	80	0	0	0	250	0	0
9	80	0	0	0	250	0	0
10	80	0	0	0	250	0	0
11	80	0	0	0	250	80	120
12	80	0	0	0	250	80	120
13	80	0	0	0	250	0	0
14	80	0	0	0	170	0	0
15	80	0	0	0	170	0	0
16	80	0	0	7.6	170	0	0
17	80	0	0	7.6	170	0	0
18	80	0	0	0	250	80	120
19	80	0	0	0	250	80	120
20	80	0	0	0	250	80	120
21	80	0	0	0	250	80	120
22	80	0	0	0	170	0	0
23	80	0	0	0	170	0	0
24	80	0	0	0	170	0	0

As shown in Tables V and VI, as lignite units 1-2 operate below their maximum output during the early morning hours (hours 4-7 and 3-7, respectively) they can contribute in primary-up reserve to their maximum capability (R_i^{1+}) so as to increase profits. In addition, in hours 16-17, the open cycle unit's opportunity cost (the system marginal price minus the unit's marginal cost) is lower than the high primary-up reserve price (see Table III) and so it is more profitable for this unit to reduce slightly its output, so as to contribute in primary-up reserve in these hours. Please note that hydro units do not contribute in primary-up/down reserves and lignite units do not contribute in AGC.

Contrary to the generating schedule of the previous case, the operation of the hydro units is not restricted in the high SMP hours only. As shown in Tables V and VI, hydro units operate in their minimum power output under AGC ($P_i^{\min,AGC}$) during hours 11-12 and 18-21, so as to have the capability to provide a higher amount of secondary-up reserve in these hours and increase profits.

Table VII presents the total daily profits of the Producer in each of the two cases studied. In the second case the profit is higher than in the first case due to the extra revenues from the reserves markets. The difference between profits depends on the clearing prices obtained in the reserves markets, which, in our case study, are considerably low.

Table VIII gives the size of the resulting MILP problem. In both cases our model results in proven optimal solution and the computing time required for the two cases studied is negligible (below 0.3 sec).

TABLE VII
PRODUCER PROFITS

Markets	Total Profit [€]
Day-ahead Energy Market	721,606
Day-ahead Energy and Reserves Markets	724,279

TABLE VIII
SIZE OF THE MILP MODEL

Number of Equations	Number of Variables	Binary Variables
6,283	7,788	1,604

V. CONCLUSIONS

This paper presented a MILP solution to the hydrothermal electricity producer self-scheduling problem in day-ahead energy and reserves markets. The producer self-scheduling problem is formulated and solved as a 0/1 mixed integer linear program (MILP). It provides a tool to enable a price-taker producer to decide his optimal degree of involvement in each market so as to maximize his profits, based on forecasts of the day-ahead energy and reserves market clearing prices. It can be also used as a first step in developing optimal bidding strategies. The model presented may also be applied in the MILP formulation of the traditional unit commitment problem solved by the ISO in real-size power systems with numerous thermal and hydro units, as the Greek one.

VI. REFERENCES

[1] J. M. Arroyo, A. J. Conejo, "Optimal Response of a Thermal Unit to an Electricity Spot Market", *IEEE Trans. Power Syst.*, vol. 15, no. 3, pp. 1098-1104, August 2000.

[2] J. M. Arroyo, A. J. Conejo, "Modeling of start-up and shut-down power trajectories of thermal units", *IEEE Trans. Power Syst.*, vol. 19, no. 3, pp. 1562-1568, August 2004.

[3] J. M. Arroyo, A. J. Conejo, "Optimal Response of a Power Generator to Energy, AGC, and Reserve Pool-Based Markets", *IEEE Trans. Power Syst.*, vol. 17, no. 2, pp. 404-410, May 2002.

[4] J. M. Arroyo, A. J. Conejo, "Self-scheduling of a hydro producer in a pool-based electricity market", *IEEE Transactions on Power Systems* vol. 17, no. 4, pp. 1265-1272, November 2002.

[5] T. Li, M. Shahidehpour, "Price-based unit commitment: A case of lagrangian relaxation versus mixed-integer programming", *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 2015-2025, November 2005.

[6] S. Bisanovic, M. Hajro, M. Dlakic, "Hydrothermal self-scheduling in a day-ahead electricity market", *Electric Power Systems Research,* 78 (2008) 1579-1596.

[7] The Regulatory Authority of Greece, "The Greek Grid and Exchange Code", online at http://www.rae.gr/en/codes/main.html.

[8] A. G. Bakirtzis, N. P. Ziogos, A. C. Tellidou, G. A. Bakirtzis, "Electricity Producer Offering Strategies in Day-Ahead Energy Market with Step-Wise Offers", *IEEE Transactions on Power Systems* vol. 22, no. 4, pp. 1804-1818, November 2007.

[9] P. N. Biskas, C. E. Zoumas, M. Philippou, "Co-optimization of commodities in the Greek energy market", presented at the 6th Mediterranean Conference and Exhibition on Power Generation, Transmission and Distribution, MedPower 2008, Thessaloniki, Greece, 2008.

[10] D. Rajan and S. Takriti, "Minimum up/down polytopes of the unit commitment problem with start-up costs", IBM Research report, RC 23628, June 2005.

[11] F. N. Lee, A.M. Breipohl, "Reserve constrained economic dispatch with prohibited operating zones", *IEEE Trans. Power Syst.*, vol. 8, no. 1, pp. 246-254, Feb. 1993

[12] The Hellenic Transmission System Operator, online at http://www.desmie.gr.

[13] H. Koetzier, W. van der Veen, T. J. Mclnally, "Setting of technical parameters for LRMC of CCGT", Final report Revision 1, KEMA Consulting, Arnhem, December 2006. Available on-line at http://www.ema.gov.sg/doc/22120603413630KEMAfinalreport.pdf

[14] All Island Modeling generator data, 2005, AIP-SEM-124-05.xls, Available on-line at: http://www.allislandproject.org/en/modelling%20group-info-documents.aspx

[15] A. Brooke, D. Kendrick, A. Meeraus, "GAMS User's Guide", Redwood City, CA: The Scientific Press, 1990. Available on-line at http://www.gams.com/docs/gams/GAMSUsersGuide.pdf

VII. BIOGRAPHIES

Christos K. Simoglou received the Dipl. Eng. Degree from the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Greece, in 2005. He is currently a Ph.D. candidate with the same university. His research interests are in power system operation and energy bidding strategies. He is a member of the Technical Chamber of Greece and IEEE.

Pandelis N. Biskas was born in Serres, Greece, in March 1977. He received the Dipl. Eng. and the Ph.D. Degrees from the Department of Electrical and Computer Engineering, Aristotle University of Thessaloniki, Greece, in 1999 and 2003, respectively. From March 2004 till August 2005 he was Post Doctorate Fellow with the Aristotle University. Since March 2005 he is with the Hellenic Transmission System Operator as a Power Systems Specialist in the Market Operation Branch. His research interests are in power system operation & control, in electricity market operational and regulatory issues, and in transmission pricing. He is a member of the Technical Chamber of Greece and of IEEE.

Anastasios G. Bakirtzis (S'77-M'79-SM'95) received the Dipl. Eng. Degree from the Department of Electrical Engineering, National Technical University, Athens, Greece, in 1979 and the M.S.E.E. and Ph.D. degrees from Georgia Institute of Technology, Atlanta, in 1981 and 1984, respectively. Since 1986 he has been with the Electrical Engineering Department, Aristotle University of Thessaloniki, Greece, where he is currently Professor and director of the Power Systems Laboratory. His research interests are in power system operation, planning and economics.