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Chaotic Behavior Observations in a Power System Model

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***Abstract*—Chaotic behavior in power systems has been studied in relatively simple and theoretical system models, where some particular assumptions are made to represent the system as a set of ordinary differential equations (ODE), using "special" nonlinear system analysis tools. In this paper, chaotic behavior on the IEEE 14-bus benchmark system, using a transient stability model and its associated differential-algebraic equations (DAE), is demonstrated and studied based on classical time-domain simulations, without the use of specialized software or simplifying assumptions. The dynamic behavior of the test system is studied for normal operating conditions and for a single contingency case, and the onset of chaos is verified through a Fourier analysis and Lyapunov exponents. The addition of a power system stabilizer (PSS) to the system is shown to remove the observed chaotic behavior.**

***Index Terms*—Chaos, crisis, Hopf bifurcations, period doubling, voltage collapse, power system stabilizer.**

I. Introduction

CHAOTIC phenomena in relatively simple power system models has been observed and studied for the past two decades. For example, in [1]-[3], chaotic behavior of a simple 3-bus power system is studied in detail, where the single system load is represented with a dynamic model to characterize the system using ordinary differential equations (ODE). In [4], on the other hand, a larger 9-bus test system is studied using an ODE model (with dynamic loads), illustrating the interactions of chaotic motions and system dynamic components; transmission system controllers are then used to prevent or eliminate chaotic oscillations. In these papers, basically two routes that lead to chaos are observed, i.e. cascading period doubling bifurcations (PDB) and torus bifurcations (TB).

The current paper presents and discusses the chaotic behavior observed on the IEEE 14-bus benchmark system [5], which is modeled based on "standard" differential algebraic equations (DAE) utilized in transient stability studies. With the help of time domain simulations using a standard power system analysis program (PSAT [6]), the associated transient-stability model is shown to have a period-doubling path to chaos. Furthermore, the chaotic attractor is shown to disappear through a blue-sky bifurcation phenomenon.

The main objectives of the current paper are: Present and discuss with enough detail chaotic behavior observed in a DAE power system model used in stability studies of power systems; analyze the observed chaotic and dynamic behavior using standard power system analysis tools, without using specialized tools that have been designed for ODE system models; and discuss in some detail interesting and not previously reported dynamic and chaotic behavior of a IEEE test system widely used by researchers studying stability issues in power systems.

It should be mentioned that this paper is of a theoretical nature, like all previous publications discussing chaos in power system models (e.g. [1]-[4]), since in real systems, as soon as sustained oscillations of any kind appear, load, generator and/or line protections act to try to eliminate these undesirable system conditions. Hence, some of the bifurcation and chaotic phenomena presented and discussed here cannot be observed in reality.

II. Bifurcations and Chaos

Period-doubling cascading is one of the most well-known routes to chaos, and consists of an infinite sequence of PDBs leading to chaos. In this case, a gradual doubling, at specific parameter values or PDB points, of the period of the oscillations triggered by an initial Hopf bifurcation (HB) eventually lead to the oscillation period becoming effectively infinity, thus resulting in a chaotic attractor [7].

Another important phenomenon in chaos, relevant to the current paper, is chaotic crises, where the nature of the chaotic dynamics changes abruptly as system parameters vary quasi-statically due to a collision between a chaotic attractor and coexisting unstable fixed points or periodic orbits [3]. A crisis involving the sudden destruction of a strange attractor through a collision with a saddle point, an unstable periodic orbit, or its associated stable manifold is known as a boundary crisis. In this context, blue sky bifurcations (BSBs) of periodic orbits are of particular interest to this paper; these bifurcations are characterized by the sudden disappearance of a limit cycle through a collision with a saddle equilibrium point.

The study of chaotic behavior in nonlinear systems is typically carried out using a combination of analyses techniques [7]. For a system with a single quasi-static varying parameter (co-dimension one bifurcation problems), these studies typically start by analyzing the changes in system equilibria as the parameter changes. From these analyses,

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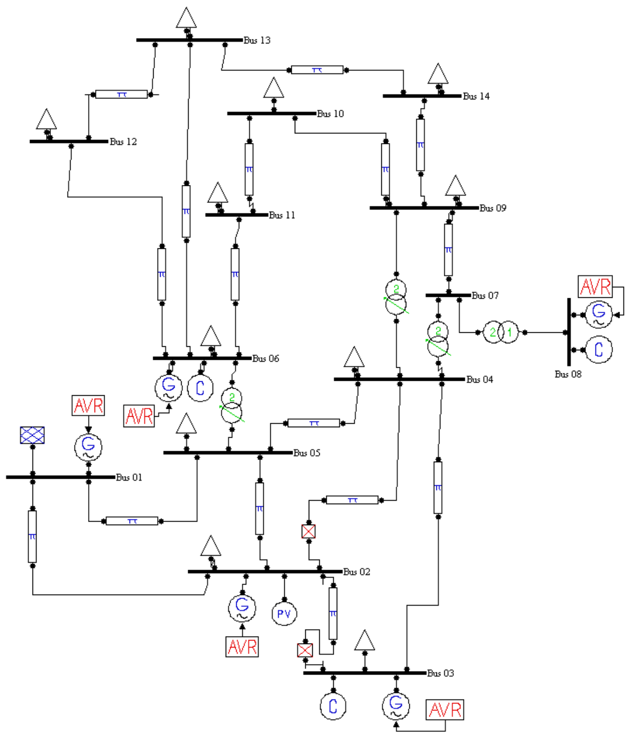

Fig. 1. EEE 14-bus benchmark system

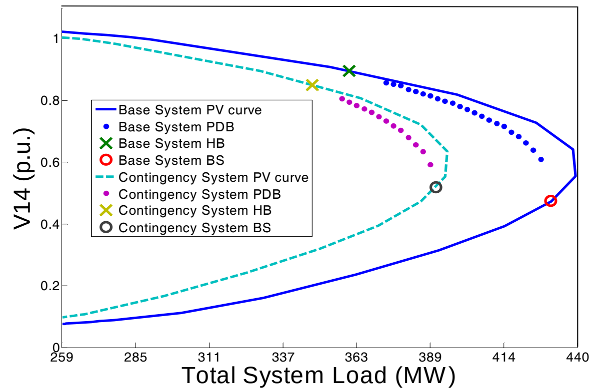

Fig. 2. PV curves (bifurcation diagrams) at Bus 14 for the 14-bus IEEE benchmark system.

diagrams of relevant system variables with respect to the varying parameter can be obtained; these diagrams are typically referred to as bifurcation diagrams. These plots in power systems are known as PV or “nose” curves, illustrating the presence of saddle-node and limit-induced bifurcations (also referred to maximum loading points) in typical DAE models, and are obtained with the help of continuation power flow (CPF) techniques [8].

Eigenvalue studies are typically used to determine whether the equilibrium points in the bifurcation diagrams are stable or unstable. Thus, this allows for the determination of the onset of HBs, which are associated with a conjugate pair of eigenvalues crossing the imaginary axis in the complex plane as the single parameter changes. These HB points have been associated in power systems with the onset of undesirable oscillations (e.g. interarea oscillations) [9].

The bifurcation of periodic orbits to determine, for example, PDB points, can be studied using a variety of branch tracing methods as discussed in [7]. Since these techniques have been developed and implemented mostly for relatively small ODE systems, their use in large DAE power system models is an issue [10]. However, time-domain simulations with respect to a varying parameter can be used to approximately study these types of bifurcations, as demonstrated in this paper, since it is relatively easy to approximately determine the points at which, for example, the frequency of an oscillation doubles, if the variations in the parameter are closely monitored and controlled.

The onset of a chaos through the appearance of a strange attractor in the time-domain trajectories can be verified though the use of Fourier analysis and/or Lyapunov exponents [7]. Thus, Fast Fourier Transforms (FFT) applied to these time trajectories (time series) allow to determine the frequency spectrum of these signals, with a wide spectrum indicating the onset of chaos. Furthermore, Lyapunov exponents, which evaluate the sensitivity of the system to initial conditions by estimating the exponential divergence of nearby orbits when at least one of these exponents becomes positive, can also be approximately computed from the time series as explained in [7] and [11].

III. Results

All results presented in this paper are obtained for the IEEE 14-bus benchmark system, which is described in detail in [5]. The system consists of five synchronous machines, three of which are synchronous compensators used only for reactive power support, represented with subtransient models and with IEEE type-1 exciters. There are 11 loads in the system, totaling 259 MW and 81.3 Mvar, represented as constant power loads (PQ loads). Each machine is equipped with an Automatic Voltage Regulator (AVR). The single-line diagram of this system is illustrated in Fig. 1.

Following classical stability/security analysis of power systems, two types of system operating conditions of the test system are studied here: normal/base and contingency operation, where a “critical” transmission line is removed (Line 2-4). The usual PV curves (bifurcation diagram) depicted in Fig. 2 for both system conditions were obtained by uniformly increasing the system load at all buses, assuming constant power factors; thus, the single bifurcation parameter used here is directly correlated to the total system load. Observe that the voltage magnitudes at this and other “remote” load buses fall below the standard 0.95 p.u. value passed certain loadings levels; this can be resolved by adding compensation at these buses, which would change the corresponding PV curves and the loading levels at which the different bifurcations shown appear. However, and given the theoretical nature of the discussions presented here, static compensation is not consider here, since in principle this will not change the nature of the bifurcation and chaotic phenomena presented here, affecting only the voltage and loading values at which the bifurcation appear.

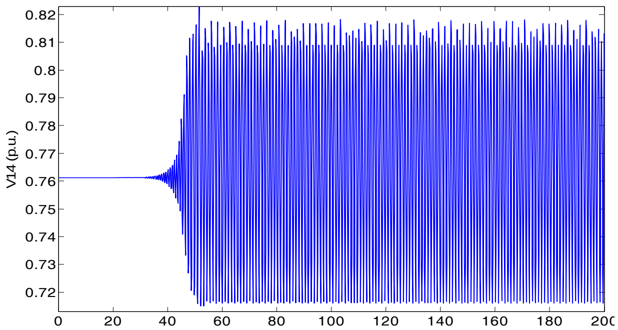

Fig 3. PDB for the base system at 414.40 MW loading.

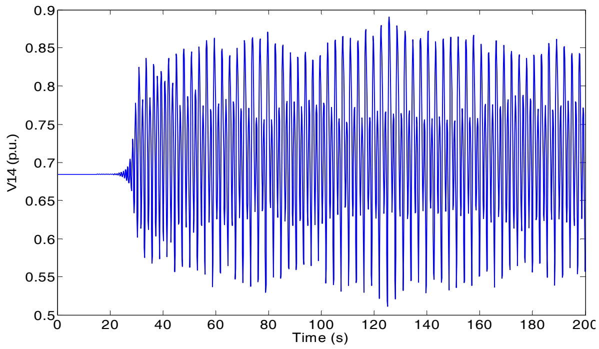

Fig. 4. Chaotic behavior for the base system at 430.717 MW loading.

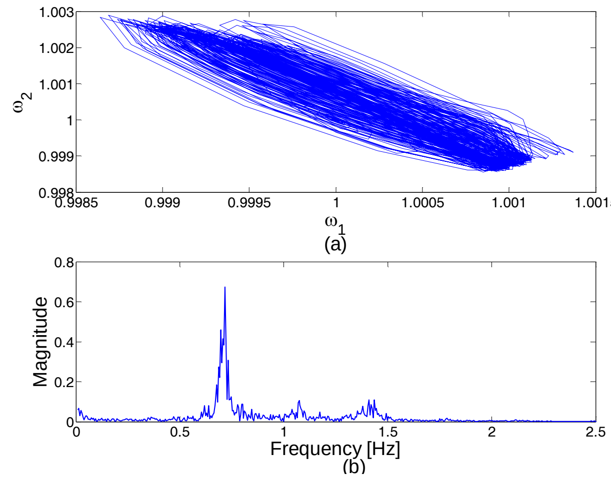

Fig. 5. Chaos results: (a) chaotic attractor projection for the base system at 430 MW loading (rotor speeds ω1 and ω2 of the main generators at Buses 1 and 2), and (b) Fast Fourier Transform (FFT) analysis of ω2.

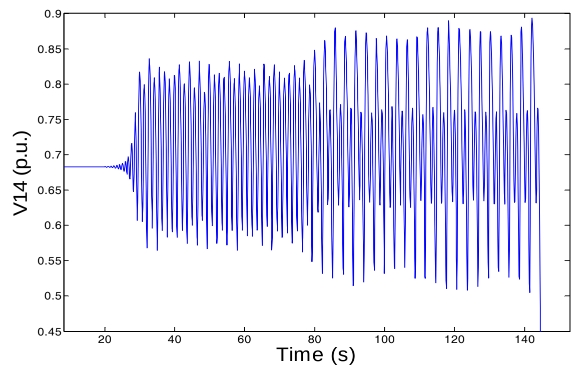

Fig. 6. Boundary crisis for the base system at 430.98 MW loading.

As illustrated in Fig. 2, an initial HB is observed for both system conditions at 360 MW and 347 MW of total loading, respectively; as a result of these HBs, corresponding stable limit cycles of approximately 1 Hz can be observed. A series of PDBs are then detected as the system load is increased, with the first one appearing at 373 MW of loading for the “base” system, and 357 MW for the “contingency” system. Figure 3 depicts the time trajectory of the voltage at the “remote” load Bus 14 (V14) for the base system at one of these PDBs for the base system; although this is an algebraic variable, since the system time trajectories are “invertible” (the algebraic equations’ Jacobian is invertible along these trajectories), the algebraic variables are directly associated with the state variables [13].

The PDBs eventually lead to the appearance of chaos for both system conditions, as shown in the time trajectory depicted in Fig. 4. This behavior can also be illustrated through the strange attractor shown in Fig. 5(a), which is very similar to the attractors shown in [1]-[4]. The corresponding Fast Fourier Transform (FFT) is shown in Fig. 5(b), which proves the presence of chaos, since strange attractors are characterized by a relatively wide frequency spectrum as the one depicted here, which would be considered wide in the context of transient stability studies.

From the time series shown in Fig. 4, the largest Lyapunov exponents were also calculated to confirm the onset of chaos using the techniques described briefly in Section II. Thus, the largest Lyapunov exponent is a negative value (-5.0061e-004) at 414.40 MW loading for the base system, while this exponent becomes positive (0.0149) at 430.717 MW loading, thus confirming the presence of chaotic behavior for the latter loading conditions. For the contingency case, the largest Lyapunov exponent is -0.0112 at 362.6 MW loading, and 0.0038 at 372.96 MW loading.

A boundary crisis through a BSB is observed at approximately 430.98 MW loading for the base system, and about 390.57 MW loading for the contingency system. Figure 6, which is similar to comparable time-domain profiles depicted for the 3-bus ODE system model in [3], illustrates the boundary crisis effect on the voltage at Bus 14 for the base system.

To study the effect of opening Line 2-4 on the benchmark system, the line was tripped at 2 s of operation of the base system. In this case, the system presented a different time-domain behavior that the one expected from the aforementioned studies for the contingency case, i.e. different equilibrium points and limit cycles (oscillations) were obtained in this case. Thus, the new PV curve (bifurcation diagram) depicted in Fig. 7 was obtained through a series of time domain simulations. Observe the differences in the voltage and loading values shown in this figure for the line trip event with respect to those shown in Fig. 2 for the contingency case; for example, the BSB for the contingency system occurs at about 390.57 MW, whereas for the line-trip system it is observed at approximately 415.73 MW, and the maximum “power-flow” loading point, which corresponds to a saddle-node bifurcation, is observed at approximately 394 MW loading for the

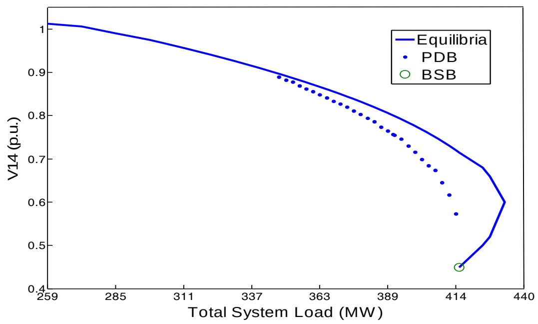

Fig. 7. PV curve or bifurcation diagram for the line trip system.

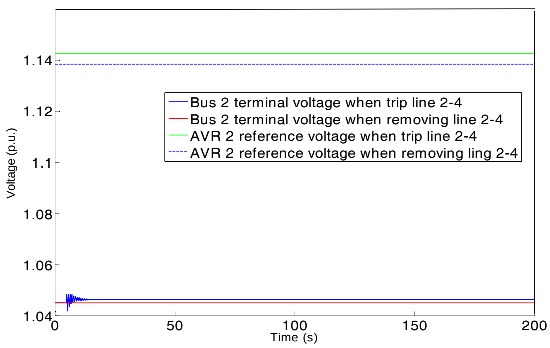

Fig. 8. Line trip system Boundary crisis at 415.73MW loading

contingency system, whereas this occurs at about 433 MW loading for the line-trip case.

The difference between Figs. 2 and 7 is due to the fact that in power systems, unlike other nonlinear systems, power flow models, which are simplified models of the dynamic system to represent typical operating strategies, are used to determine equilibrium points of the associated dynamic models. Therefore, when there are changes in the system, such as the line trip (contingency) discussed here, the corresponding power flow solution or operating point and the actual equilibrium point of the dynamic system model differ during time domain simulations, due mainly to the generators' voltage regulators (AVRs) droops, thus making the reference voltage (associated with the equilibrium point of the dynamic model) different from the terminal voltage (obtained from the power flow solution). This is clearly illustrated in Fig. 8, which depict the terminal voltages at Bus 2 (Generator 2) and the corresponding AVR reference voltages for both the system with Line 2-4 removed and the system with this line tripped during the simulation; these differences between terminal and reference voltages in the system generators are the main reason for the differences between Figs. 2 and 7.

A PSS was added to Generator 1 to remove the HBs in all cases, as discussed in [12], resulting in the removal of all chaotic phenomena. It is interesting to note that the equilibria depicted in Fig. 7 were obtained via time-domain simulations of a line trip on the base system with a PSS.

It is well known in stability studies that load models significantly affect the dynamic behavior of power systems [15]. Thus, the presented oscillations, bifurcation and chaotic phenomena will be affected by the load models used. For example, for a simple impedance load model, the presented phenomena do not appear. On the other hand, for the recovery dynamic load model proposed in [16], similar oscillations and transient behavior were observed for various values of the exponents used to represent the voltage dependency. Observe that although loads are not typically modeled as sole constant power in practice, but rather as part of, for example, a ZIP load model, these models are typically used in stability studies to model stressed system conditions and form the basis of certain stability analysis techniques (e.g. [17], [18]).

IV. Conclusions

The appearance of a chaotic attractor on a standard DAE model of the 14-bus IEEE benchmark system was discussed in detail in this paper. Using standard power system analysis tools and time-domain simulations, a series of PDBs was shown to lead to chaos in this case, and the sudden disappearance of the chaotic attractor through a BSB phenomenon was demonstrated. The onset of chaos was verified through FFT analyses of the time series obtained from the time domain simulations, as well as the computation of the largest Lyapunov exponents from these time trajectories. A PSS controller was used to remove the HBs and thus eliminate chaos.

It is important to highlight the fact that all results were obtained without the need of specialized nonlinear system analysis tools and without making any particular modeling assumptions or simplifications. This is not the case in all previous publications reporting on chaotic observations in simplified ODE power system models, which have made use of standard analysis tools for the study of bifurcations and chaos in nonlinear dynamic systems. Finally, even though chaos studies are of a theoretical nature in power systems, since in practice system protections do not allow the presence of undesirable sustained oscillations, bifurcation and chaotic studies do yield insights into the dynamic behavior and characteristics of power systems, thus allowing improving their design, control and operation.

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