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# Fast Decoupled Steady-State Solution for Power Networks Modeled at the Bus Section Level

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***Abstract*— Conventional tools to provide steady-state power network solutions rely on bus-branch models, in which a "bus" is actually the result of merging internal electrical nodes pertaining to given substation. The corresponding network solutions are thus unable to readily provide information about variables internal to the substations, such as power flows through circuit breakers and bus-section nodal voltages. Since the knowledge of such variables is important to several applications, such as real-time topology estimation and corrective switching methods, the need then arises to re-examine the power flow formulation in order to obtain detailed solutions at the substation level. This paper addresses that problem by extending the decoupled power flow formulation in order to allow the representation of selected parts of the network at the substation level. For that purpose, the state vector is expanded so as to include power flows through switching branches as new state variables, in addition to the conventional nodal voltages. Moreover, information regarding the status of switching branches is taken into account as additional linear equations to be solved along with the traditional power flow equations. Simulating results considering several substation layouts and two IEEE test systems are used to illustrate and evaluate the proposed approach.**

***Index Terms*— Power Network Modeling, Steady-State Power Network Solution, Fast Decoupled Power Flow.**

## I. Introduction

Structural changes in the power industry and the continued evolution of electrical networks have brought about additional complexities to power system analysis, stressing the need for the development of automated modeling procedures. Determining the power flow distribution over switching branches within a substation constitutes a typical case. The traditional way of determining such a flow distribution over substation devices relies on the conventional bus-branch model for the electrical network, according to which substation nodes are merged to form a single bus under the assumption that substation arrangements are perfectly known. Since by doing so switches and circuit breakers do not explicitly appear in the model, that procedure avoids some pitfalls, such as numerical problems caused by the often employed artifice of representing closed/open status of switching devices by atypically small/high impedances, respectively. On the other hand, all operational information related to substation components, such as power flows through circuit breakers, bus section voltages, etc., are not readily available. As the knowledge on such variables is often of great interest, the operator has to resort to additional procedures to determine the corresponding values. In addition, such procedures have to be repeated whenever a change of substation arrangement occurs.

Real-time modeling is an example of a subarea where one clearly identifies a current trend towards the representation of parts of the power network at the bus-section level. The trend is driven by the emergence of topology estimators [1] and the development of new topology error identification algorithms relying on the explicit representation of switching devices [2], [3], [4], [5], [6], [7], [8], [9]. In both cases, the search of the correct network topology involves analyzing distinct substation arrangements for the same problem, something which is not easily carried out through the use of the conventional bus-branch model.

Representing substations at the physical level is also relevant in other areas, such as corrective switching studies. In such case, one looks for proper switching strategies to alleviate or eliminate system overloads [10], [11], [12]. Since the search may involve bus-splitting, the need then arises to explicitly represent circuit breakers. In the past, artifices have been employed to accommodate that need within the limitations of the bus-branch model. Thus, a circuit breaker would be represented through a fictitious branch of very low (breaker closed) or very high (breaker open) reactance, or through jumper lines of equal reactance value but opposite signs, both connected to a fictitious node [10], [11], [12]. However, those artifices tend to cause numerical problems in the course of power flow solutions.

The approach adopted in this paper builds on previous research efforts towards the representation of switching branches in power system state estimation [2], [3], which have paved the way for the emergence of the Generalized State Estimation concept [4]. This framework allows the explicit representation of switching devices in state estimation studies and underlies a number of recently proposed methodologies for processing topology errors in substation arrangements [5]-[8]. More recently, preliminary efforts towards extending the switching branch representation presented in [2], [3] to power flow studies via Newton's method have been reported [13].

The main objective of this paper is to demonstrate that the decoupled power flow method can be equally extended to accommodate detailed bus-section models for the network substations. This applies to both the original decoupled formulation and any of its Fast Decoupled Power Flow (FDPF) versions. As in [13], modeling selected parts of the network down to the substation level implies augmenting the state vector in order to include power flows through switching branches as new state variables, in addition to the conventional nodal voltages. Information regarding the status of switching

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branches is included into the resulting Extended Fast Decoupled Power Flow (XFDPF) problem as additional linear relationships. This paper provides details on how the required changes should be embedded in the conventional decoupled power flow formulation in order to generate XFDPF.

In addition to determining the power flow distribution through substation components, the proposed method is also able to quickly update results whenever changes are introduced on the status configuration of a given substation. Thus, the impact of alternative switching strategies can be readily evaluated. The IEEE 24 and 30 bus test systems are used to illustrate and validate the results of the proposed approach.

This paper is organized as follows.....

## II. Power Flow at Substation Level

As discussed above, the traditional way of determining the system power flow distribution relies on the conventional model of the electrical network, known as bus-branch model, where the substation arrangements is previously determined and the substation nodes are merged to form a single bus. This procedure avoids the explicit representation of switches and circuit breakers and the consequent numeric problems caused by adopting small and high impedances to represent closed and open status of such devices. Besides, the basic network matrices employed for steady-state analysis, such as the bus admittance matrix, bus-branch incidence matrices, etc., can be readily built from the model. On the other hand, lack of information on substation physical arrangements prevents the prompt determination of the power flow distribution through circuit breakers and other substation devices. Whenever such a flow distribution is required, as it is the case in studies connected to on-line topology estimation [5]-[8] and corrective switching studies [10]-[12], ad-hoc post-processing procedures have to be conducted.

In this section, the conventional power flow analysis based on the network bus-branch model is extended to include the explicit modeling of circuit breakers and switches, thus enabling it to deal with networks modeled at bus section level. This extension is motivated by the trend towards a more detailed representation of the power network in state estimation studies. As in Generalized State Estimation [4], the above mentioned numerical problems are avoided by modeling switching branch power flows as new state variables. Information regarding the devices' status is embedded in the power flow problem as new (and linear) relationships, producing a solvable non-redundant set of algebraic equations [13], as described in the sequel.

### A. Physical Level Representation

Before addressing the procedures to embed the representation of selected substations in the network model used in power flow studies, the conventional problem formulation must be reviewed to accommodate the required changes. First of all, the state vector is extended in order to include the power flows through switching branches as new state variables, in addition to the conventional nodal voltages states. The extended state vector is thus given by:

$$\bar{\mathbf{x}} = \left[ \begin{array}{cccc} \boldsymbol{\theta}^T & \mathbf{V}^T & \mathbf{t}^T & \mathbf{u}^T \end{array} \right]^T \tag{1}$$

where $\boldsymbol{\theta}$ and $\mathbf{V}$ are the vectors of bus voltage phase angles and magnitudes, respectively, whereas $\mathbf{t}$ and $\mathbf{u}$ are the active and reactive power flow vectors for the modeled switching branches, respectively.

Simultaneously, the information regarding the status of the explicitly represented switching devices is included into the power flow formulation as new sets of equations to be solved along with the network equations. Thus, if a switching branch connecting nodes $k$ and $m$ is closed, then the voltage angle difference $f^{cl}_{\theta,km}$ and the voltage drop $f^{cl}_{v,km}$ across this device are zero, that is

$$f^{cl}_{\theta,km} = \theta_k - \theta_m = 0 \quad \text{and} \quad f^{cl}_{v,km} = v_k - v_m = 0 \tag{2}$$

The above set of equations is referred to as *closed breaker operational conditions.* Assuming that $N_{cl}$ switching branches are closed, those conditions are expressed in compact form by the $2N_{cl}$-dimensional vector

$$\mathbf{f}^{cl}_o(\boldsymbol{\theta}, \mathbf{V}) = \left[ \begin{array}{c} \mathbf{f}^{cl}_\theta(\boldsymbol{\theta}) \\ \mathbf{f}^{cl}_v(\mathbf{V}) \end{array} \right] = \mathbf{0} \tag{3}$$

where $\mathbf{f}^{cl}_\theta(\boldsymbol{\theta})$ and $\mathbf{f}^{cl}_v(\mathbf{V})$ are the vector forms of the expressions defining the closed breaker conditions in Eq. (2).

On the other hand, if the device is open then the active and reactive power flow through it are zero, that is

$$t_{km} = 0 \quad \text{and} \quad u_{km} = 0 \tag{4}$$

If $N_{op}$ is the number of open circuit breakers, the *open breaker operational conditions* are expressed in compact form by the $2N_{op}$-dimensional vector

$$\mathbf{f}^{op}_o(\mathbf{t}, \mathbf{u}) = \left[ \begin{array}{c} \mathbf{t}^{op}(\mathbf{t}) \\ \mathbf{u}^{op}(\mathbf{u}) \end{array} \right] = \mathbf{0} \tag{5}$$

where $\mathbf{t}^{op}(\mathbf{t})$ and $\mathbf{u}^{op}(\mathbf{u})$ are $N_{op}$-vectors corresponding to the active and reactive open breaker operational conditions, respectively.

The above discussed changes on the set of state variables and the new set of equations describing the network at the bus-section level imply that the power injection equations must also be modified to comply with those changes. The injections at a bus $k$ can be expressed as the sum of power flows through the branches incident to it. The power flow through conventional branches (i.e., transmission lines and transformers) is calculated exactly as in the conventional formulation, that is, as a function of the complex voltages at the branch terminal nodes. On the other hand, power flows through switching branches are directly expressed in terms of the new state variables. Therefore, the active and reactive equations are expressed as:

$$P_k = \sum_{m \in \Omega_k} t_{km}(V_k, V_m, \theta_k.\theta_m) + \sum_{\ell \in \Gamma_k} t_{k\ell} \tag{6}$$

$$Q_k = \sum_{m \in \Omega_k} u_{km}(V_k, V_m, \theta_k.\theta_m) + \sum_{\ell \in \Gamma_k} u_{k\ell} + Q^{sh}_k(V_k) \tag{7}$$

where $\Omega_k$ is the set of buses connected to the bus/bus-section $k$ through conventional branches; $\Gamma_k$ is the set of buses connected to bus/bus-section $k$ through switching branches, and $Q^{sh}_k$ is the reactive power flow through shunt branches (capacitors or reactors) incident to bus $k$ (by convention, $Q_k^{sh} > 0$ when it flows from bus $k$ to the ground).

Therefore, the set of extended power flow equations is composed by the set of bus power mismatches at each node, just as in the conventional formulation, augmented by the linear operational equations which bring to the problem the substation level information, that is:

$$\bar{\mathbf{f}}(\bar{\mathbf{x}}) = \begin{bmatrix} \mathbf{P}^{spec} - \mathbf{P}(\boldsymbol{\theta}, \mathbf{V}, \mathbf{t}) \\ \mathbf{Q}^{spec} - \mathbf{Q}(\boldsymbol{\theta}, \mathbf{V}, \mathbf{u}) \\ \mathbf{f}_o^{cl}(\boldsymbol{\theta}, \mathbf{V}) \\ \mathbf{f}_o^{op}(\mathbf{t}, \mathbf{u}) \end{bmatrix} = \mathbf{0} \tag{8}$$

If $N_{PV}$ $(N_{PQ})$ is the number of $PV$ $(PQ)$ buses, the nonlinear algebraic system given by Eq. (8) comprises $N_{PV} + 2N_{PQ} + 2N_{cl} + 2N_{op}$ equations.

## III. Extended Fast Decoupled Power Flow: XFDPF

In this paper, the extended nonlinear system of equations given by Eq. (8) is solved by the well known fast decoupled power flow (FDPF) method [14]. It is assumed that the basic condition for $P - \theta/Q - V$ decoupling applies, that is, the $X/R$ ratio for all conventional branches is sufficiently large. Furthermore, the additional operating conditions allowing the adoption of *fast* decoupled schemes [14] are also assumed valid. Accordingly, Eqs. (8) can be solved through an iterative algorithm which sequentially solves in each major iteration the two linear systems:

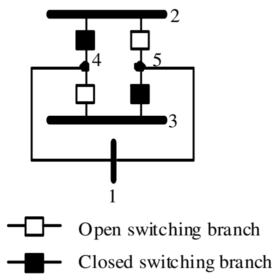
$$\mathbf{B}'_{ext} \begin{bmatrix} \boldsymbol{\Delta\theta} \\ \boldsymbol{\Delta}\mathbf{t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Delta}\mathbf{P}(\boldsymbol{\theta}^{\upsilon}, \mathbf{V}^{\nu}, \mathbf{t}^{\upsilon})/\mathbf{V}^{\upsilon} \\ \mathbf{f}_{\theta}^{cl}(\boldsymbol{\theta}^{\upsilon}) \\ \mathbf{t}^{op}(\mathbf{t}^{\upsilon}) \end{bmatrix} \tag{9}$$

$$\mathbf{B}''_{ext} \begin{bmatrix} \boldsymbol{\Delta}\mathbf{V} \\ \boldsymbol{\Delta}\mathbf{u} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Delta}\mathbf{Q}(\boldsymbol{\theta}^{\upsilon+1}, \mathbf{V}^{\upsilon}, \mathbf{u}^{\upsilon})/\mathbf{V}^{\upsilon} \\ \mathbf{f}_{v}^{cl}(\mathbf{V}^{\upsilon}) \\ \mathbf{u}^{op}(\mathbf{u}^{\upsilon}) \end{bmatrix} \tag{10}$$

where $\mathbf{B}'_{ext}$ and $\mathbf{B}''_{ext}$ are the (constant) fast decoupled coefficient matrices [14] extended to account for the switching branch operational conditions given by Eqs. (3) and (5), as illustrated in the next subsection. Superscript $\upsilon$ indicates values of the corresponding variables in iteration $\upsilon$, which are updated by the iterative algorithm, as described in Subsection III-B.

### A. Structure of XFDPF Matrices: Illustrative Example

Fig 1 illustrates the modifications on the structure of the FDPF coefficient matrices needed to solve XFDPF problems. In the 5-node test system depicted in Fig. 1*(a)*, node 1 is a conventional, non-detailed bus representation, whereas nodes 2 through 5 correspond to bus sections of a substation represented at the physical level. Power flows through the switching branches are defined as state variables, generating the extra columns of $\mathbf{B}'_{ext}$ in Fig. 1*(b)*. In the same figure, symbol "$*$" represents real-valued entries obtained from the admittance of the conventional network branches. Finally, the last four rows of $\mathbf{B}'_{ext}$ correspond to the statuses of the four circuit breakers, according to the concepts presented in Section II. The structure of matrix $\mathbf{B}''_{ext}$ can be similarly obtained.




*(a)*

| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $t_{24}$ | $t_{25}$ | $t_{34}$ | $t_{35}$ |
|---|---|---|---|---|---|---|---|---|---|
| $P_1$ | * | 0 | 0 | * | * | 0 | 0 | 0 | 0 |
| $P_2$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $P_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $P_4$ | * | 0 | 0 | * | 0 | −1 | 0 | −1 | 0 |
| $P_5$ | * | 0 | 0 | 0 | * | 0 | −1 | 0 | −1 |
| $\theta_2 - \theta_4$ | 0 | 1 | 0 | −1 | 0 | 0 | 0 | 0 | 0 |
| $\theta_3 - \theta_5$ | 0 | 0 | 1 | 0 | −1 | 0 | 0 | 0 | 0 |
| $t_{25}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $t_{34}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

*(b)*

Fig. 1. Example of XFDPF coefficient matrices : *(a)* Five-node test system; *(b)* Structure of the corresponding $B'_{ext}$ matrix.

### B. XFDPF Algorithm

Once the physical level representation is incorporated into the power flow formulation, as discussed in Section II, and the assumptions of FDPF are observed, Eqs.9 and 10 are valid and the corresponding coefficient matrices and right-hand side vectors can be easily built. In addition, the well known FDPF algorithm [14] can be adapted to accommodate the detailed bus section representation, thus leading to the following XFDPF algorithm:

1) initialize $\upsilon \leftarrow 0$;
2) Compute angle corrections and active power flow through the modeled switching branches using Eq. (9);
3) Update active state vectors: $\boldsymbol{\theta}^{\upsilon+1} \leftarrow \boldsymbol{\theta}^{\upsilon} + \boldsymbol{\Delta\theta}$ and $\mathbf{t}^{\upsilon+1} \leftarrow \mathbf{t}^{\upsilon} + \boldsymbol{\Delta}\mathbf{t}$;
4) Perform full convergence test. If convergence is reached, go to step 9, otherwise continue to step 5;
5) Compute voltage magnitude corrections and reactive power flow through modeled switching branches using Eq. (10);
6) Update reactive state vectors: $\mathbf{V}^{\upsilon+1} \leftarrow \mathbf{V}^{\upsilon} + \boldsymbol{\Delta}\mathbf{V}$ and $\mathbf{u}^{\upsilon+1} \leftarrow \mathbf{u}^{\upsilon} + \boldsymbol{\Delta}\mathbf{u}$;
7) Perform full convergence test. If convergence is reached, go to step 9, otherwise contiue to step 8;
8) Update $\upsilon \leftarrow \upsilon + 1$ and go to Step 2;
9) Compute final values for nodal power injections and power flows through conventional branches.

**Remarks:**

1) Notice that $\boldsymbol{\theta}^{\upsilon+1} = \boldsymbol{\theta}^{\upsilon} + \boldsymbol{\Delta\theta}$ is computed as soon as the solution of the first linear system is available and immediately used to update the right-hand side of Eq. (10). In general, the most recently computed values should be employed to update the right-hand sides of Eqs. (9) and (10);
2) Full convergence of the XFDPF algorithm, which is checked in steps 4 and 7, actually requires successive convergence of both the $P - \delta - t$ and $Q - V - u$ subproblems (not necessarily in that order). While that does not occur, the two half-iterations continue to be alternately performed. In both cases, convergence tests are based on the magnitudes of power injection residuals;
3) It is important to remark that the switching branch operational conditions given by Eqs. (3) and (5), as well

as the additional terms in Eqs. (6) and (7), are *linear*. As a consequence, their simultaneous solution along with the conventional power flow equations does not cause any increase on the number of iterations for convergence with respect to the conventional FDPF algorithm.

## IV. Simulation Results

An XFDPF computer program has been developed in MATLAB by applying the formulation proposed in Sections II and III to represent switching branches in the network model, and the IEEE 24-bus and 30-bus test-systems are employed to evaluate the performance of the proposed approach. The one-line diagrams and network data of the bus-branch models for both test systems can be found, for instance, in references [15], [16] and [17].

Simulation results are grouped into two cases, A and B. Case A refers to the 24-bus test system and considers two substations exhibiting distinct circuit-breaker arrangements. Case B is based on the IEEE 30-bus test-system and also illustrates the computation of power flows through switching devices of two substations whose arrangements are similar to each other, but are distinct from those of case A and involve a somewhat larger number of circuit-breakers. Case A also investigates the impact of changing the status of some circuit breakers on the power flows through switching devices of the detailed substations.

### *A. Case A: IEEE 24-bus - Substations 14 and 16*

This case considers that one is interested on the power flow distribution through the switching devices of substations 14 and 16 of the IEEE 24-bus test system. To that end, those substations are represented at the bus-section level, as shown in Fig. 2, while the conventional bus-branch representation is maintained for the remaining substations. It is assumed that the arrangement of substation 14 is of the ring-bus type, whereas a breaker-and-a-half scheme is employed for substation 16. The resulting network is modeled as described in Sections II and III. The proposed XFDPF method is then applied to obtain a steady-state solution for the network.

This case considers two operating conditions for substations 14 and 16. The first operating condition, referred to as Case A1, is the same presented in reference [6], where the line that connects substations 14 and 16 (line $27-28$) is in operation, that is, breakers $14-27$ and $27-28$ are closed. Case A2 considers the same operating condition, except that now both breakers $14-27$ and $27-28$ have been switched open.

*1) Results for Case A1:* Power flow results obtained with XFDPF for substations 14 and 16 and adjacent buses are shown in Tables I and II. Table I is divided into two sections, one for each substation. Each of those is in turn subdivided into two subsections, one with nodal results for the substation component nodes and the other with the power flows through branches connecting the substation to *external* nodes. For the sake of conciseness, only the component nodes with nonzero injections are represented in Table I. Although not all bus-section voltages are shown, those connected through closed switching branches exhibit the same voltage, as expected. This is illustrated by the sample nodal results shown in Table I.

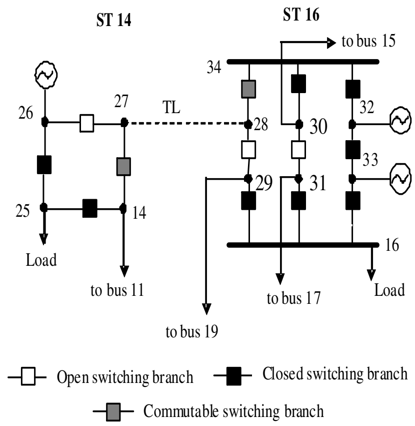



Fig. 2. Bus-Section model for substations 14 and 16 of the IEEE 24-bus test system

Power flow results *internal* to each substation, that is, the flows through a substation's switching devices, are presented in Table II. Again for conciseness, zero power flows (through open circuit breakers) are omitted.

As one can readily verify, the results in Tables I and II are consistent, both in terms of power balance for each substation node and the power flow distribution over switching branches. XFDPF successfully converges after 4 iterations, which is the same number of iterations required when a conventional FDPF algorithm is applied to the bus-branch model of the original test-system, considering line $27-28$ in operation. This behavior has indeed been observed in all tests conducted with the extended power flow formulation. As discused in Subsection III-B, that should be expected, since the additional terms and equations are linear functions of the variables contained in the extended state vector.

*2) Results for Case A2:* This case considers that transmission line $27-28$ has been switched out of service by simultaneously opening circuit breakers $14-27$ and $27-28$. Convergence of the XFDPF algorithm is successfully obtained after 6 iterations. That differs from Case A1, since the resulting operating condition for Case A2 is somewhat more severe. However, it is the same number of iterations required when a conventional FDPF is applied to the bus-branch model of the test-system considering the outage of line $27-28$, as expected.

The results are presented in Tables III and IV, and lead to the same conclusions of Case A1 regarding power balance and power flow distribution. Besides, they clearly show the expected changes on power flow distribution in substations 14 and 16 when a change of substation topology occurs, thus emphasizing the importance of XFDPF as an adequate tool to perform a fast analysis of the impact of topology changes. In addition, if one is interested in obtaining new results for additional configuration changes in substations 14 and 16, it suffices to inform the new desired status for the modeled switching branches.

TABLE I

LOAD FLOW RESULTS FOR CASE A1

| *Substation 14* | | | |
|---|---|---|---|
| Component nodes with nonzero injections | | | |
| | **Injection** | **Nodal voltage** | |
| **Node** | $\mathbf{P}_k$+**j** $\mathbf{Q}_k$ | $\|V_k\|\,(p.u.)$ | $\angle V_k$ |
| 25 | $-194.0 - j\ 39.0$ | 0.9800 | $13.92^o$ |
| 26 | $0.0 + j\ 94.089$ | 0.9800 | $13.92^o$ |
| Flows to adjacent buses / substations | | | |
| **Branch** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ | | |
| $14 - 11$ | $235.215 - j\ 34.135$ | | |
| $27 - 28$ | $-429.215 + j\ 89.224$ | | |
| *Substation 16* | | | |
| Component nodes with nonzero injections | | | |
| | **Injection** | **Nodal voltage** | |
| **Node** | $\mathbf{P}_k$+**j** $\mathbf{Q}_k$ | $\|V_k\|\,(p.u.)$ | $\angle V_k$ |
| 16 | $-100.0 - j\ 20.0$ | 1.0170 | $23.80^o$ |
| 32 | $77.5 - j\ 105.0851$ | 1.0170 | $23.80^o$ |
| 33 | $77.5 - j\ 105.0851$ | 1.0170 | $23.80^o$ |
| Flows to adjacent buses / substations | | | |
| **Branch** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ | | |
| $30 - 15$ | $-89.966 - j\ 70.741$ | | |
| $31 - 17$ | $-314.667 - j\ 78.581$ | | |
| $29 - 19$ | $20.772 - j\ 58.494$ | | |
| $28 - 27$ | $438.861 - j\ 22.53$ | | |

TABLE II

POWER FLOW DISTRIBUTION OVER SWITCHING BRANCHES - CASE A1

| **Breaker** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ |
|---|---|
| *Substation 14* | |
| $14 - 25$ | $194.000 - j\ 55.089$ |
| $14 - 27$ | $-429.215 + j\ 89.224$ |
| $25 - 26$ | $0.000 - j94.089$ |
| *Substation 16* | |
| $16 - 29$ | $20.772 - j\ 58.494$ |
| $16 - 31$ | $-314.667 - j\ 78.581$ |
| $16 - 33$ | $193.895 + j117.075$ |
| $28 - 34$ | $-438.861 + j\ 22.353$ |
| $30 - 34$ | $89.966 + j\ 70.741$ |
| $32 - 33$ | $-271.395 - j\ 11.990$ |
| $32 - 34$ | $348.895 - j\ 93.094$ |

### B. Case B: IEEE 30-bus - Substations 12 and 15

In this case, substations 12 and 15 of the IEEE 30-bus test system have been selected as the substations of interest, and both are assumed to be of the double-bus type, involving multiple circuits. As before, the conventional bus-branch model is employed for the remaining substations. The loading condition employed in this case study is based on the one employed in reference [16]. Substations 12 and 15 of the original test system are modeled at the bus-section model, as shown in Figure 3.

The power flows through closed devices determined by XFDPF are consistent with those through conventional branches and the injections at end bus sections. Therefore, the results comply with the power balance equations for each substation. Also, power flows through open devices are correctly computed as zero (although this information is omitted in Table VI for the sake of conciseness). Convergence is obtained in 5 iterations. Again, this is the same number of iterations required by the FDPF method applied to the test system's bus branch model.

TABLE III

LOAD FLOW RESULTS FOR CASE A2

| *Substation 14* | | | |
|---|---|---|---|
| Component nodes with nonzero injections | | | |
| | **Injection** | **Nodal voltage** | |
| **Node** | $\mathbf{P}_k$+**j** $\mathbf{Q}_k$ | $\|V_k\|\,(p.u.)$ | $\angle V_k$ |
| 25 | $-194.000 - j\ 39.000$ | 0.9800 | $-5.70^o$ |
| 26 | $0.000 + j\ 66.909$ | 0.9800 | $-5.70^o$ |
| Flows to adjacent buses / substations | | | |
| **Branch** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ | | |
| $14 - 11$ | $-194.000 + j\ 27.909$ | | |
| $27 - 28$ | $0.0 + j\ 0.0$ | | |
| *Substation 16* | | | |
| Component nodes with nonzero injections | | | |
| | **Injection** | **Nodal voltage** | |
| **Node** | $\mathbf{P}_k$+**j** $\mathbf{Q}_k$ | $\|V_k\|\,(p.u.)$ | $\angle V_k$ |
| 16 | $-100.000 - j\ 20.000$ | 1.0170 | $46.56^o$ |
| 32 | $77.500 - j115.684$ | 1.0170 | $46.56^o$ |
| 33 | $77.500 - j115.684$ | 1.0170 | $46.56^o$ |
| Flows to adjacent buses / substations | | | |
| **Branch** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ | | |
| $30 - 15$ | $33.872 - j\ 86.881$ | | |
| $31 - 17$ | $-284.955 - j\ 85.643$ | | |
| $29 - 19$ | $306.083 - j\ 78.843$ | | |
| $28 - 27$ | $0.0 + j\ 0.0$ | | |

TABLE IV

LOAD FLOW RESULTS FOR CASE A2

| **Breaker** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ |
|---|---|
| *Substation 14* | |
| $14 - 25$ | $194.000 - j\ 27.909$ |
| $25 - 26$ | $0.000 - j66.909$ |
| *Substation 16* | |
| $16 - 29$ | $306.083 - j\ 78.843$ |
| $16 - 31$ | $-284.955 - j\ 85.643$ |
| $16 - 33$ | $-121.128 + j144.486$ |
| $30 - 34$ | $-33.872 + j\ 86.881$ |
| $32 - 34$ | $33.872 - j\ 86.881$ |
| $32 - 33$ | $43.628 - j\ 28.803$ |

## V. CONCLUSIONS

This paper introduces a method to extend the fast decoupled power flow formulation in order to allow the explicit representation of substation switching devices in steady-state power network studies. The proposed approach is able to model selected parts of the electric network at the bus section level, so that active and reactive power flows through switching branches are readily obtained as part of the output of the power flow study.

The changes required to represent circuit breakers in the fast decoupled power flow formulation are based on the definition of new state variables associated with switching branches, as detailed in the paper. An important feature of the proposed methodology is that the operational conditions defined to represent the status of switching branches are linear. As a result, the convergence rate of the extended fast decoupled power flow is not degraded by the inclusion of the switching branch representation into the power flow problem.

The paper describes the application of the proposed method to the IEEE 24-bus and 30-bus test systems. For each network, substations with different arrangements are selected and then modeled at the bus section level. Results of several case

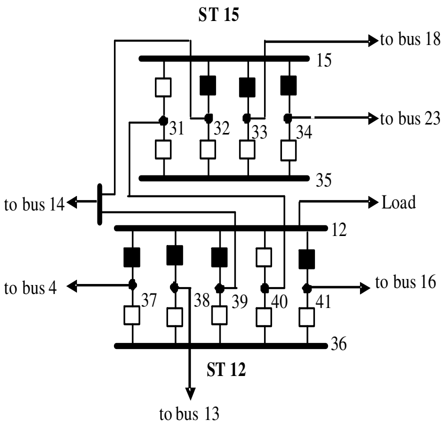



Fig. 3. Bus-section model for substations 12 and 15 of IEEE 30-bus test system

TABLE V

LOAD FLOW RESULTS FOR CASE B

| *Substation 12* | | | |
|---|---|---|---|
| Component nodes with nonzero injections | | | |
| | **Injection** | **Nodal voltage** | |
| **Node** | $\mathbf{P}_k$+**j** $\mathbf{Q}_k$ | $\lvert V_k\rvert\,(p.u.)$ | $\angle V_k$ |
| 12 | $-11.2 - j\ 7.5$ | 1.0230 | $-16.52^o$ |
| Flows to adjacent buses / substations | | | |
| **Branch** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ | | |
| $37-4$ | $-39.92 + j\ 9.52$ | | |
| $38-13$ | $0.000 - j\ 35.09$ | | |
| $39-14$ | $18.075 + j\ 9.555$ | | |
| $41-31$ | $0.000 + j\ 0.000$ | | |
| $41-16$ | $10.65 + j\ 8.52$ | | |
| *Substation 15* | | | |
| Component nodes with nonzero injections | | | |
| | **Injection** | **Nodal voltage** | |
| **Node** | $\mathbf{P}_k$+**j** $\mathbf{Q}_k$ | $\lvert V_k\rvert\,(p.u.)$ | $\angle V_k$ |
| 15 | -8.2-j2.5 | 0.938 | -18.95$^o$ |
| Flows to adjacent buses / substations | | | |
| **Branch** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ | | |
| $31-40$ | $0.000 + j\ 0.000$ | | |
| $32-14$ | $-10.97 - j\ 6.56$ | | |
| $33-18$ | $2.00 + j\ 3.54$ | | |
| $34-23$ | $0.769 + j\ 0.521$ | | |

studies illustrate the benefits of the proposed tool. In real-time applications, for instance, it provides a fast means to analyzing the impact of circuit breaker operation on the power flow distribution throughout the network, including substation components. In addition to real-time modeling, the extended power flow method can also be instrumental to other areas, such as corrective switching studies.

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TABLE VI

POWER FLOW DISTRIBUTION OVER SWITCHING BRANCHES - CASE B

| **Breaker** | $\mathbf{P}_{kl}$+**j** $\mathbf{Q}_{kl}$ |
|---|---|
| *Substation 12* | |
| $12-37$ | $-39.923 + j\ 9.521$ |
| $12-38$ | $0.000 - j\ 35.094$ |
| $12-39$ | $18.075 + j\ 9.555$ |
| $12-41$ | $10.649 + j\ 8.518$ |
| *Substation 15* | |
| $15-32$ | $-10.972 - j\ \ 6.562$ |
| $15-33$ | $2.003 + j\ \ 3.541$ |
| $15-34$ | $0.769 + j\ \ 0.521$ |

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