

PMU placement on the basis of SCADA measurements for fast load flow calculation in electric power systems

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Abstract.¹ The paper considers the problem of PMU placement in such a way that the volume of initial information based on the SCADA and PMU measurements is sufficient to determine all the state vector components for load flow calculations without iterations. The PMU number in this case should be minimal. The problem of PMU placement is solved by the simulated annealing method.

Index Terms – load flow, state vector, SCADA, phasor measurement units (PMUs).

I. INTRODUCTION

The problem of steady state (SS) calculation (load flow) that is one of the first problems arisen in creation of Computer-Aided Systems of Dispatching Control plays a leading role in the complex of control problems of electric power system (EPS) operation and expansion. The problem of SS calculation is an independent problem and also forms a base which allows the solution of other, more sophisticated problems such as optimization of normal state of EPS, stability analysis, reliability estimation, etc.

In the most general form the SS calculation problem can be formulated as a problem of solving nonlinear equations describing EPS state, which are applied to calculate voltage phasors at the buses by the given powers or currents at the buses. Since the 1950s and up to nowadays numerous methods for load flow calculation have been devised [1-4]. Despite a great variety of these methods the SS calculations have problems in convergence and provision of the required high-speed of algorithms. In the early 1970s development and perfection of the tools for collection, processing and transmission of measurements in EPS to calculate SS gave rise to application of the state estimation (SE) methods. The SE problem also consists in calculation of the voltage phasors at the buses, but on the basis of measured values of different state parameters rather than only parameters of the powers at the buses. The measurements for SE are data of the SCADA system: power flows in transmission lines, voltage magnitudes, currents and powers at the buses. Here the vector of voltage phasors at the buses is called a state vector. The SE problem is solved by using the overdetermined system of nonlinear equations that reflects dependences of the measured variables on the state vector. The SE methods make it possible to construct a more accurate model of the current EPS state. However, in the case of large volume of information that is often substantially inhomogeneous due to great distinctions in measurement accuracy, they do not solve the problems of

convergence and high speed of the computational algorithms, and besides aggravate them in some cases [5].

A possible way to solve these problems is to devise non-iterative or direct methods for solving the problem of SS calculation in general and the problem of SE in particular. The works of both Russian and foreign authors are devoted to solution of this problem [6-8]. The main idea of these methods is to apply the graph theory for reduction of the calculated scheme to the form convenient for successive calculation of the state vector components by the known SS equations and available measurement information. In this case only minimum necessary measurements are used for calculation, but not all available ones as in the traditional SE methods. Similar methods are widely applied to calculate SS and SE of distribution networks with the radial configuration [9,10].

The paper suggests an algorithm of fast SS calculation based on the measurements of the SCADA system. It applies the procedure of reducing the EPS calculated scheme to a tree and the accurate synchronized measurements of voltage magnitudes and phases at the buses that are received from the phasor measurement units (PMUs). The first section presents a problem formulation for fast load flow calculation on the basis of PMU data. The second section describes an algorithm of EPS scheme reduction to a tree and presents an idea of its realization by the minimum number of PMUs. In the next section an algorithm for calculation of the state vector components for the scheme in the form of a tree based on the SCADA measurements and the PMU data is presented. Then the algorithm of PMU placement for fast load flow calculation by the method of annealing simulation is considered. The next section presents calculation results for a real system fragment that are obtained in the simulation experiment (at measurement modeling) and by the real SCADA measurements. The results obtained by the algorithm of fast load flow calculation and one of the traditional SE methods are compared.

II. PROBLEM FORMULATION

The current power system state can be determined on the basis of the known minimal set of state variables that will specify unambiguously the rest of state variables. Such a set of state variables is called a state vector. In the work these are voltage magnitudes and phases $x = \{U, \delta\}$.

The calculating speed of current system state depends on the way of determining all the state vector components. Traditionally the state vector components are calculated in the SE by the iterative methods. In the extreme cases the speed of achieving the result may turn out insufficiently fair.

Measurement of these values by PMU is the most attractive way of determining all the state vector components. PMU placed at the bus can measure the voltage

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which all δ , U can be calculated with the least computer time and the minimum number of required PMUs.

Possibility of calculating δ , U at each bus is verified by transition from bus to bus along all the tree branches, provided the unknown δ , U can be calculated in addition by using different combinations of the measured and computed state variables. If δ , U can not be determined, PMU is installed at the bus.

There are several variants for moving along the tree. Depending on location of the reference bus in the scheme the process of determining the unknown δ , U will be more successful or less successful. The successful process implies that a combination of the maximum number of measurements applied in calculations, the minimum number of pseudo-measurements and the minimum number of PMUs to be installed in the case of impossibility to determine δ , U at the considered bus by using the SCADA measurements and pseudo-measurements.

B. Algorithm for calculation of δ , U

1. Reduction of the scheme to the tree (removal of all loops from the network graph).
2. Determination of the tree vertex (the reference bus), i.e. the bus, from which the voltages are calculated by the tree branches. The reference bus is searched for by the simulated annealing method. PMU is installed at the reference bus.
3. Calculation of all the state vector components by the tree branches on the basis of the known relations between the EPS variables.
4. Calculation of voltages at the buses that are not included in the tree scheme. To do this, for each of such buses the scheme state is estimated. Here the number of buses is equal to $m+1$, where m – number of lines adjacent to the removed bus.
5. Comparison of the values of state vector components that are obtained at point 4 with the values obtained at point 3. If the difference is less than some threshold, then go to point 7. Otherwise to point 6.
6. Generation of signal about the error in measurements. Go to point 8.
7. Adjustment of the values of flows in the boundary lines. The boundary lines are the lines limited by the buses, at which PMU is installed. Adjustment of the values of injections in the buses at which the voltage is corrected.
8. The end of algorithm work.

V. PMU PLACEMENT FOR NON-ITERATIVE LOAD FLOW

A. Approaches to PMU placement

PMU placement in the power system network is a combinatorial problem on placement of K transducers at N network buses on the basis of placement criteria to be chosen. The problem can be solved by the complete enumeration of placement variants with the constraints, for example, not to install PMU at the terminal bus. For large-scale networks the complete enumeration requires much computer time. The logic of reasoning about the lacking need to check absolutely all the variants of PMU placement allows elimination of the variants containing terminal buses, all the combinations of two neighboring buses, etc. from

consideration and application of various heuristic approaches.

In [12], [13] the authors assume that before solving the PMU placement problem there are no measurements in the network. It is provided with measurements, when PMUs are installed at network buses. The overview of possible variants of PMU placement and discussion of the problem, when the mathematically optimal placement ceases to be such with changing conditions of power system operation are presented in [12]. Besides the authors suggest solving the PMU placement problem by the double search method. In [14] this problem is solved by using the genetic algorithm.

In the present work the problem of PMU placement is solved by the simulated annealing (SA) method. Its idea is a gradual system transition from the initial state to some final one based on the predetermined criterion. PMUs installed in the initial configuration can be moved to other buses, if a new configuration of PMUs leads to the optimal solution.

B. The simulated annealing (SA) method [15].

The path revealing the optimal reference bus is searched for by the simulated annealing method. The initial information is preliminarily processed to reduce the time of solving the problem. Based on the mix of measurements in the scheme the lacking pseudo-measurements for voltages, injections and power flows are calculated in addition by using the bichromatic graph² [16]. A generalized list of measurements is formed based on the measurements and calculated pseudo-measurements.

The annealing method determines an optimal solution by searching for only in the direction of the objective function decrease and by avoiding the local optima on the basis of the probability of taking an incorrect decision. The objective function of the SA method in our problem has the form:

$$\min E = \frac{K1_{PMU} + K2_{step} + K3_{double}}{K4_{success}} \quad (1)$$

where $K1_{PMU}$ – number of additionally installed PMUs;

$K2_{step}$ – maximum number of steps from the reference to the final bus resulting in calculation of δ, U ;

$K3_{double}$ – number of doubling counts due to the fact that δ, U at the current bus can be calculated with the help of different measurements;

$K4_{success}$ – index of successful measurements of voltage magnitudes and phases by spreading along the graph.

In turn, $K4_{success} = n_1\alpha_1 + n_2\alpha_2 + n_3\alpha_3 + n_4\alpha_4$, with $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$,

where α_i (are set by the researcher) represent scores for application of n_1 accurate measurements from PMU, n_2 measurements from SCADA, n_3 pseudo-measurements, n_4 pseudo-measurements obtained from the formulas containing pseudo-measurements squared.

Solution to the problem starts with the choice of an arbitrary bus considered as a reference one, assignment of PMU at it with the accurate measurements of

² The bichromatic graph sets up a correspondence between the steady state equations and SCADA measurements in the scheme graph to calculate pseudo-measurements of state variables.

$\delta_{PMU}, U_{PMU}, I_{ij_{PMU}}, I_{ij_{PMU}}$ and motion from it along the tree branches (count of $K4_{success}$). In this case the longer the path from the reference to the terminal bus, the greater the error accumulated during intermediate calculations. It means the necessity to choose the reference bus so that it was located approximately at an equal distance from the final buses (count of $K2_{step}$). Hence, it is more preferable to start choosing a reference bus from the more connected buses.

If the result E_{new} obtained by using (1) after the next iteration is lower than the accepted earlier optimal result ($E_{new} < E_{opt}$), a new variant of assignment is taken $E_{opt} = E_{new}$. If not, the return to the previous step or not depends on the probability value $P(\Delta E)$ of taking an incorrect decision

$$P(\Delta E) = e^{-\Delta E / (k_b T)}, \quad (2)$$

where $\frac{\Delta E}{k_b T}$ – the Metropolis criterion that is an analog to the Boltzmann factor [15].

Then one of the neighboring buses is assigned as a candidate to be accepted as a reference bus. The voltage magnitudes and phases are recalculated. Thus, different buses are successively treated as candidates and the objective function value is calculated. The next more successful result is stored.

Provided on the path there is the bus, at which calculation of δ, U is impossible because of lacking measurements in the transmission lines incident to it, an additional PMU is installed there (count of $K1_{PMU}$). Then PMU with highly accurate values of δ, U , and currents in all connected lines is installed at the bus. Hence it is possible to recalculate δ, U at the ends of these lines. And if such measurements have already been obtained earlier, they are taken as doubling ones (count of $K3_{double}$).

The annealing process is controlled by the temperature T that gradually decreases to zero. The lower the T , the less probable is an incorrect decision (2). As the result the optimal solution is obtained at T that is very close to zero.

The criterion for completion of work is $\min E$.

VI. PRACTICAL RESULTS

The example of the 13-bus scheme (Fig. 2) illustrates the possibility for obtaining all the state vector components on the basis of measurements from the SCADA system and the data of one PMU. Calculations were performed on the simulated and real data.

The network scheme is reduced to a tree by removal of bus 3 as the most connected one from the graph (Fig. 3). The branches that are not included in the tree are shown by dashed lines. The use of the simulated annealing algorithm reveals that bus 13 is the optimal reference bus where PMU is installed. Additional PMUs are not required. The tree obtained has three branches.

The voltage vectors at all the buses are calculated starting with bus 13.

Point 1. A PMU is installed at bus 13. The voltage magnitudes and phases are calculated at neighboring buses 1, 2, 12 by means of PMU data by the following formulas.

$$U_j = U_i - I_{ij}(R \cos \varphi_{ij} + X \sin \varphi_{ij}) \quad (3)$$

$$\delta_j = \delta_i - \arctg \frac{I_{ij}(X \cos \varphi - R \sin \varphi)}{U_i - R \cos \varphi - X \sin \varphi} \quad (4)$$

Voltage magnitudes and phases of other buses are calculated by the following measurements from the SCADA system:

Point 2. At buses 4, 6, 10 by the active and reactive power flows at the line beginning:

$$U_j = \sqrt{\left(U_i - \frac{P_{i-j} * R + Q_{i-j} * X}{U}\right)^2 + \left(\frac{P_{i-j} * X - Q_{i-j} * R}{U}\right)^2} \quad (5)$$

$$\delta_j = \delta_i - \delta_{i-j}; \quad \delta_j = \delta_i - \arctg \frac{P_{i-j} * X - Q_{i-j} * R}{U_i^2 - P_{i-j} * R - Q_{i-j} * X} \quad (6)$$

Point 3. At buses 11, 5, 7, 8 by the active and reactive power flows at the line end and voltage:

– calculation of reactive power flow at the line beginning

$$Q_{i-j} = Q_{j-i} + \frac{P_{j-i}^2 + Q_{j-i}^2}{U_j^2} * X - B U_j^2 \quad (7)$$

– calculation of U_j, δ_j by formulas (5), (6).

Point 4. At bus 9 by all available measurements:

$$\text{– from } \Delta P = P_{10-9} - P_{9-10} = \frac{P_{10-9}^2 + Q_{10-9}^2}{U_{10}^2} * R \quad (8)$$

the reactive power flow module is calculated at the line beginning Q_{10-9} ;

– calculation of U_j by formula (5) at different values of reactive power flows (Q_{10-9} , $-Q_{10-9}$). The choice is made of the flow, at which the obtained value of voltage magnitude corresponds to the measured one.

– calculation of δ_j by formula (6) at the chosen value of reactive power flow.

The components of objective function (1) for the considered scheme are calculated in the following way (Table 1):

$K1_{PMU} = 0$, since there is no need in additional PMUs and hence, $K3_{double} = 0$. $K2_{step}$ increases by 1 at each step from bus to bus.

For components $K4_{success}$ the following points of calculations are applied:

n_1 - P.1 (3)-(4), n_2 - P.2 (5)-(6), n_3 - P.3 (7), n_4 - P.4 (8).

Table 1
Calculation of the objective function by branch of the tree

№	Branches	$K1_{PMU}$	$K2_{step}$	$K3_{doubl}$	$K4_{success}$
1	13-1-11	0	2	0	$1*\alpha_1 + 1*\alpha_3$
2	13-2	0	1	0	$1*\alpha_1$
3	13-12-5- 4-6-7-8- 10-9	0	8	0	$1*\alpha_1 + 3*\alpha_2 + 3*\alpha_3 + 1*\alpha_4$

$$\text{Finally, } E_{opt} = \frac{0+8+0}{3\alpha_1 + 3\alpha_2 + 4\alpha_3 + \alpha_4}$$

The voltage at the remote bus is calculated by separation of the 5-bus scheme from the scheme shown in Fig. 3 (Fig. 4).

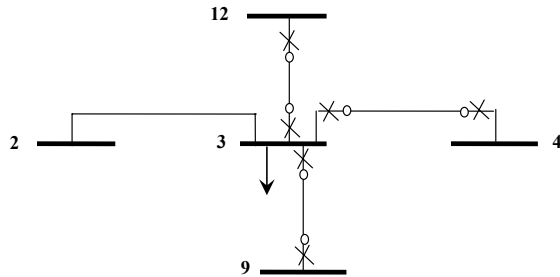


Fig. 4. A 5-bus scheme

For the obtained scheme SE is performed by the available measurements. Voltages calculated by formulas (3)-(8) are taken as initial data. If the difference between the obtained voltage values and the initial data at the boundary buses of the scheme is less than some threshold, the injections in buses are adjusted.

A. Simulation calculation

The SS variables (y_{yp}) are noised by using the random number generator. The values of measurements are calculated by the formula

$$y_{noise} = y_{ss} + x_{ran} * \sqrt{\sigma^2},$$

where σ^2 – measurement variance, $x_{ran} \rightarrow N(0,1)$.

The voltages in the tree branches are calculated by formulas (5), (6). For SE of the 5-bus scheme the modeled measurements are applied as initial data. Table 2 illustrates the calculation results.

Table 2
Calculation results based on simulated data

№ of bus	U_{ss}	U_{noise}	U_{calc}	U_{se}
1	2	3	4	5
2	753	752	756	
3	748	748	751	751
4	503	503	503	505
5	506	509	508	508
6	505	508	507	
7	500	501	502	
9	502	503	504	
10	507	505	509	
11	500	499	502	502
12	507	505	509	
326	743	742	745	
453	502	502	504	504
454	740	743	743	

The second column presents voltage magnitudes in the steady state. The third column shows voltage values obtained due to noise. The fourth column presents voltage values calculated by the tree branches. The fifth column illustrates voltage values obtained as a result of SE of the 5-bus scheme. The Table shows that the voltage values are calculated by the tree branches differ negligibly from the voltages of steady state.

B. Calculation based on real data

The real verified measurements and the measurements from PMU installed at bus 13 are applied as initial data. Voltages of the tree branches are calculated by formulas (3)-(8). SE of the 5-bus scheme is performed by the measurements given in Fig. 4. The calculation results are shown in Table 3.

Table 3
Calculation results based on real data

№	Measurements	Estimate		Non-iterative method	
	2	3		4	
	\bar{U}	\hat{U}	$\hat{\delta}$	U_{calc}	δ_{calc}
1	753	753.5	0	753	0
2	746	747.5	-0.1032	747.5	-0.1032
3	517	502.9	-0.1182	503.0	-0.1182
4	502	505.5	-0.1590	505.4	-0.1608
5	505	505.3	-0.1716	505.2	-0.1737
6	498	500.9	-0.1530	501.6	-0.1549
7	497	504.0	-0.1148	504.6	-0.1175
8	512	507.9	-0.104	508.6	-0.1070
9	515	500.3	-0.0967	500.8	-0.0992
10	515	502.2	-0.1022	508.7	-0.1048
11	741	743.5	-0.1934	743.3	-0.1919
12	512	502.6	-0.1133	502.6	-0.1133
13	785	740.3	-0.0835	740.3	-0.0835

The second column presents values of measurements. The third column shows the SE results for the 13-bus scheme. This calculation was done to determine accuracy of the results obtained by applying the suggested technique. The fourth column gives voltage values calculated by the suggested algorithm.

VII. DETERMINATION OF THE QUALITY OF RESULTS

The quality of results is determined by the objective function value of state estimation:

$$\varphi(x) = \sum_{i=1}^m \frac{(\bar{y}_i - y_i(\hat{x}))^2}{\sigma_i^2}.$$

\bar{y} – vector of measurements, \hat{x} – estimates (or calculated values by the tree branches) of the state vector components.

The values of the criteria for different calculations are presented in Table 4.

Table 4
The values of criteria φ

Simulated data		Real data	
SE	Non-iterative method	SE	Non-iterative method
6.3	7.1	20.35	22.70

The Table shows that the values of criteria are approximately the same for all the calculation methods. It means that the EPS state can be calculated by the non-

iterative method almost with the same accuracy as the SE method.

VIII. CONCLUSION

1. The algorithm for fast SS calculation that is constructed by reduction of the calculated EPS scheme to a tree is suggested. It applies a minimum number of PMUs and an optimal set of measurements obtained from the SCADA system.
2. The algorithm for PMU placement by the SA method is worked out. The algorithm determines an optimal reference bus in the tree scheme and an optimal set of measurements that makes it possible to calculate the state vector for all the scheme buses with the minimum accumulation of calculation errors.
3. SS is calculated for a fragment of the real system based on the simulated and real data. The state vector calculated by the suggested method is shown to coincide with the results of state estimation.
4. The speed of determining EPS state by different methods is analyzed for comparison. The problem of calculating EPS state variables for the 13-bus scheme by the non-iterative method proves to be solved 0.01s faster than the state estimation problem.

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