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Parameter Identification of Unsymmetrical Transmission Lines

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***Abstract*—Today transmission lines are protected by protection relays at their terminating stations. If a fault appears, the protection relays of the faulty line as well as of adjacent lines react and record measurement data. Until today these records are often unexploited. The objective of this work is to analyse these records with the goal to identify equipment parameters in particular to estimate parameters of unsymmetrical transmission lines by using time-varying phasors. Due to the characteristic of the assumed line model only synchronously sampled fault data of two terminating stations can be used for this type of identification.**

This paper presents a theory for the identification of transmission line parameters based on time-varying phasors. By assuming equal admittances at both ends of a π-section model it is possible to calculate the currents through the line using the measured currents of the fault records. Afterwards the line parameters are estimated using the least squares method. It is shown that only measurements including transients can be used for the identification. The proposed approach is verified in two steps. At first fault records are generated by simulating a line model and after that real measurements on a three phase dynamic network model are used for a verification of the algorithm.

***Index Terms*—parameter identification, transmission line model, time-varying phasors.**

I. Introduction

THE global trend to transmit electrical energy via large distances e. g. from offshore wind parks or from dams in china into load centres, requires a reliable operation of transmission lines. These tendencies lead also to an operation of the equipment more and more near to its thermal and stability limits. In consequence the network is more sensitive and requires an accurate setting of the protection relays. Their algorithms are based on exact parameters of the equipment; e. g. the impedance matrix of a not completely transposed transmission line.

The main function of protection relays is to switch off the transmission line after the occurrence of a fault. The protection relays of adjacent lines react as well during a fault, which is farther away and record the electrical values. Fig. 1 shows a typical situation. The starting protection relays record the voltage and current signals, which can be used in an offline analysis.

Fault records contain information about the fault type, the fault location, they can be used to verify the function of the protection relay and furthermore to estimate equipment parameters. E. g. the impedance matrix of the marked transmission line in Fig. 1 could be identified. The purpose of this work is to estimate the whole impedance matrix of a not completely transposed transmission line. Due to technical restrictions this is often the case. In the future this matrix could be used for a more exact determination of the fault location as well as base of power system analysis.

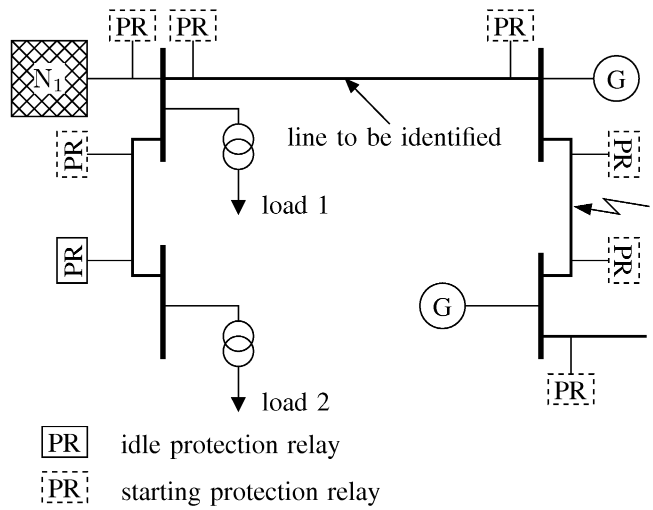

Fig. 1. Typical network with a fault and a line to be identified

The presented approach is based on time-varying phasors of voltages and currents of the terminating stations of a transmission line, which have to be time-synchronised. In section II the assumed model structure is presented and motivates why the data preprocessing described in section III is necessary. With respect to the model structure an estimation equation based on the least squares method (LS) is derived in section IV. Afterwards some results are presented in section V.

II. Modeling the Transmission Line

In general a process identification is based on a predefined model. In the case of a transmission line there exist basically two different kinds of models. Since a transmission line has a spatial extent it is possible to treat them as a system with distributed parameters. This yields in the case of a symmetric (completely transposed) transmission line after decoupling the system states (e. g. symmetric components) the telegraph equation. A restriction to the steady state operation leads to a relation between the voltage and current values of two apart positions. If the line is not completely transposed a decoupling of the states in symmetrical components is not achievable. Hence a partial differential equation system is obtained, which is not easy to treat.

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The other approach is the assumption of a system with lumped parameters. The corresponding line model is shown in Fig. 2 and will be applied in this paper. Basically the model contains three coupled π-sections. As can be seen in Fig. 2

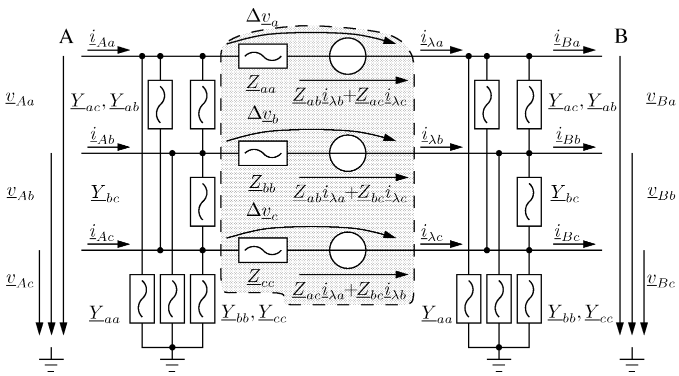

Fig. 2. Model of an unsymmetric transmission line in natural components

this model allows different self impedances $\underline{Z}_{aa}, \underline{Z}_{bb}, \underline{Z}_{cc}$ and different couplings $\underline{Z}_{ab}, \underline{Z}_{ac}, \underline{Z}_{bc}$ between the conductors. The objective is to determine these parameters which are highlighted with the grey-coloured box by using the least squares (LS) method. In order to apply this method, appropriate equations have to be formulated. Each voltage drop $\Delta\underline{v}_{a,b,c}$ yields a relation to the wanted impedances and the currents $\underline{i}_{\lambda a,b,c}$. As can be seen the voltages and currents are complex-valued and are based on time-varying phasors [1]. The relation of the time-varying phasor $\underline{X}(t)$ corresponding to the real-valued and measurable signal $x(t)$ is

$$x(t) = \mathrm{Re}\left\{\underline{X}(t)\, e^{j\,\omega\, t}\right\} = \mathrm{Re}\left\{\underline{x}(t)\right\}\,. \tag{1}$$

The practical determination of the time-varying phasor is based on the so called classical demodulation technique [1], [2]. Using phasors has the following advantages:

- The algorithm to determine the phasor implicates a low pass filtering and therefore removes high frequency noise.
- The fundamental frequency is removed from the phasor. Hence the phasor is slow compared to the measured signal and the necessary time derivatives are easier to determine.

In order that the demodulation technique is applicable, the real-valued signal need to have the following structure

$$x(t) = X(t)\,\cos(\omega\, t + \varphi(t)), \tag{2}$$

whereas $X(t)$ and $\varphi(t)$ are low-pass functions. After deriving the so-called in-phase and quadrature components according to [2] the time-varying phasor is

$$\underline{X}(t) = X_d(t) + j\,X_q(t)\,. \tag{3}$$

That means every signal occurring in Fig. 2 has after transforming in the phasor domain the structure of eq. (3).

III. Data Preprocessing

As mentioned in section II the objective is to identify the elements inside the grey-colored box of Fig. 1. This requires the determination of the currents $\underline{i}_{\lambda a,b,c}$ which can be done by assuming equal admittances $\underline{\boldsymbol{Y}}$ on both terminating stations A and B. After collecting $N \geq 3$ measurement values during a fault, it is possible to derive the currents to ground at stations A and B according to

$$\underline{\boldsymbol{I}}_{CA} = \underline{\boldsymbol{Y}}\,\underline{\boldsymbol{V}}_A \tag{4}$$

$$\begin{pmatrix} \underline{I}_{CAa,1} & \cdots & \underline{I}_{CAa,N} \\ \underline{I}_{CAb,1} & \cdots & \underline{I}_{CAb,N} \\ \underline{I}_{CAc,1} & \cdots & \underline{I}_{CAc,N} \end{pmatrix} = \underline{\boldsymbol{Y}} \begin{pmatrix} \underline{V}_{Aa,1} & \cdots & \underline{V}_{Aa,N} \\ \underline{V}_{Ab,1} & \cdots & \underline{V}_{Ab,N} \\ \underline{V}_{Ac,1} & \cdots & \underline{V}_{Ac,N} \end{pmatrix}$$

$$\underline{\boldsymbol{I}}_{CB} = \underline{\boldsymbol{Y}}\,\underline{\boldsymbol{V}}_B, \quad \underline{\boldsymbol{V}}_A, \underline{\boldsymbol{V}}_B, \underline{\boldsymbol{I}}_{CA}, \underline{\boldsymbol{I}}_{CB} \in \mathbb{C}^{3\times N}\,. \tag{5}$$

By eliminating $\underline{\boldsymbol{Y}}$ a relation between the two matrices, including the currents $\underline{\boldsymbol{I}}_{CA}$, $\underline{\boldsymbol{I}}_{CB}$ can be established.

$$\underline{\boldsymbol{I}}_{CA} = \underline{\boldsymbol{I}}_{CB}\,\underline{\boldsymbol{V}}_B^T \left(\underline{\boldsymbol{V}}_B\,\underline{\boldsymbol{V}}_B^T\right)^{-1} \underline{\boldsymbol{V}}_A = \underline{\boldsymbol{I}}_{CB}\,\underline{\boldsymbol{K}}(\underline{\boldsymbol{V}}_A, \underline{\boldsymbol{V}}_B) \tag{6}$$

Finally the relations

$$\Delta\underline{\boldsymbol{I}} := \underline{\boldsymbol{I}}_A - \underline{\boldsymbol{I}}_B = \underline{\boldsymbol{I}}_{CA} + \underline{\boldsymbol{I}}_{CB} \quad \text{and} \tag{7}$$

$$\underline{\boldsymbol{I}}_\lambda = \underline{\boldsymbol{I}}_A - \underline{\boldsymbol{I}}_{CA} = \underline{\boldsymbol{I}}_B + \underline{\boldsymbol{I}}_{CB} \tag{8}$$

lead to the wanted currents

$$\underline{\boldsymbol{I}}_\lambda = \begin{pmatrix} \underline{I}_{\lambda a,1} & \cdots & \underline{I}_{\lambda a,N} \\ \underline{I}_{\lambda b,1} & \cdots & \underline{I}_{\lambda b,N} \\ \underline{I}_{\lambda c,1} & \cdots & \underline{I}_{\lambda c,N} \end{pmatrix} = \underline{\boldsymbol{I}}_B + \Delta\underline{\boldsymbol{I}}\left(\boldsymbol{I}_N + \underline{\boldsymbol{K}}(\underline{\boldsymbol{V}}_A, \underline{\boldsymbol{V}}_B)\right)^{-1}\,, \tag{9}$$

where $\boldsymbol{I}_N$ denotes the identity matrix of dimension N.

IV. Parameter Estimation

A. Identifiability of the System

As parameter estimator the least squares method is applied [3]. In order to determine the parameters the so-called information matrix is to be inverted. This matrix can be used to analyse the identifiability; more precisely which operating status is necessary for a sufficient excitation. In the steady state operation the currents $\underline{i}_{\lambda a,b,c}$ yield the voltage drops $\Delta\underline{v}_{a,b,c}$ according to the following equation.

$$\begin{pmatrix} \Delta\underline{v}_a(t) \\ \Delta\underline{v}_b(t) \\ \Delta\underline{v}_c(t) \end{pmatrix} = \begin{pmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ab} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ac} & \underline{Z}_{bc} & \underline{Z}_{cc} \end{pmatrix} \begin{pmatrix} \Delta\underline{i}_{\lambda a}(t) \\ \Delta\underline{i}_{\lambda b}(t) \\ \Delta\underline{i}_{\lambda c}(t) \end{pmatrix} \tag{10}$$

After transforming (10) into a structure convenient for the LS method the data matrix for one sampling is

$$\underline{\boldsymbol{\psi}} = \tag{11}$$

$$\begin{pmatrix} \underline{I}_{\lambda a} & \underline{I}_{\lambda b} & \underline{I}_{\lambda c} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{I}_{\lambda a} & \underline{I}_{\lambda b} & \underline{I}_{\lambda c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{I}_{\lambda a} & \underline{I}_{\lambda b} & \underline{I}_{\lambda c} \end{pmatrix}.$$

Since the LS method relies on real-valued data the data matrix (11) has to be split into real and imaginary part. To show the necessary excitation of the system (10) the following system of two conductors shall be investigated.

$$\begin{pmatrix} \Delta\underline{v}_a \\ \Delta\underline{v}_b \end{pmatrix} = \begin{pmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} \\ \underline{Z}_{ab} & \underline{Z}_{bb} \end{pmatrix} \begin{pmatrix} \underline{i}_{\lambda a} \\ \underline{i}_{\lambda b} \end{pmatrix} \tag{12}$$

After transforming (12) and splitting into real (index $()_\alpha$) and imaginary part (index $()_\beta$) eq. (13) is derived.

$$\Delta\boldsymbol{v} = \boldsymbol{\Psi}\boldsymbol{p} \tag{13}$$

After collecting N measurements the components of eq. (13) are as follows.

$$\Delta\boldsymbol{v} = \begin{pmatrix} \Delta v_{a\alpha,1} \\ \Delta v_{b\alpha,1} \\ \vdots \\ \hdashline \Delta v_{a\beta,1} \\ \Delta v_{b\beta,1} \\ \vdots \end{pmatrix} \in \mathbb{R}^{4N}, \qquad \boldsymbol{p} = \begin{pmatrix} Z_{aa\alpha} \\ Z_{aa\beta} \\ Z_{ab\alpha} \\ Z_{ab\beta} \\ Z_{bb\alpha} \\ Z_{bb\beta} \end{pmatrix} \quad \text{and}$$

$$\boldsymbol{\Psi} = \begin{pmatrix} i_{a\alpha,1} & -i_{a\beta,1} & i_{b\alpha,1} & -i_{b\beta,1} & 0 & 0 \\ 0 & 0 & i_{a\alpha,1} & -i_{a\beta,1} & i_{b\alpha,1} & -i_{b\beta,1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hdashline i_{a\beta,1} & i_{a\alpha,1} & i_{b\beta,1} & i_{b\alpha,1} & 0 & 0 \\ 0 & 0 & i_{a\beta,1} & i_{a\alpha,1} & i_{b\beta,1} & i_{b\alpha,1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \in \mathbb{R}^{4N\times 6}$$

Analysing the identifiability of the impedance matrix in eq. (12) results in checking the rank of the symmetrical information matrix

$$\boldsymbol{\Psi}^T\boldsymbol{\Psi} = \begin{pmatrix} \alpha & 0 & \gamma & \delta & 0 & 0 \\ & \alpha & -\delta & \gamma & 0 & 0 \\ & & \alpha+\beta & 0 & \gamma & \delta \\ & & & \alpha+\beta & -\delta & \gamma \\ & & & & \beta & 0 \\ & & & & & \beta \end{pmatrix}, \tag{14}$$

whereas

$$\alpha = \sum_{k=1}^{N} i_{a\alpha,k}^2 + i_{a\beta,k}^2 \qquad \gamma = \sum_{k=1}^{N} i_{a\alpha,k}\, i_{b\alpha,k} + i_{a\beta,k}\, i_{b\beta,k}$$

$$\beta = \sum_{k=1}^{N} i_{b\alpha,k}^2 + i_{b\beta,k}^2 \qquad \delta = \sum_{k=1}^{N} i_{a\beta,k}\, i_{b\alpha,k} - i_{a\alpha,k}\, i_{b\beta,k}\,.$$

In the steady state operation the currents $\underline{i}_{a,k}$ and $\underline{i}_{b,k}$ may be as follows.

$$\underline{i}_{a,k} = \hat{I}_a e^{j\,\omega\, k\,\Delta T} \tag{15a}$$

$$\underline{i}_{b,k} = \hat{I}_b e^{j(\omega\, k\,\Delta T - \varphi)} \tag{15b}$$

These assumptions yield the elements of matrix (14).

$$\alpha = N\,\hat{I}_a^2 \qquad \gamma = N\,\hat{I}_a\,\hat{I}_b\,\cos(\varphi)$$

$$\beta = N\,\hat{I}_b^2 \qquad \delta = N\,\hat{I}_a\,\hat{I}_b\,\sin(\varphi)$$

The determinant of $\boldsymbol{\Psi}^T\boldsymbol{\Psi}$ is

$$\det\left(\boldsymbol{\Psi}^T\boldsymbol{\Psi}\right) = (\alpha+\beta)^2(\gamma^2+\delta^2-\alpha\,\beta)^2\,. \tag{16}$$

In the case of an excitation like (15) (steady state) the determinant (16) vanishes and the system is not identifiable.

The analysis of the identifiability of the system yields that it is insufficient to use measurement data of the steady state. The consequence is to use fault records for identifications as was mentioned in [4] as well. During these events DC-parts occur in the currents and lead to a regular information matrix.

B. Identification of the Impedance Matrix

In order to describe a fault correctly, derivatives of the time-varying current phasors need to be built. By using complex-valued signals according to (1) the derivative of a time-varying phasor is given by

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\underline{x}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\underline{X}(t)\, e^{j\,\omega\,t}\right) \\ &= \left(\dot{\underline{X}}(t) + j\,\omega\,\underline{X}(t)\right) e^{j\,\omega\,t}\,. \end{aligned} \tag{17}$$

With respect to this and the definition of a time-varying phasor (1) an estimation equation using the voltage drops $\Delta\underline{v}_{a,b,c}$ is derived at time t_i as follows:

$$\Delta\underline{\boldsymbol{v}}_i = \underline{\boldsymbol{\psi}}_i\,\boldsymbol{p}, \tag{18}$$

where

$$\underline{\boldsymbol{\psi}}_i^T = \begin{pmatrix} \underline{I}_{\lambda a,i} & 0 & 0 \\ j\omega\underline{I}_{\lambda a,i}+\dot{\underline{I}}_{\lambda a,i} & 0 & 0 \\ \underline{I}_{\lambda a,i} & \underline{I}_{\lambda a,i} & 0 \\ j\omega\underline{I}_{\lambda b,i}+\dot{\underline{I}}_{\lambda b,i} & j\omega\underline{I}_{\lambda b,i}+\dot{\underline{I}}_{\lambda b,i} & 0 \\ \underline{I}_{\lambda c,i} & 0 & \underline{I}_{\lambda a,i} \\ j\omega\underline{I}_{\lambda c,i}+\dot{\underline{I}}_{\lambda c,i} & 0 & j\omega\underline{I}_{\lambda a,i}+\dot{\underline{I}}_{\lambda a,i} \\ 0 & \underline{I}_{\lambda b,i} & 0 \\ 0 & j\omega\underline{I}_{\lambda b,i}+\dot{\underline{I}}_{\lambda b,i} & 0 \\ 0 & \underline{I}_{\lambda c,i} & \underline{I}_{\lambda b,i} \\ 0 & j\omega\underline{I}_{\lambda c,i}+\dot{\underline{I}}_{\lambda c,i} & j\omega\underline{I}_{\lambda b,i}+\dot{\underline{I}}_{\lambda b,i} \\ 0 & 0 & \underline{I}_{\lambda c,i} \\ 0 & 0 & j\omega\underline{I}_{\lambda c,i}+\dot{\underline{I}}_{\lambda c,i} \end{pmatrix},$$

$$\boldsymbol{p} = \begin{pmatrix} R_{aa} \\ L_{aa} \\ R_{ab} \\ L_{ab} \\ R_{ac} \\ L_{ac} \\ R_{bb} \\ L_{bb} \\ R_{bc} \\ L_{bc} \\ R_{cc} \\ L_{cc} \end{pmatrix}, \qquad \Delta\underline{\boldsymbol{v}}_i = \begin{pmatrix} \Delta\underline{V}_{a,i} \\ \Delta\underline{V}_{b,i} \\ \Delta\underline{V}_{c,i} \end{pmatrix}.$$

The relation of the elements of the parameter vector $\boldsymbol{p}$ in (18) to the impedances used in Fig. 2 is $\underline{Z}_{ij} = R_{ij} + j\,\omega\,L_{ij}$. Since $e^{j\,\omega\,t}$ occurs in the complex-valued signals as well as in their derivatives this term was canceled in (18) and hence the equations are constituted only of the time-varying phasors denoted with underlined capital letters.

After applying the well-known LS method, the parameter vector $\boldsymbol{p}$ can be calculated as follows.

$$\Delta\underline{\boldsymbol{v}} = \underline{\boldsymbol{\Psi}}\,\boldsymbol{p}, \quad \Delta\underline{\boldsymbol{v}} \in \mathbb{C}^{3N},\ \underline{\boldsymbol{\Psi}} \in \mathbb{C}^{3N\times 12}$$

$$\begin{pmatrix} \Delta\underline{\boldsymbol{v}}_1 \\ \vdots \\ \Delta\underline{\boldsymbol{v}}_N \end{pmatrix} = \begin{pmatrix} \underline{\boldsymbol{\psi}}_1 \\ \vdots \\ \underline{\boldsymbol{\psi}}_N \end{pmatrix} \boldsymbol{p} \tag{19}$$

Since the so called data matrix $\underline{\boldsymbol{\Psi}}$ is complex, the real-valued parameter vector can be derived according to

$$\hat{\boldsymbol{p}} = \begin{pmatrix} \mathrm{Re}\{\underline{\boldsymbol{\Psi}}\} \\ \mathrm{Im}\{\underline{\boldsymbol{\Psi}}\} \end{pmatrix}^{+} \begin{pmatrix} \mathrm{Re}\{\Delta\underline{\boldsymbol{v}}\} \\ \mathrm{Im}\{\Delta\underline{\boldsymbol{v}}\} \end{pmatrix}, \qquad (20)$$

where '$^+$' denotes the Moore-Penrose matrix inverse.

V. Verification of the Algorithm and Results

A. Simulative Experiment

In order to verify the algorithm simulations were done in MATLAB® and SIMULINK®. The algorithms to derive the time-varying phasors as well as the identification were implemented in Matlab. In Simulink a model for generating test data was built. As can be seen in Fig. 3 this model contains a three-phase voltage source on the left-hand side, the adjoining Line I, which is to be identified and a second line, which is loaded by $R_{La,b,c}$. On the right-hand side a fault can occur on the bus bar.

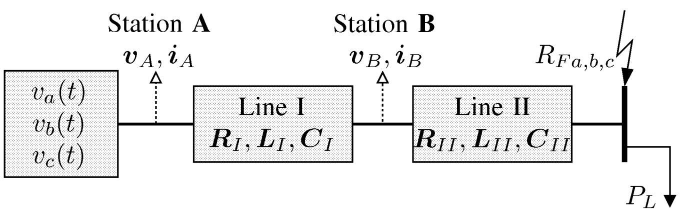

Fig. 3. Schematic of the Simulink model with the Line I to be identified

The simulation was done by using the parameters stated in Table I, whereas the lines are described with the matrices (21)-(23). Both lines have equal per-unit-length line parameters but different lengths l_I and l_{II}.

TABLE I
SIMULATION PARAMETERS

voltage	$V_{a,b,c} = 380\,\mathrm{kV}/\sqrt{3}$
line lengths	$l_I = 2\,l_{II} = 100\,\mathrm{km}$
resistance matrix	$\boldsymbol{R}_I = l_I\,\boldsymbol{R}'$ $\boldsymbol{R}_{II} = l_{II}\,\boldsymbol{R}'$
inductance matrix	$\boldsymbol{L}_I = l_I\,\boldsymbol{L}'$ $\boldsymbol{L}_{II} = l_{II}\,\boldsymbol{L}'$
capacitance matrix	$\boldsymbol{C}_I = l_I\,\boldsymbol{C}'$ $\boldsymbol{C}_{II} = l_{II}\,\boldsymbol{C}'$
fault resistance	$R_{Fa,b,c} = \;\; 3.8\,\Omega$
load	$P_L = 580\,\mathrm{MW}$

$$\boldsymbol{R}' = \begin{pmatrix} 58.221 & 47.104 & 47.063 \\ 47.104 & 58.224 & 47.104 \\ 47.063 & 47.104 & 58.221 \end{pmatrix} \mathrm{m}\Omega/\mathrm{km} \qquad (21)$$

$$\boldsymbol{L}' = \begin{pmatrix} 1.6876 & 0.8652 & 0.7267 \\ 0.8652 & 1.6876 & 0.8652 \\ 0.7267 & 0.8652 & 1.6876 \end{pmatrix} \mathrm{mH}/\mathrm{km} \qquad (22)$$

$$\boldsymbol{C}' = \begin{pmatrix} 11.305 & -2.446 & -0.820 \\ -2.446 & 11.775 & -2.446 \\ -0.820 & -2.446 & 11.305 \end{pmatrix} \mathrm{nF}/\mathrm{km} \qquad (23)$$

These parameters arise from the standard example of the *SimPowerSystems*-Toolbox [5]. The lines considered here have no ground wires and are operated at $50\,\mathrm{Hz}$. In order to implement the line, a block simulating a mutual inductance was used to model the impact of parameters (21) and (22). The capacitances between the wires and ground can be determined from (23). They are divided into one half at the beginning and at the end of the line and finally implemented as admittances (cf. Fig. 2).

The typical currents in Line I during a phase 'b'-to-ground fault at the bus bar are shown in Fig. 4. Like in practice the

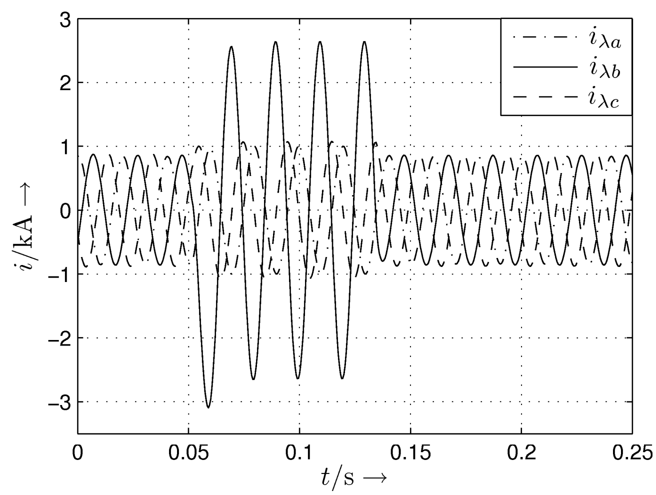

Fig. 4. Currents through the healthy Line I during a ground fault of phase 'b' at the bus bar cf. Fig. 3

current and voltage signals at stations A and B (cf. Fig. 3) are available and "sampled" in the simulation every $0.5\,\mathrm{ms}$. To get more realistic "samples" the data acquisition process is modeled as well. Therefore the impact of the analog-to-digital converters is considered. The simulated values are quantized due to the analog to digital conversion and have an additional, equally distributed error of $\pm$ half a quantization step. During testing a protection relay with reference signals it was found out that a further error occurs. The inaccuracy of the three low-order bits cause an additive normally distributed error (± 4 quantization steps).

Finally a phase 'b'-to-ground fault at the bus bar is used for testing the algorithms presented in sections III and IV. As results of the identification the relative errors of the parameters

$$\delta\hat{\boldsymbol{p}}_i/\% = 100\frac{\hat{\boldsymbol{p}}_i - \boldsymbol{p}_i}{\boldsymbol{p}_i} \qquad (24)$$

are shown in Fig. 5.

For simplification it was assumed that the diagonal elements of matrices (21) and (22) are equal and hence denoted in Fig. 5 with R_{aa} and L_{aa} respectively. As it is well-known the fault inception time has an effect on the DC-part of the currents and it seems that it has an effect on the estimated parameters as well. It is obvious that the coupling resistances are estimated inferior to the coupling inductances. It is quite probably that the reason is the major magnitude of the reactances. That means that the assumed disturbance model of the analog to digital conversion has a major influence on the resistances. Furthermore it is striking that $\hat{R}_{ac}$ was estimated inferior to

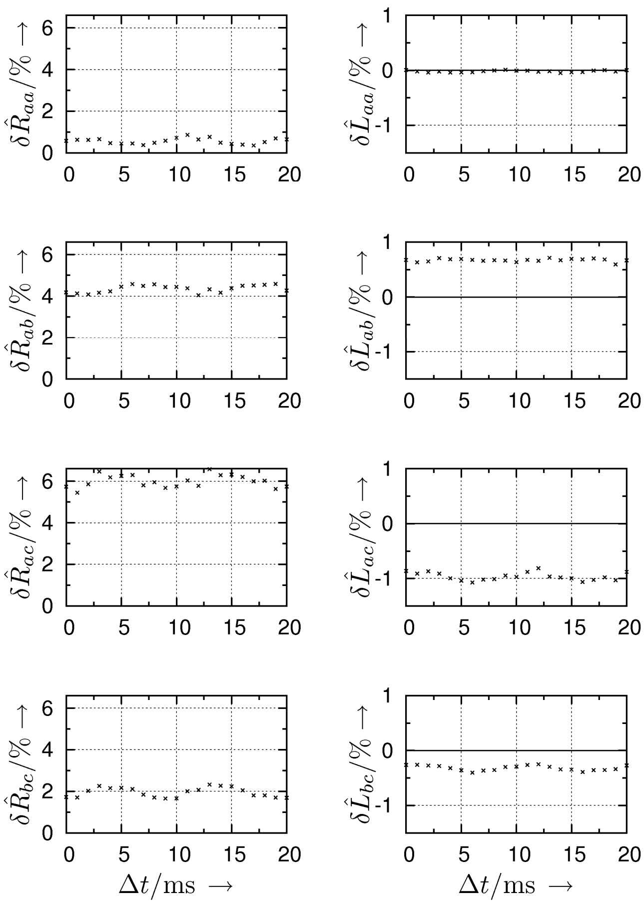

Fig. 5. Relative errors of the parameters depending on the time difference Δt between the last zero-crossing and the fault inception time

the other parameters $\hat{R}$, $\hat{R}_{ab}$ and $\hat{R}_{bc}$. Generating a phase 'a' or phase 'c' to ground fault yield no bad values for $\delta\hat{R}_{bc}$ respectively $\delta\hat{R}_{ab}$ compared to the other ones as one would expect. This phenomenon is further to investigate.

B. Measurements on a Three Phase Dynamic Network Model

After acceptable simulation results measurements were done on a physical three phase dynamic network model in our lab. In the 220 kV-level the model consists of two turbo generators each 63 MVA, a 150 km/220 kV double line and three substations. It is equipped with digital protective relays and a remote control system. The voltage scale is $r_V = 500$ and the current scale is $r_I = 20$. Fig. 8 shows the model structure.

The objective of the experiment was to estimate the parameters of Line I (Line II was insulated) by generating a fault record. Line I was loaded by a constant load at side A. At an arbitrary point of time a single phase fault was generated on the bus bar in substation A for a specific time.

1) Experiment: By consideration of Fig. 8 the procedure shall be explained. The whole network model is supplied via substation C at the bottom of Fig. 8. Switching on E2 and C2 in Block II supplies substation B. Switching on B3, B1, A1 and A3 provides the opportunity to load the Line I via Block I of substation C. In this case Line I was loaded by switching on C1 and V3. Furthermore a facility was programmed which generates arbitrary phase-to-ground faults for 100 ms at the bus bar of station A. The electrical values were recorded time synchronously. Therefore the current and voltage transducers in A1 and B1 were used.

2) Measurement Values: The electrical values were acquired with a sampling time of $T = 10\,\mu$s. Due to errors of the analog-to-digital converters or other disturbing couplings the measured values of currents (cf. Fig. 6) and of voltages (cf.

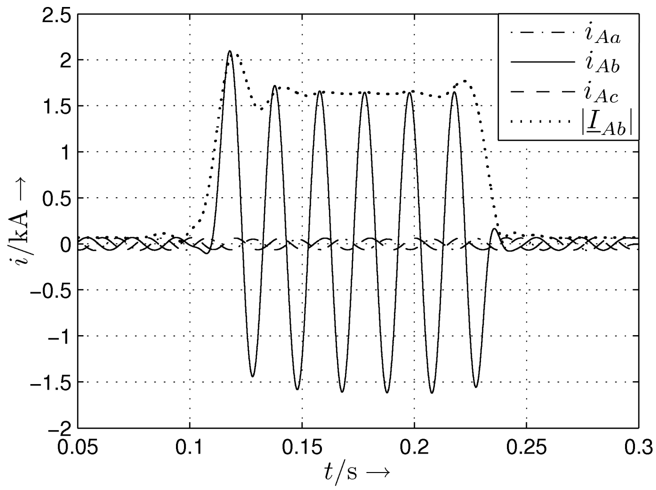

Fig. 6. Filtered currents at station A during a phase 'b'-to-ground fault. 4th order butterworth filter with a cutoff frequency of 100 Hz (primary values)

Fig. 7) had to be filtered. In Fig. 6 additionally the absolute value of the time-varying phasor $\underline{I}_{Ab}(t)$ (eqs. (1) and (3)) is displayed as well. In principle the absolute value is the envelope of the time signal $i_{Ab}(t)$ [2].

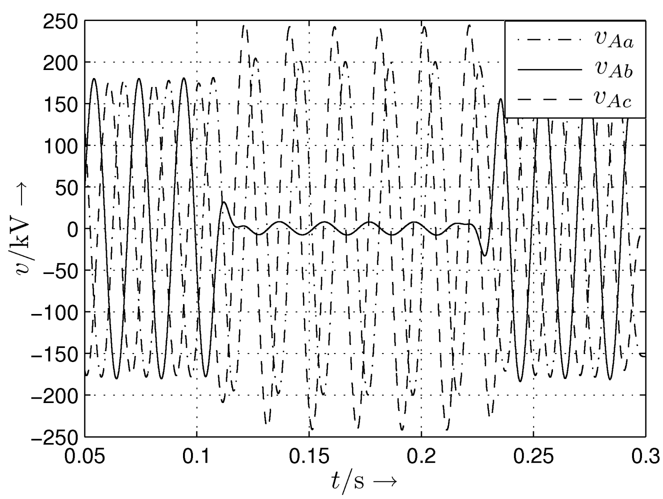

Fig. 7. Filtered voltages at station A during a phase 'b'-to-ground fault. 4th order butterworth filter with a cutoff frequency of 100 Hz (primary values)

The voltage and current signals were recorded for every single phase-to-ground fault ('a' - GND, 'b' - GND, 'c' - GND).

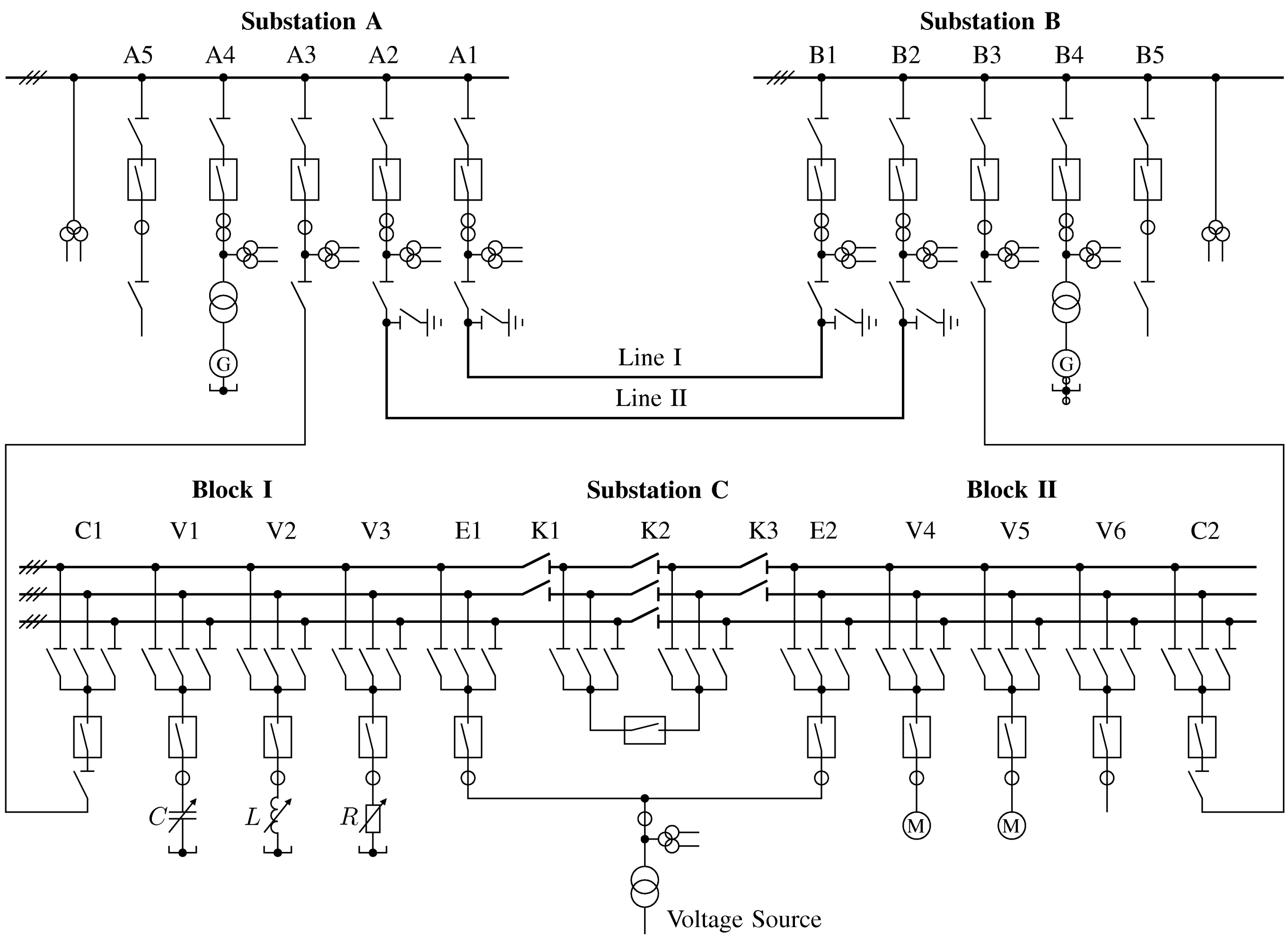

Fig. 8. Three phase dynamic network model with three substations and a 150 km/220 kV double line

3) Estimated Line Parameters: In order to assess the estimation results it is necessary to have reference values of the line parameters. Therefore additional measurements were done to determine the self impedances and coupling impedances. These reference values were determined by measuring every phase-to-ground loop and every phase-to-phase loop respectively. The second column of Table II contains the references denoted with $\boldsymbol{p}_i/\Omega\,\mathrm{km}^{-1}$. At the first step the self impedances are assumed to be equal. The remaining columns contain the estimated parameters denoted with $\hat{\boldsymbol{p}}_i/\Omega\,\mathrm{km}^{-1}$ and their relative errors according to eq. (24) as well. Like in the simulation the reactances are estimated better than the resistances (cf. Fig. 5). Furthermore it is striking that couplings between the two conductors which are not drained by the short-circuit current (e. g. R'_{ac} during a phase 'b'-to-ground fault) are estimated inferior to the impedances belonging to the conductor which is drained by the short-circuit current (R'_{ab} and R'_{bc} in this case). These cases are marked in Table II. Light grey means a relatively good estimated parameter; dark grey marks a bad estimated parameter.

TABLE II
ESTIMATED PARAMETERS ASSUMING EQUAL SELF IMPEDANCES (PRIMARY VALUES, 220 kV-LEVEL)

	ref.	fault 'a'-GND		fault 'b'-GND		fault 'c'-GND	
	$\boldsymbol{p}_i$	$\hat{\boldsymbol{p}}_i$	$\delta\hat{\boldsymbol{p}}_i$	$\hat{\boldsymbol{p}}_i$	$\delta\hat{\boldsymbol{p}}_i$	$\hat{\boldsymbol{p}}_i$	$\delta\hat{\boldsymbol{p}}_i$
R'	0.110	0.111	1.5	0.106	-3.6	0.112	2.3
X'	0.587	0.581	-0.9	0.581	-1.1	0.581	-1.0
R'_{ab}	0.007	0.009	32.4	0.004	-50.7	0.002	-72.1
X'_{ab}	0.340	0.340	-0.2	0.340	-0.1	0.336	-1.4
R'_{ac}	0.008	0.010	31.7	0.042	447.5	0.010	23.1
X'_{ac}	0.340	0.342	0.5	0.338	-0.7	0.340	-0.3
R'_{bc}	0.009	0.004	-55.5	0.005	-45.0	0.008	-12.0
X'_{bc}	0.349	0.377	8.1	0.337	-3.3	0.341	-2.2

Again it has to be remarked that the reactances are estimated more accurately than the resistances since their values are much bigger (e. g. $X_{ab} = 150\,\mathrm{km}\,X'_{ab} = 51.00\,\Omega \gg R_{ab} = 1.05\,\Omega$). Hence measuring errors have less influence on the reactances.

In Table III the estimated parameters are shown by assuming different self impedances. Like in the case of assuming equal self impedances (Table II) the reactances are estimated more accurately than the resistances. The grey-colored cells in Table III show again the fact described above.

Compared with the results of the simulation the quality of the estimated reactances is similar in cases marked with light grey in Tables II and III. Due to quite small resistance values and hence a wide influence of measurement errors the quality of these estimates is inferior to the simulation results.

TABLE III
ESTIMATED PARAMETERS ASSUMING DIFFERENT SELF IMPEDANCES (PRIMARY VALUES, 220 kV-LEVEL)

	ref.	fault 'a'-GND		fault 'b'-GND		fault 'c'-GND	
	$\boldsymbol{p}_i$	$\hat{\boldsymbol{p}}_i$	$\delta\hat{\boldsymbol{p}}_i$	$\hat{\boldsymbol{p}}_i$	$\delta\hat{\boldsymbol{p}}_i$	$\hat{\boldsymbol{p}}_i$	$\delta\hat{\boldsymbol{p}}_i$
R'_{aa}	0.110	0.112	1.5	0.080	-27.3	0.180	63.8
X'_{aa}	0.587	0.581	-0.9	0.530	-9.6	0.554	-5.6
R'_{bb}	0.109	0.177	62.7	0.106	-2.5	0.070	-35.6
X'_{bb}	0.592	0.540	-8.7	0.581	-1.9	0.524	-11.4
R'_{cc}	0.106	0.024	-77.2	0.153	43.3	0.112	5.6
X'_{cc}	0.597	0.512	-14.3	0.540	-9.7	0.581	-2.7
R'_{ab}	0.007	0.009	21.2	0.004	-43.7	0.018	153.8
X'_{ab}	0.340	0.340	-0.2	0.341	0.0	0.268	-21.2
R'_{ac}	0.008	0.009	21.2	0.052	571.4	0.008	9.7
X'_{ac}	0.340	0.343	0.7	0.286	-15.9	0.340	-0.2
R'_{bc}	0.009	0.003	-63.5	0.005	-49.5	0.008	-17.1
X'_{bc}	0.349	0.285	-18.1	0.338	-3.0	0.342	-1.9

By means of the estimated parameter $\hat{R}'_{ab} = 18\,\mathrm{m\Omega/km}$ (Table III, fault 'c'-GND) the difficulty to identify a coupling resistance shall be demonstrated. The relative error of this estimate is $\delta R'_{ab} = 153.8\,\%$; the absolute error is $\Delta\hat{R}'_{ab} = \hat{R}'_{ab} - R'_{ab} = 11\,\mathrm{m\Omega/km}$ respectively. With respect to the voltage and current scales (introduced in section V-B) and the line length of $l = 150\,\mathrm{km}$ the per-unit-length absolute error $\Delta\hat{R}'_{ab} = 11\,\mathrm{m\Omega/km}$ can be transformed into the physical model domain.

$$\Delta\hat{R}_{ab,M} = \Delta\hat{R}'_{ab}\, l\, \frac{r_I}{r_V} = 66\,\mathrm{m\Omega} \tag{25}$$

These quite small values can be caused by different contact resistances during the experiment. Hence they have a wide influence on the estimates (dark grey colored in Tables II and III).

VI. CONCLUSION

This paper deals with the modelling of unsymmetrical transmission lines and their identification using the least squares method. The well-known impedance matrix is used to model a not completely transposed transmission line. The extension of the impedance system with admittances serves for an approximated calculation of the actual currents trough the impedances. Using time-varying phasors is advantageous since noise, higher frequencies and the fundamental frequency as well are removed. In a first step test data is generated by simulating a line model with known parameters and an occurring single phase-to-ground fault. After applying the algorithms the parameters are estimated within an acceptable error range. Due to the different order of dimension in the resistances and reactances a sophisticated validation of the results is necessary. The resistances are identified within a relative error range of $\pm 6\%$. Reactances are estimated much better; in a range of $\pm 1\%$.

In order to verify the algorithm measurements were done on a physical three phase dynamic network model. By using the generated fault records every parameter of the impedance matrix was able to identify. As was expected the quality of the estimates of the resistances is quite bad. Since the resistances of the network model are quite small ($< 100\,\mathrm{m\Omega}$) contact resistances have a wide influence on the estimates. In certain cases the reactances are able to identify within an error range of $\pm 3\%$. In general it became apparent while analysing the experimental results that the two couplings belonging to a conductor drained by the single phase short-circuit current are estimated more accurately than the remaining coupling between the healthy conductors.

The obtained parameters can be used to parameterize protection relays, for power system analysis as well as for a more accurate fault location.

Since up to now the measurements were assumed to be sampled time-synchronised the next steps in this project are to develop algorithms to resynchronise real fault records and afterwards testing the identification using this data.

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