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The Impact of a Given Trading Limit on a Two-Area Test System

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***Abstract*—This paper uses methods from stochastic analysis and stochastic modeling to determine the impact of a certain trading limit on the transfer between the two areas of a benchmark two-area power system. We also try to state which uncertainties are important to consider when calculating this power transfer.**

***Index Terms*—Ornstein-Uhlenbeck Process, Power Market, Power Transfer, Trading Limit.**

I. Introduction

In a deregulated electricity market with area-pricing it is often desirable to be able to transfer as much electric power as possible between the areas of the system, since this is the most cost efficient way to use the power grid. However, due to limitations in the system there is a maximal amount of power that is possible to transfer between the areas of the system, this limit is called the total transfer capacity (TTC). When the transmission system operator (TSO) is setting the trading limits between the different areas of a multi-area power system he cannot, however, consider only the TTC but also has to consider the uncertainties in the system. In this paper we will try to give an example of what impact a certain trading limit can have on the transfer between two areas of a power system and which uncertainties are important to consider when calculating this power transfer. The treatment in this paper does not consider any component specific power system but focuses on the trading between different actors at the market and the resulting power flows in the system. An electricity market is somewhat different from other markets since electric power is generated at the same time that it is consumed. A common market-structure is given below.

The Market

Electrical power is traded in energy per trading period. Usually, trading is performed hourly as on the European Energy Exchange [1], PJM [2], and in the Nordic system [3]. Also, half-hour periods are used in some systems such as for example in UK [4]. To resemble the Nordic power market, in this project we assume a trading period of one hour, but the suggested models and methods can be applied to any period length. The market structure assumed in this project includes the following possibilities of physical trading of power:

- Day-ahead market: This is where most of the electric energy is traded. The day-ahead market closes at noon on the day before the trading period during which the actual exchange of energy will take place, thus the name day-ahead market. Hence, energy at this market is traded 12 to 36 hours before it is actually delivered, and accurate forecasts of the electric energy consumption and the state of the power grid are thus needed.
- Intra-day trading: Intra-day trading offers a possibility to adjust the traded quantities from the day-ahead market according to updated forecasts or to non-accepted bids on the day-ahead market. It is possible to continuously trade power on this market after that the day-ahead market has closed and the traded quantities and prices have been published, until one hour before the start of the actual hour.
- Real-time balancing market: On this market, the system operator can trade power in order to keep a balance between production and consumption in the system when necessary. This is a part of the frequency control. In situations with excess power, the system operator can accept bids corresponding to decreasing the net production in the system (downward balancing). In the opposite situation, the system operator calls bids corresponding to increasing the net production in the system (upward balancing). Bids to this market can be submitted shortly before the start of the operational hour. Let's assume that in our market bids can be submitted up till 10 minutes before the start of the operational hour (as is the case on the Nordic market [3]). These bids are then activated during the operation hour in order to keep the frequency close to the nominal frequency. Bids are usually activated in order of their cost but some considerations will also have to be taken to the stability constraints, i.e. the NTC.
- Imbalance settlement: The difference between the actual energy consumption / production and the traded quantity is called an imbalance. At the end of each hour the transmission system operator (TSO) checks the imbalance for each actor on the electricity market. If a consumer has consumed more than he has bought on the ahead markets he has a negative imbalance that he will have to pay for, if he on the other hand has consumed less power than

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he has bought on the ahead markets he will get payed for his positive imbalance. The price for positive and negative imbalances are the same, if not, say that the price for a positive imbalance is less than the price for a negative imbalance, then it will be profitable to buy more power than the actor's forecast, which is not totaly certain, tells the actor that he/she will use in order to avoid a negative imbalance. This would render much problems for the TSO, but is avoided by keeping the same prices on both positive and negative imbalances.

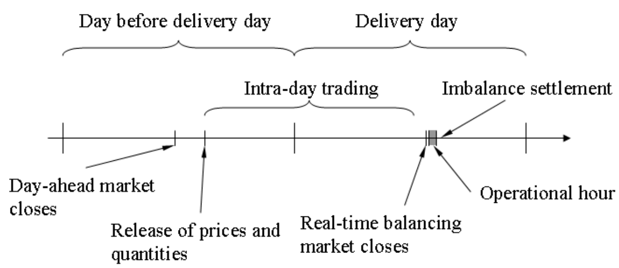

Fig. 1. Time line for physical trading of electricity.

A time-line showing the above trading possibilities is shown in Figure 1. As mentioned above the large quantities are usually traded on the day-ahead market, while the intra-day market can be used to decrease the imbalance between quantities traded on the day-ahead market and the expected production according to updated production forecasts. However when analyzing the transfers between the areas of the system the market whose structure is of primary interest is the regulating power market. This market is quite unique for the Nordic system. In most electricity markets this is handled by an automatic system called Automatic Generation Control (AGC). AGC is preferable when the underlying grid has a meshed structure in which case analyzing the resulting power flows when altering the production in a production unit becomes very complicated. However in some systems, like the Nordic system the areas are not meshed and the secondary control can be allowed to be a market which will lead to a more efficient use of the system assets than AGC would.

In this paper we will, as mentioned above, investigate the power flows between the two areas of a two area benchmark power system with few actors. First, the system and the actors will be presented, then some arising questions concerning which uncertainties are important will be put, and each question will be discussed in the following sections. The last section is devoted to a numerical example where, using Monte Carlo Simulations, the distribution of the maximal power flow between the areas of our test system during a specified time-period will be estimated.

II. Arising Questions

Assume that we have two areas, Area 1 and Area 2. In Area 1 the producer A has a hydropower plant. In Area 2 the trader B sell energy to a small village and a factory, and the producer C has a fossil fuel power plant. The price of electricity in Area 1 is less than the price in Area 2 and therefore B wants to by as much power as possible to cover the demand of his customers for from A in Area 1. The demand, $(D_t^B,\ 0 \leq t \leq 24)$ MW, of the customers of B during the day is modeled by:

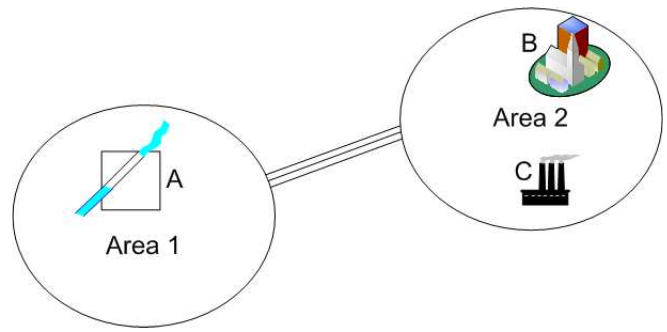

Fig. 2. A schematic sketch of the system considered.

$$D_t^B = m_t + X_t = m_t + e^{-\alpha t}(D_0 - m_0 + \int_0^t \sigma_s e^{\alpha s} dB_s), \quad (1)$$

where m_t is the mean of the load and $(B_t\ ,t \geq 0)$ is standard Brownian motion [5], This model was used to model the electric power consumption in the Swedish power system in [6]. As an example a similar model is commonly used to model temperature fluctuations when pricing weather derivatives [7]. Since the electric power consumption is very temperature dependent it will thus seem reasonable to use a similar model for the electric power consumption.

The energy consumed during each hour (trading period) is

$$S_k^B = \int_{k-1}^{k} D_t^B dt \text{ MWh, for } k = 1, 2, ..., 24. \quad (2)$$

However, assume that the transmission system operator (TSO) tell A that he is not allowed to sell more than τ_k MWh during hour k, for $k = 1, 2, ..., 24$, to consumers in Area 2, due to limitations in the power system. When B hears about this he decides to buy τ_k MWh from A and the remaining $S_k^B - \tau_k$ MWh from C during hour k, for $k = 1, 2, ..., 24$. Some important questions that arises when contemplating the event described above are:

1) How will the trading of B at the ahead-markets (i.e. the day-ahead market and the intra-day trading market) look when B does an optimal deal?
2) How will the variations within the hour be balanced? Only by using frequency control or stockpiled at the real-time balancing market?
3) Where in the system is the frequency control placed?
4) What will be the distribution of the maximal power flow between the two areas?

These questions will now be dealt with, one for each of the following four sections.

III. THE TRADING OF B AT THE AHEAD-MARKETS

In some power systems the TSO does not make instantaneous measurements of the electric power consumption at the nodes but rather measurements of the energy consumed at the nodes of the system in short time intervals (often of lengths 10-30 minutes). Lets assume that B gets information of the total energy consumption of its customers for every 10 minutes that pass. This means that B will never have perfect information about its customers power consumption, unless it is constantly zero for an entire 10 minute interval which seems very unlikely.

When forecasting the load, B will want to forecast the hourly energy consumptions ($S_k^B = \int_{k-1}^{k} D_t^B dt,\ k = 1, ..., 24$) rather than ($D_t^B,\ 0 \leq t \leq 24$), since this is the traded quantity. Let ($\mathcal{F}_t,\ t > -\infty$) be the sigma algebras containing the information that B has at time t (observe that this is a filtration for a discrete time process). What B wants is to find first $E\left(S_k^B|\mathcal{F}_{-12}\right)$ for trading at the day-ahead market and then $E\left(S_k^B|\mathcal{F}_{k-2}\right)$ for trading at the intra-day market.

Since $I_t^B = \int_0^t D_s ds$ does not have the Markov property like X_t does, we will have to use all the measurements done up to the time of the decision when calculating the sought mean values. That I_t^B does not have the Markov property can be realized by the fact that if S_k^B is around $\int_{k-1}^{k} m_t dt$ and S_{k-1}^B is very small then $E[S_{k+1}^B|S_k^B, S_{k-1}^B]$ is well above $\int_{k}^{k+1} m_t dt$, but if S_{k-1}^B is very large then $E[S_{k+1}^B|S_k^B, S_{k-1}^B]$ is well below $\int_{k}^{k+1} m_t dt$, hence S_t^B does not have the Markov property and thus cannot be a diffusion.

How can then the above sought expected values be found? Well, we start by noting that by Itô's formula of Partial Integration,

$$\int_0^T D_t dt = \int_0^T m_t dt + \frac{1}{\alpha}\left(\left(D_0 - m_0\right)\left(1 - e^{-\alpha T}\right) + \sigma e^{-\alpha T} \int_0^T \left(e^{\alpha T} - e^{\alpha s}\right) dB_s\right) \quad (3)$$

where the first integral is the deterministic part, $(D_0 - m_0)\left(1 - e^{-\alpha T}\right)$ originates from the initial distribution of (D_t ,$t \geq 0$) and the last integral is a purely stochastic part depending on the trajectory of the Brownian motion (B_t ,$t \geq 0$). Observe that by $t = 0$ in this part we mean the time that the measurements started. Since this is very long ago (i.e. T is very large), and we are interested in integrals over intervals of the form $(T, T + \Delta)$, where Δ is 10 minutes, the part originating from the initial distribution of (D_t ,$t \geq 0$) can be neglected. Note also that $\int_0^t \sigma_s e^{\alpha s} dB_s$ is a stochastic integral with a $\mathcal{F}_s^B$-measurable integrand and hence, normally distributed, and that the same reasoning also apply for $\int_0^T \left(e^{\alpha T} - e^{\alpha s}\right) dB_s$. Now, since the sum of, or the difference between, any combinations of integrals of the form (3) over intervals $(T_1, T_1 + \Delta), (T_2, T_2 + \Delta), ..., (T_n, T_n + \Delta)$ with $T_i, i = 1, 2, ..., n$ sufficiently large will be normally distributed, the vector of increments in energy-consumption during the 10-minute intervals will be a normal vector. Hence, finding the conditional mean values above is reduced to computing an integral.

IV. BALANCING VARIATIONS WITHIN THE HOUR

When the consumption in the system increases the system frequency [8], which in the Nordic system is normally 50Hz [9], will decrease. This causes power plants that have a certain amount of production capacity reserved for what is known as the primary frequency control to increase their production. In the same manner a decrease in consumption or an increase in production will lead to an increased system frequency and thus that the power plants with a capacity reserved for primary control decrease their production. All power plants that are used in the primary control have a certain gain, R[MW/Hz], which indicates how the generation in this plant is changed when the frequency changes, the production of the primary control will thus only depend on the system frequency. The distribution of the gain amongst the different areas of a power system will thus decide how the transfers between the areas changes when the load changes in some part of the system or one of the generating units is suddenly taken out of service. Observe that the primary control only terminates the change of frequency. When this angular acceleration/deceleration is stopped we will end up with a different system frequency. What then happens is that the secondary control steps in to take the frequency back to its nominal value. As mentioned above this secondary control is a market called the regulating power market in the Nordic power system. How this market works will thus have a influence on the size of the transfers in the system. If the TSO, which monitor the transfers through all the critical sections in the system can steer the secondary control to always keep the transfers below some limit a smaller limit for Transfer Reliability Margin (TRM) can be kept than if the no transfer limit is considered when activating bids from the real-time balancing market. In this paper the aim is to find the impact of a certain trading limit on the transfer between the two areas of a small test power market and the ability for the TSO to stockpile energy at the real-time market will not be considered.

V. LOCATION OF THE POWER PLANTS THAT CONTRIBUTE TO THE FREQUENCY CONTROL

In close connection to the subject of the previous section is the question of where the frequency control power plants are located and in what area available capacity can be bought at the real-time market. If for example half of the frequency control reserve is located in Area 1 then, when neglecting losses, half of the load variations will be compensated from Area 1 and thus contribute to the power flow between the two areas. In reality, with the size of the power systems of today, the TSO often does not know the distribution, or even the size, of the frequency control reserve in the system. Therefore, when simulating or calculating the power transfers between the different areas of a multi-area system one has to use a multi-dimensional probability distribution, with one dimension for each area in the system, to represent the frequency control reserve in the system. In this paper, however, there are only two areas and the problem becomes less complicated.

VI. The distribution of the maximum power flow between Area 1 and Area 2

As we have seen, the distribution of the r.v. $P_{k,max} = \sup_{k-1\le t\le k} |P_t^{1,2}|$ may depend on, how B trades on the ahead-markets, how the variations in the demand of B's customers are taken care of, the placement of the frequency control power plants and from where (if at all) electric energy can be bought at the real-time market, and of course what the trading limit τ_k is set to be.
In this section we are going to explain how to find this distribution for our simple test system. We assume that the deviations of D_t^B from its mean value are small enough to be handled by the frequency control or rather that the frequency control reserve is large enough to handle the load variations, that $100\beta\%$ of the frequency control reserve is located in Area 1, and that D_t^B is modeled as described above.

Let V_k^C be the amount of energy that B has bought from C at the ahead markets, we also assume that $V_k^C > 0$ (i.e. that the expected (forecasted) demand of the customers of B for the hour in question is more than τ_k MWh). C will probably want to run its power plant on a constant rate so that its production throughout the hour will be V_k^C. Now, the power flow between Area 1 and Area 2, $P_t^{1,2}$ can be written as

$$P_t^{1,2} = \tau_k + \beta(D_t^B - V_k^C). \tag{4}$$

Hence $P_{k,max} = \sup_{k-1\le t\le k} |\tau + \beta(D_t^B - V_k^C)|$. Under normal circumstances the maximum is attained in the direction from Area 1 to Area 2, i.e. $P_{k,max} = \tau + \beta \sup_{k-1\le t\le k} \left\{D_t^B - V_k^C\right\}$. But V_k^C is constant, hence

$$P_{k,max} = \tau + \beta\left(\sup_{k-1\le t\le k} \left\{D_t^B\right\} - V_k^C\right). \tag{5}$$

Left is only the problem of finding the distribution of $\sup_{k-1\le t\le k} \left\{D_t^B\right\}$ and its connection to V_k^C.

A. *Finding V_k^C given a general set of measurements*

Before the intra-day market closes, one hour before the start of the operational hour, B will have to use the information at hand to make a forecast of his customers demand during the operational hour (i.e. a forecast of S_k^B). The information at hand is the energy consumption of B's customers in 10-minute intervals made up till one hour before the start of the operational hour. Lets call this series $Y_i, i = 1, 2, ..., N$, with Y_1 the last measure made. Furthermore let T be the time that measurements have been made (i.e. $T = N/6$ hours), and remember that T is very large. The random variables Y_i are given by

$$Y_i = \int_{T-i\Delta}^{T-(i-1)\Delta} D_t dt \approx \int_{T-i\Delta}^{T-(i-1)\Delta} m_t dt + \tag{6}$$

$$\frac{\sigma}{\alpha}e^{-\alpha(T-(i-1)\Delta)} \int_0^{T-(i-1)\Delta} \left(e^{\alpha(T-(i-1)\Delta)} - e^{\alpha s}\right) dB_s$$

$$-\frac{\sigma}{\alpha}e^{-\alpha(T-i\Delta)} \int_0^{T-i\Delta} \left(e^{\alpha(T-i\Delta)} - e^{\alpha s}\right) dB_s$$

Now let $I_t^B = \int_0^t X_s ds$, as before. What we want to do is to find the covariance-matrix for the normal vector

$$\left(S_k^B, Y_1, Y_2, ..., Y_N\right). \tag{7}$$

The mean vector is obviously given by

$$\left(\int_{k-1}^{k} m_t dt, \int_{T-\Delta}^{T} m_t dt, \int_{T-2\Delta}^{T-\Delta} m_t dt, ..., \int_{T-N\Delta}^{T-(N-1)\Delta} m_t dt\right). \tag{8}$$

Since,

$$\begin{aligned} &Cov(Y_i, Y_j) = \\ &Cov(I_{T-(i-1)\Delta} - I_{T-i\Delta}, I_{T-(j-1)\Delta} - I_{T-j\Delta}) = \\ &Cov(I_{T-(i-1)\Delta}, I_{T-(j-1)\Delta}) - \\ &Cov(I_{T-(i-1)\Delta}, I_{T-j\Delta}) - \\ &Cov(I_{T-i\Delta}, I_{T-(j-1)\Delta}) + Cov(I_{T-i\Delta}, I_{T-j\Delta}), \end{aligned}$$

and $S_k^B = I_k^B - I_{k-1}^B$, we only need to compute Cov(I_u, I_t) for all $u, t \in [0, T]$. This is given by

$$\begin{aligned} &\text{Cov}(I_u, I_t) = \\ &= \frac{\sigma^2}{\alpha^2}\left((u\wedge t) - \frac{1}{\alpha}\left(e^{-\alpha u} + e^{-\alpha t}\right)\left(e^{\alpha(u\wedge t)} - 1\right) + \right. \\ &\left. \frac{1}{2\alpha}\left(e^{\alpha(u\wedge t - u\vee t)} - e^{-\alpha(u+t)}\right)\right). \end{aligned}$$

Now that the mean vector and the covariance-matrix are found, V_k^C is , due to the market structure where positive and negative imbalances have the same price, the mean of $S_k^B|Y_1, Y_2, ..., Y_N$, which can be calculated from the conditional normal-distribution

$$f_{S_k^B|Y_1=y_1,Y_1=y_1,...,Y_N=y_N}(x) = \frac{f_{S_k^B,Y_1,Y_2,...,Y_N}(x, y_1, y_2, ..., y_N)}{f_{Y_1,Y_2,...,Y_N}(y_1, y_2, ..., y_N)}.$$

B. *The distribution of D_{k-1}^B given a general set of measurements*

To calculate the distribution of the maximal transfer between the two areas during the operational hour we also need to know the distribution of $D_{k-1}^B|Y_1, Y_2, ..., Y_N$. We start by noting that also

$$\left(D_{k-1}^B, Y_1, Y_2, ..., Y_N\right), \tag{9}$$

is a normal-vector, and hence, the same method as when computing the distribution of $S_k^B|Y_1, Y_2, ..., Y_N$ can be used. What is needed here is in addition the variance of D_{k-1}^B and $\text{Cov}(D_{k-1}^B, I_t)$ for $t \in [0, T]$. These can be calculated straight forwardly by applying the Itô isomorphism as shown below:

$$\text{Var}\left[D_{k-1}^B\right] = E\left[\left(\sigma e^{-\alpha(T+1)} \int_0^{T+1} e^{2\alpha s} dB_s\right)^2\right] = \qquad (10)$$
$$= \sigma^2 e^{-2\alpha(T+1)} \int_0^{T+1} e^{2\alpha s} ds = \frac{\sigma^2}{2\alpha}\left(1 - e^{-2\alpha(T+1)}\right),$$

and

$$\text{Cov}(D_{k-1}^B, I_t) = \frac{\sigma^2}{\alpha^2}\left(\left(e^{\alpha(t-(T+1))} - e^{\alpha(T+1)}\right)\right. \qquad (11)$$
$$\left. - \frac{1}{2}\left(e^{\alpha(t-(T+1))} - 1\right) e^{-\alpha(T+1+t)}\right).$$

The distribution of $D_{k-1}^B|Y_1, Y_2, ..., Y_N$ can now be calculated using the conditional distribution just as was made in the previous subsection.

C. The distribution of $P_{k,max}$

There are several different ways of finding the distribution of $P_{k,max}|Y_1, Y_2, ..., Y_N$, one way would be to use the same method that was used in [6]. This method rewrites (D_t^B, $k-1 \leq t \leq k$) as a time shifted Brownian motion with initial distribution $D_{k-1}^B|Y_1, Y_2, ..., Y_N$ and then uses a method developed by Axel Lehman [10] to find the distribution of $\sup_{k-1 \leq t \leq k} \{D_t^B\}$. The distribution of (5) is then found by simple addition and subtraction of stochastic variables (observe that V_k^C and D_{k-1}^B are independent). At last $P_{k,max}$ is found as the marginal distribution of $P_{k,max}|Y_1, Y_2, ..., Y_N$. An alternative method is to use Monte Carlo Simulation and randomize outcomes of D_t^B and use the method presented above to determine V_k^C and thus get $P_{k,max}$.

VII. NUMERICAL EXAMPLE

In this Section a numerical example will be given to illustrate how to find the amount of energy that B has bought from C at the ahead markets ,V_k^C, in reality, and then how to use Monte Carlo Simulations to find the distribution of $P_{k,max}$. In the load model for this example a thousandth of the load model in the Swedish power system is used, this model was derived in [6]. This means that for our model of the demand of the customers of B $\alpha = 0.0296$, $\sigma = 0.196$ MW, and m_t is shown in Figure 3.

A realization of the consumption of the customers of B for a period of Monday to Friday is given in Figure 4.

In real life B might not use all the earlier measurement to estimate V_k^C, because of the numerical difficulties involved and the small difference in prediction between using a lot of values and just a few. Since D_t^B is an Itô diffusion, B only needs an estimate of the load at the time of the decision to find the best possible forecast. To find this value it is not of such great importance to know the energy consumption

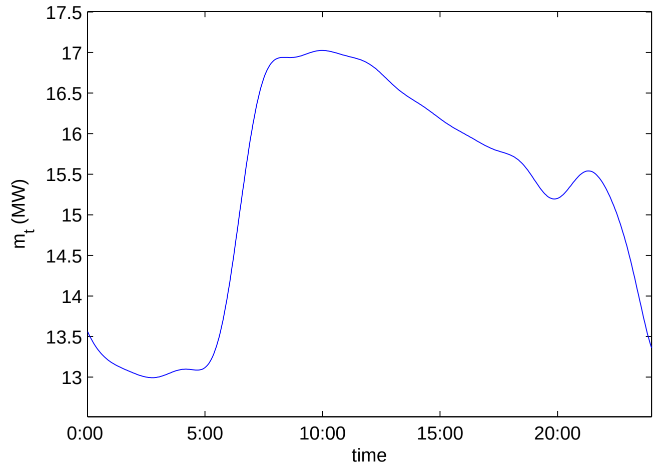

Fig. 3. The mean demand of the customers of B during a working weekday.

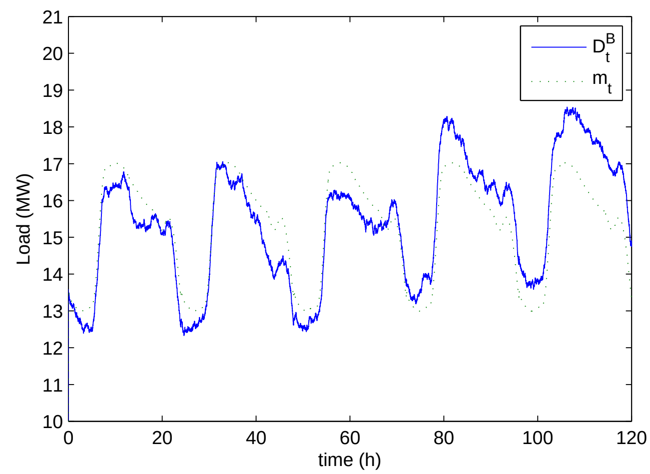

Fig. 4. A realization of the demand of B's customers for one working week.

during some time interval yesterday if you already know the energy consumption during the last six 10-minute periods. What we want is to estimate a function (the load) given the values of its integral (energy consumption) at certain points of time. In this numerical example B uses six values (i.e. the consumption for all the 10-minute intervals of the last hour) at time $T = 1$ to make his forecast of the energy consumption during the hour $[2, 3]$. Observe that now B will have to take into account also the part of (3) originating from the initial distribution of (D_t ,$t \geq 0$), which is the asymptotic distribution of (D_t ,$t \geq 0$), since we will use no prior information of the load. In the covariances of the energy consumptions we will thus have to add a factor $\text{Var}(D_0)\left(1 - e^{-\alpha u}\right)\left(1 - e^{-\alpha t}\right) = \sigma^2\left(1 - e^{-\alpha u}\right)\left(1 - e^{-\alpha t}\right)/2\alpha^3$ to $\text{Cov}(I_u, I_t)$. The trading period that we have chosen to simulate is the hour between 6 and 7 in the morning. The main reason for this choice was that this hour has one of the largest variations in the mean load curve m_t , it will thus often contain one of the largest variations between trading limit and maximal power transfer. The result of the simulation with $\tau_7 = 10$ MWh/h, and all frequency control reserve in Area 1 (i.e. β=1) is depicted in Figure 5 where a Monte Carlo estimate of $F_{P_{max,7}}$ is plotted and a 99% upper confidence bound for $P_{max,7}$ is given by the dashed line.

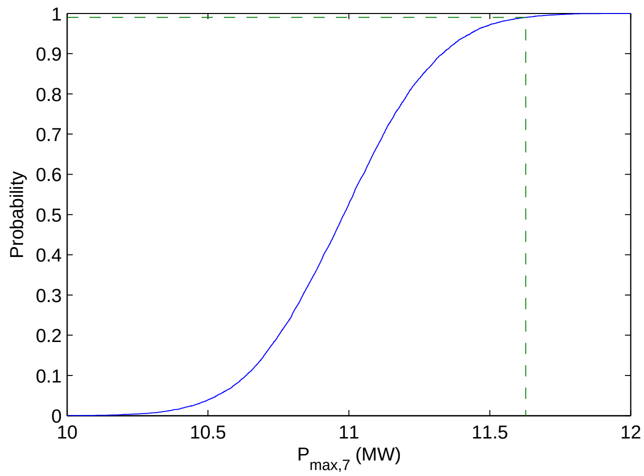

Fig. 5. A Monte Carlo estimate of $F_{P_{max,7}}$ and a 99% upper confidence bound.

What is interesting to note is that even though the trading limit is set to 10 MWh/h, the 99% upper confidence bound is more than 11.6 MW. Hence it is of great importance to consider the factors described in this paper when setting the trading limits of a system.

VIII. Conclusions

In this paper the impact a certain trading limit has on the transfer between two areas of a power system and which uncertainties are important to consider when calculating this power transfer was investigated. To reach the final general result as explained in Section VI-C we will, however, have to compute a integral of many dimensions which is numerically cumbersome. Therefore Monte Carlo Simulations might be a better alternative. What is interesting to note when it comes to the physical behavior of the system is that for different hours, the same trading limit will give different distributions of the maximal transfer due to the daily variation of the mean curve. In the problem formulated there are very few actors on the electricity market and to get a more realistic model of the impact of a certain trading limit a market with more actors would be needed.

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