Computation of Spectral Components in System with Wind Generation Unit

P. Janik, T. Lobos, J. Rezmer, P. Schegner, *Member, IEEE,* Z. Waclawek

***Abstract*--Signal parameters estimation is an important prerequisite for assessment of power quality (PQ) indices. Nowadays, large amounts of measured data need to be automatically processed for appropriate and useful data mining in PQ. Especially, modern wind generators are often seen as sources of PQ disturbances, which should be constantly supervised. The authors propose an application of modified Singular Value Decomposition (SVD) method for signal parameters estimation. Results of the proposed method are compared with broadly used Fourier Transform. Additionally,, results from Prony method are presented. A mechanical model of doubly fed induction generator (DFIG), operating in various conditions was chosen as a source of disturbed signals. Research results verify the usefulness of SVD based method.**

***Index Terms*—Power Quality, SVD, interharmonics, Prony Method .**

I. Introduction

PARAMETER estimation of signals is the first step in processing rough samples of measured currents and voltages. The accuracy and efficiency of the applied parameter estimation algorithms is of outmost importance for further data processing, and finally, for the legitimacy of decisions based on power quality (PQ) monitoring [1, 2, 3].

This paper is focused on harmonic and interharmonic distortion assessment. Interharmonics are defined as non-integer harmonics of the main fundamental component and their estimation is important for PQ monitoring, control and protection tasks.

In the practice, mostly characteristic harmonics are taken into account [2], e.g. during the design process of filters and protection devices. The above implies that interharmonincs are treated not carefully enough and so may be considered more damaging than characteristic harmonics [4,5].

One of common sources of harmonics in electrical systems are induction generators with power electronic devices (DFIG) working in wind farms [6]. This type of wind generators is far more popular than conventional induction generators, connected directly to the grid. DFIG generators are equipped with frequency power converters generating a width spectrum of harmonic and interharmonic components which deteriorate PQ.

There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, and subspace methods. Most of them operate in the narrow range of frequencies and at moderate noise levels. The linear methods of spectrum estimation, based on Fourier Transform, have considerably low resolution in frequency domain. These methods usually assume that only harmonics are present, the periodicity intervals are fixed and no transients are present in the signal.

The proposed method, based on Singular Value Decomposition (SVD) [7, 8, 9], is a technique for modeling sampled data as a linear combination of exponentials. Such modeling is sensible for interharmonics and transients, occurring during every parameter change in the investigated system. Obviously, the proposed parametric method, as an parametric method, requires a prior knowledge of the number of components, often regarded as a disadvantage. This disadvantage is visible when compared with other parametric method, the Prony method.

Firstly, the SVD method has been introduced and basic equations have been listed. SVD description is followed by DFIG wind generator as a source of distorted signals. Measurements on physical model of a DFIG were carried out for various operation modes. Measured signals are analyzed with FFT and SVD. The analysis results are commented and summarized in the conclusion.

II. Spectrum Estimation Method Based on Singular Value Decomposition

The proposed method, based on Singular Value Decomposition (SVD) [7, 8, 9], is a technique for modeling sampled data as a linear combination of exponentials.

Accordingly, the signal $x(t)$ can be described mathematically as a sum of N exponential components:

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + \ldots + A_N e^{s_N t} \tag{1}$$

where: A - signal amplitude, $s = \alpha + j\omega$ - complex frequency.

For practical reasons we consider only digital signals, sampled with the interval T

$$t = [nT] \tag{2}$$

The signal samples are given by

$$x_n = A_1 (e^{s_1 T})^n + A_2 (e^{s_2 T})^n + \ldots + A_N (e^{s_N T})^n = A_1 z_1^n + A_2 z_2^n + \ldots + A_N z_N^n \tag{3}$$

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The practical realization of the proposed method imposes designing of a digital filter with finite impulse response (FIR). This filter (Fig. 1) should block all signal components. The filter transmittance in general form is given by

$$H(z)=1+h_1z^{-1}+h_2z^{-2}+...+h_Nz^{-N} \tag{4}$$

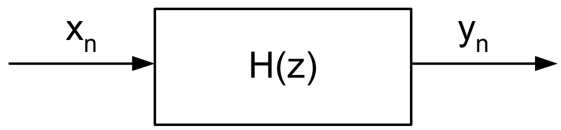

Fig. 1. Basic digital filter scheme as a practical realization of proposed analysis method

The filter response is given by

$$\mathrm{Y(z)}=\mathrm{H(z)}\cdot\mathrm{X(z)} \tag{5}$$

The blocking property of the considered filter results in

$$\begin{aligned} y_N &= x_N+h_1x_{N-1}+h_2x_{N-2}+...+h_Nx_0=0 \\ y_n &= 0 \;\; for\; n\geq N \end{aligned} \tag{6}$$

Equation (6) can also be written in a practice relevant matrix form

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -h_N & -h_{N-1} & -h_{N-2} & \cdots & -h_1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-2} \\ x_{N-1} \end{bmatrix} \tag{7}$$

or

$$\mathbf{x}_1=\mathbf{S}\mathbf{x}_0 \tag{8}$$

Generally, the realization of proposed method implies using *M*-column matrices instead of one column vectors of input signal samples x_n.

$$\mathbf{X}_1^{(M)}=\mathbf{S}\mathbf{X}_0^{(M)} \tag{9}$$

The construction of the $\mathbf{X}_1^{(M)}$ matrix is given in (10). The $\mathbf{X}_0^{(M)}$ matrix is constructed in a similar manner.

$$\mathbf{X}_1^{(M)}=\begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_{M-1} & x_M \\ x_2 & x_3 & x_4 & \cdots & x_M & x_{M+1} \\ x_3 & x_4 & x_5 & \cdots & x_{M+1} & x_{M+2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{N-1} & x_N & x_{N+1} & \cdots & x_{N+M-3} & x_{N+M-2} \\ x_N & x_{N+1} & x_{N+2} & \cdots & x_{N+M-2} & x_{N+M-1} \end{bmatrix} \tag{10}$$

Similarly, equation (3) can also be given in a modified matrix form

$$\mathbf{x}_1=\mathbf{W}_0\mathbf{Z}\mathbf{A} \tag{11}$$

where

$$\mathbf{W}_0=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ z_1 & z_1 & z_1 & \cdots & z_1 & z_1 \\ z_1^2 & z_2^2 & z_3^2 & \cdots & z_{N-1}^2 & z_N^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ z_1^{N-2} & z_2^{N-2} & z_3^{N-2} & \cdots & z_{N-1}^{N-2} & z_N^{N-2} \\ z_1^{N-1} & z_2^{N-1} & z_3^{N-1} & \cdots & z_{N-1}^{N-1} & z_N^{N-1} \end{bmatrix} \tag{12}$$

$$\mathbf{Z}=\begin{bmatrix} z_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & z_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & z_3 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & z_{N-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & z_N \end{bmatrix} \tag{13}$$

$$\mathbf{A}=\begin{bmatrix} A_1 & A_2 & \ldots & A_{N-1} & A_N \end{bmatrix}^T \tag{14}$$

Considering (8) and (11) the following equation can be formulated

$$\mathbf{W}_0\mathbf{Z}\mathbf{A}=\mathbf{S}\mathbf{W}_0\mathbf{A} \tag{15}$$

finally

$$\mathbf{S}=\mathbf{W}_0\mathbf{Z}\mathbf{W}_0^{-1} \tag{16}$$

The latest equations shows, that $z_1, z_2, z_3, ..., z_N$ are the eigenvalues of the matrix $\mathbf{S}$.

Regarding (9) the matrix $\mathbf{S}$ may be calculated

$$\mathbf{S}=\mathbf{X}_1^{(M)}\mathbf{X}_0^{(M)T}(\mathbf{X}_0^{(M)}\mathbf{X}_0^{(M)T})^{-1} \tag{17}$$

Knowing the $z_1, z_2, z_3, ..., z_N$ values the complex frequencies (3) $s_1, s_2, s_3, ..., s_N$ may be easily calculated. Similarly the values of amplitudes $A_1, A_2, A_3, ..., A_N$ may be derived from (11).

III. Prony Method for Spectral Components Estimation

The Prony method is a technique for modeling sampled data as a linear combination of exponential functions [3, 4]. Although it is not a spectral estimation technique, the Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics.

Assuming *N* complex data samples, the investigated function can be approximated by *p* exponential functions:

$$y[n]=\sum_{k=1}^{p}A_k e^{(\alpha_k+j\omega_k)(n-1)T_p+j\psi_k} \tag{18}$$

where

$n = 1, 2, ..., N$, T_p - sampling period, A_k - amplitude, α_k - damping factor, ω_k - angular velocity, ψ_k - initial phase.

The discrete-time function may be concisely expressed in the form

$$y[n] = \sum_{k=1}^{p} h_k z_k^{n-1} \tag{19}$$

where

$h_k = A_k e^{j\psi_k}$, $z_k = e^{(\alpha_k + j\omega_k)T_p}$

The estimation problem is based on the minimization of the squared error over the N data values

$$\delta = \sum_{n=1}^{N} \left|\varepsilon[n]\right|^2 \tag{20}$$

where

$$\varepsilon[n] = x[n] - y[n] = x[n] - \sum_{k=1}^{p} h_k z_k^{n-1} \tag{21}$$

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method, that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data can be made.

Consider the *p*-exponent discrete-time function:

$$x[n] = \sum_{k=1}^{p} h_k z_k^{n-1} \tag{22}$$

The *p* equations of (5) may be expressed in matrix from as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_p^0 \\ z_1^1 & z_2^1 & \cdots & z_p^1 \\ \vdots & \vdots & & \vdots \\ z_1^{p-1} & z_2^{p-1} & \cdots & z_p^{p-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[p] \end{bmatrix} \tag{23}$$

The matrix equation represents a set of linear equations, that can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial, that has the exponents as its roots:

$$F(z) = \prod_{k=1}^{p} (z - z_k) = (z - z_1)(z - z_2)\ldots(z - z_p) \tag{24}$$

The polynomial may be represented as the sum:

$$F(z) = \sum_{m=0}^{p} a[m] z^{p-m} = \\ = a[0] z^p + a[1] z^{p-1} + \ldots + a[p-1] z + a[p] \tag{25}$$

Shifting the index on (19) from *n* to *n-m* and multiplying by the parameter *a*[*m*] yield:

$$a[m] x[n-m] = a[m] \sum_{k=1}^{p} h_k z_k^{n-m-1} \tag{26}$$

Equation (26) can be modified into:

$$\sum_{m=0}^{p} a[m] x[n-m] = \\ = \sum_{k=1}^{p} h_k z_k^{n-p} \left\{ \sum_{m=0}^{p} a[m] z_k^{p-m-1} \right\} \tag{27}$$

The right-hand summation in (27) may be recognized as a polynomial defined by (25), evaluated at each of its roots yielding the zero result:

$$\sum_{m=0}^{p} a[m] x[n-m] = 0 \tag{28}$$

The equation (28) can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (25) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k .

For practical situations, the number of data points *N* usually exceeds the minimum number needed to fit a model of exponentials, i.e. $N > 2p$. In the over determined data case, the linear equation (28) should be modified to:

$$\sum_{m=0}^{p} a[m] x[n-m] = e[n] \tag{29}$$

The estimation problem is based on the minimization of the total squared error:

$$E = \sum_{n=p+1}^{r} \left|e[n]\right|^2 \tag{30}$$

IV. Wind Generation Model as Source of Disturbances

A mechanical model of the DFIG was used for the generation of distorted signals. The rotor of induction machine is connected to the grid with a back-to-back voltage source converter which controls the excitation system. This most significant feature [9] enables subsynchronous and super-synchronous operation speeds in generator mode and adjustable reactive power generation

In steady state and at fix speed the output power P_m for a lossless generator is given as a sum of power generated in stator and rotor.

In subsynchronous generation mode the stator supplies and the rotor consumes active power. In supersynchronous mode both, stator and rotor circuit supply active power. The stator active power is proportional to i_{rq} component and the reactive power to i_{rd} in the *dq* coordinate system. The reactive power generation is controlled through the power converter which enables inductive and capacitive reactive power.

V. Analysis of Measured Signals

The proposed SVD based method was applied for analysis of current signals. Currents at the point of common coupling were measured with the sampling frequency of 10 kHz. For the signal analysis a 0.2 seconds measurement window was used. However, by unchanged measurement window, a sampling frequency of 1 kHz was high enough for the SVD based method (undersampling).

Generally, all signals were highly distorted and nonstationary. It was assumed, that for those types of signals, the commonly used Fourier Transform is not sufficient enough [5].

Several spectral analyses were carried out using both, FFT and SVD based method. Additionally two final examples show the Prony results for comparison. As examples, the results obtained for three different operation modes were presented. The wind generator worked in super synchronous mode and the produced reactive power was inductive, capacitive and in the third case kept near zero. In every case the level of active power production was constant.

The amplitude and frequency of the dominant fundamental 50 Hz component was clearly detected by both methods, so in further considerations the focus was set only on harmonic content.

A. Normal operation - reactive power near zero

Usually, the wind generator should produce only active power. The current waveform in that case is shown in Fig. 1.

Fig. 2 and Fig. 3 show the spectral components obtained with FFT and SVD, respectively.

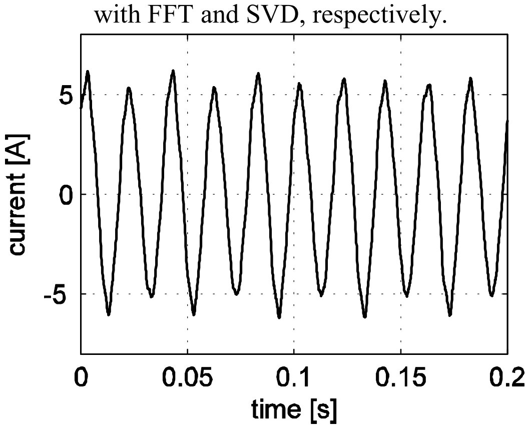

Fig. 1. Current signal at cos φ near one

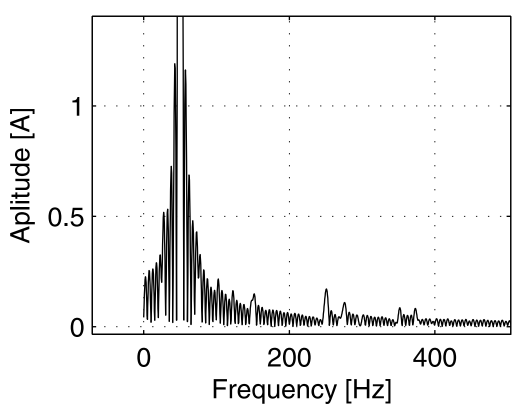

Fig.2. FFT of the measured current waveform, cos φ near one

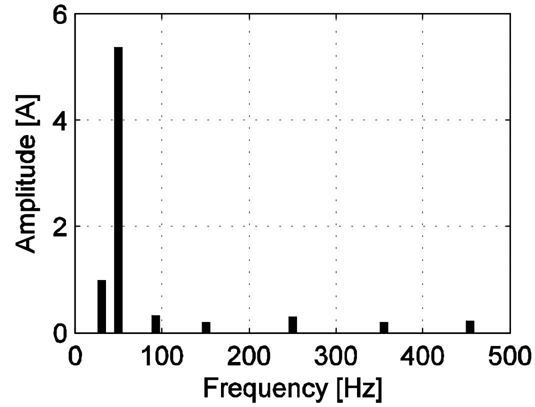

Fig.3. SVD of the measured current waveform, cos φ near one

The FFT spectrum in Fig. 2 has not a sufficient resolution. Especially the 30 Hz component as in Fig. 3 is not clearly visible.

B. Purposeful operation – inductive reactive power generation

Actually, it is not really practical to force the wind generator to introduce inductive reactive power into the system. If the cos φ is set to a high inductive value, the harmonic content differs from previous case even by unchanged active power production.

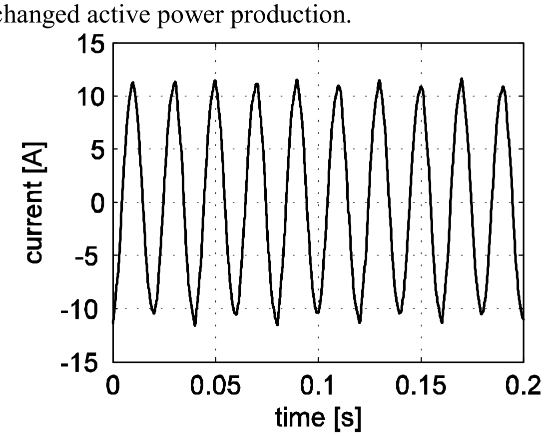

Fig. 4. Current signal at cos φ = 0.5 ind

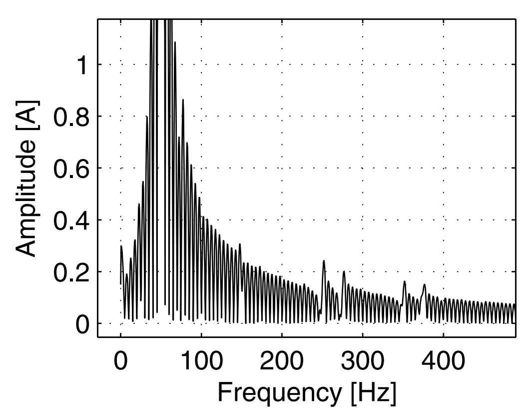

Fig.5. FFT of the measured current waveform, cos φ = 0.5 ind

The FFT spectrum (Fig. 5) of the current (Fig. 4) does not clearly indicate the interharmonics visible in the SVD spectrum (Fig. 6). Especially, the 133.6 and 263.4 Hz components are not visual.

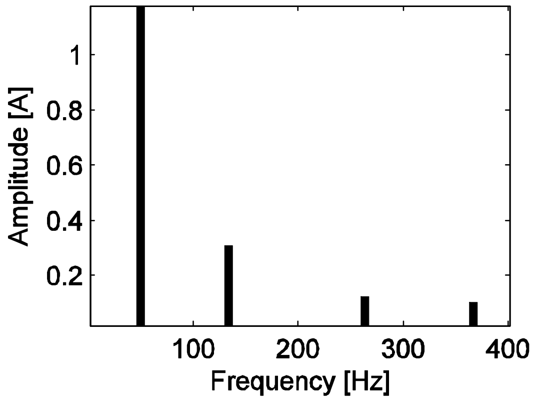

Fig. 6. SVD of the measured current waveform, cos φ = 0.5 ind

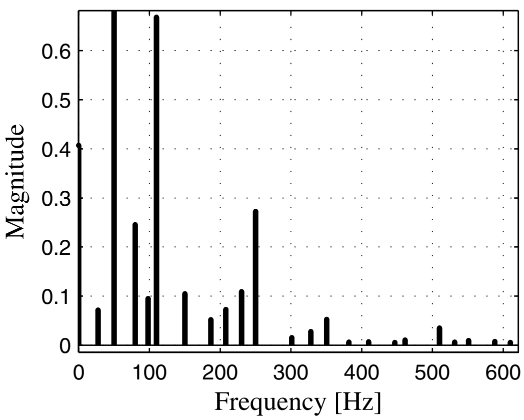

Fig. 7 Spectral estimation of current with Prony mode

The Prony method shows more components with slightly varying magnitudes (Fig. 7). That indicates the uncertainty of results for not appropriate setting of components number.

C. Purposeful operation – capacitive reactive power generation

From the practical point of view, it is in some cases advisable to introduce capacitive reactive power into the grid. Current introduced into the network at cos(φ)=0.7 is shown in Fig. 8. The current amplitude is smaller than in the previous case, due to greater cos(φ) by the same active power.

It is clearly visible, that the harmonic content in this case significantly differs from both previous cases.

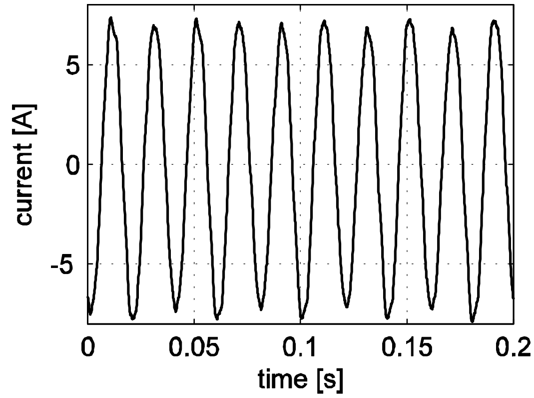

Fig. 8. Current signal at cos φ = 0.7 cap

The content of interharmonics is significantly high (Fig. 10). The FFT (Fig. 9) does not show distinctive the 91.5 spectral component, only a buckling about 100 Hz.

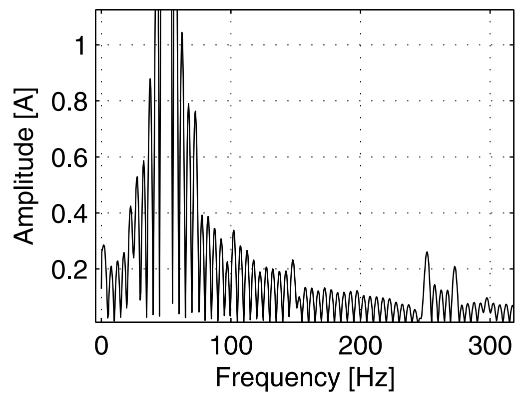

Fig.9. FFT of the measured current waveform, cos φ = 0.7 cap

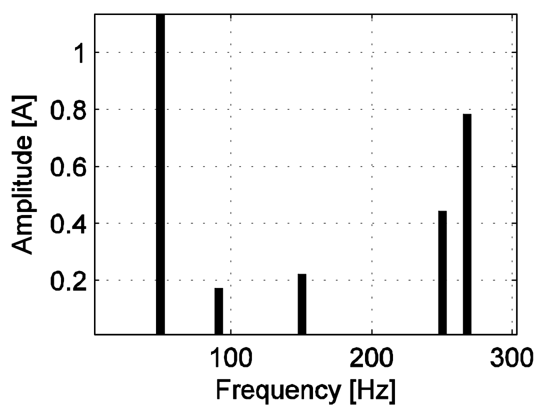

Fig.10. SVD of the measured current waveform, cos φ = 0.7 cap

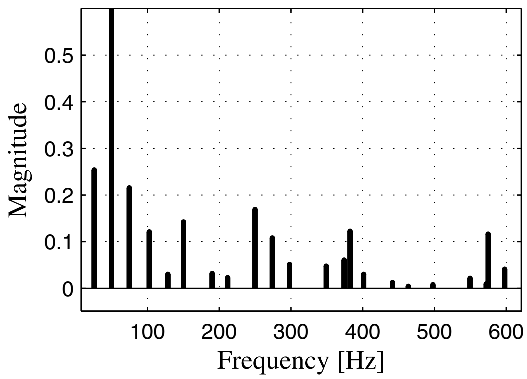

Fig. 11. Spectral estimation of current with Prony model

The 91.5 Hz spectral component is visible while using Prony method (Fig. 11). If both methods indicate the same component then a reassurance of its presence is given.

VI. Conclusion

Data mining understood as extraction of implicit and useful information from enormous measurement data sets, plays a key role in modern power quality assessment.

The first step in information extraction from rough samples of measured currents and voltages is adequate parameters estimation.

The proposed SVD based method was applied for spectral component estimation of current signals at PCC.

Preliminary signal analysis results showed a superiority of the proposed method over the commonly applied FFT. Especially, in computing spectral components, which were not an integer multiplies of the fundamental 50 Hz frequency. Similarly, the results obtained with Prony method are better then FFT. The comparison of both parametric methods indicates the problem of correct setting of the components number. In this issue FFT may be helpful.

Interesting is the relatively high content of interharmonics in the measured currents. Additionally, the interharmonic content varied with changes in the operation mode of the DFIG model. As an example, the case of constant angular velocity and produced active power was presented. The change of reactive power influenced strongly the amount of interharmonics.

The preliminary research results encourage further analysis of current signals registered at real wind turbines in order to assess the interharmonic content as a function of the operation mode of DFIG.

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VII. Biographies

Prof. Tadeusz Lobos received the M.Sc., Ph.D. and Habilitate Doctorate (Dr.Sc.) degrees, all in electrical engineering, from the Wroclaw University of Technology, Poland, in 1960, 1967, and 1975, respectively. He has been with the Department of Electrical Engineering, Wroclaw University of Technology, since 1960, where he became a Full Professor in 1989. From 1982 to 1986, he worked at the University of Erlangen-Nuremberg, Germany. His current research interests are in the areas of transients in power systems, control and protection, and especially application of neural networks and signal processing methods in power systems.. The Alexander von Humboldt Foundation, Germany awarded Dr. Lobos a Research Fellowship in 1976 and he spent this fellowship at the Technical University of Darmstadt. He received the Humboldt Research Award, Germany in 1998. Now he is a visiting Professor at the Dresden University of Technology, Germany, in the framework of Merkator Program (DFG).

Przemysław Janik graduated from the Wroclaw University of Technology in 2000. In 2005 he obtained Ph D. degree from Faculty of Electrical Engineering at Wroclaw University of Technology, where he works science then as scientist. His research activities are electrical power quality, neural networks and artificial intelligence classification systems.

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Zbigniew Wacławek. received the M.Sc. and Ph.D. degrees, both in electrical engineering, from the Wroclaw University of Technology, Poland, in 1991 and 1996 respectively. He has been with the Department of Electrical Engineering, Wroclaw University of Technology, since 1991. His current research interests include signal processing methods for power system analysis, especially application of neural networks, and optimization techniques.

Prof. Peter Schegner (M' 1999) studied Electrical Power Engineering at the Darmstadt University of Technology (Germany) where he received the Dipl.-Ing. degree in 1982. After that he worked as system engineer in the field of power system control and became a memberof the scientific staff at the Saarland University (Germany), receiving the Ph D degree in 1989 with a thesis on the earth-fault distance protection. From 1989 until 1995 he worked as head of the development department of protection systems at AEG, Frankfurt A.M., Germany. In 1995 he became a Full Professor and head of Institute of Electrical Power Systems and High Voltage Engineering of the Dresden University of Technology (Germany)