Phasor Measurement Units' Allocation in Unified Electrical Power Network of Egypt

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***Abstract*--This paper presents a comparison between Phasor Measurement Units' (PMUs') optimal allocation for system topological observability, and PMUs' optimal number and locations for minimum state estimation residual error. A newly developed technique based on Discrete Particle Swarm Optimization (DPSO) is used in finding the PMUs' optimal allocation for different system unobservability depth. For the optimal allocation of PMUs from the point of view of minimum state estimation residual error, a new methodology is used with a new hybrid state estimation technique. The comparison is applied to a large system depicted from Unified Electrical Power Network (UEPN) of Egypt. The simulation results are discussed in details.**

***Index Terms*--Discrete particle swarm optimization (DPSO), hybrid parameters state estimation, optimal allocation, phasor measurement units (PMUs), topological observability, tree search method (TSM), unobservability depth.**

I. INTRODUCTION

One of the recent measuring devices is Synchronized Phasor Measurement Unit (PMU) [1]–[6]. Due to the strong correlation between PMUs and the Global Positioning Satellites (GPS), PMUs began to spread widely after the great improvement in the satellite techniques and communications. Since the middle of 1980's, some researchers have paid attention to this great subject, and there are others, everyday, who like to go through this way [1].

PMUs are used in different electrical power engineering applications such as measurements, protection, control, observation,… etc. In measurements, it has unique ability to provide synchronized phasor measurements of voltages and currents from widely dispersed locations in an electric power grid, to be collected at control center for analysis. It is an electronic device that uses state-of-the-art digital signal processors that can measure 50/60Hz AC waveforms (voltages and currents) typically at a rate of 48 samples per cycle. A phase-locked oscillator along with a Global Positioning System (GPS) reference source provides the needed high-speed synchronized sampling with 1 microsecond accuracy. Line frequencies are also calculated by the PMU at each site. This method of phasor measurement yields a high degree of resolution and accuracy. The resultant time tagged phasors can be transmitted to a local or remote receiver at rates up to 50/60 samples per second. PMUs come in different sizes. Some of the larger ones can measure up to 10 phasors plus frequency while others only measure from one to three phasors plus frequency [2].

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Fig. 1 shows the PMU hardware block diagram for the previous illustrated procedures.

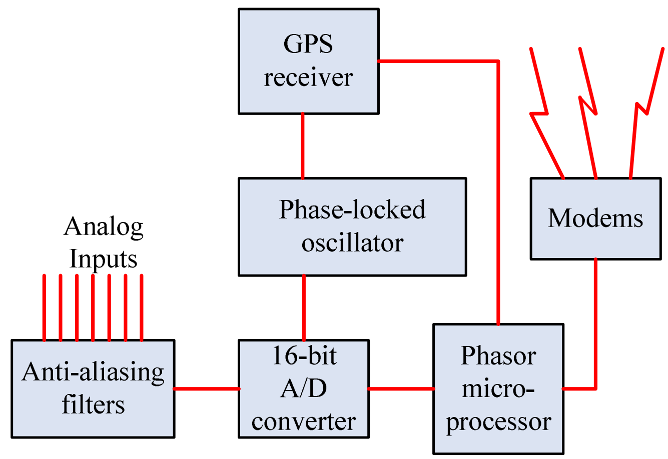

Fig. 1. Phasor Measurement Unit (PMU) hardware block diagram [2]

PMUs revolutionize the way of power systems monitoring and control. This revolution benefits the Wide Area Monitoring System (WAMS) technology.

In protection and control, PMUs are used in many applications for measuring the synchronized phasor parameters needed for taking a decision or an action.

It is not economically, up till now, to spread the PMUs allover the power system buses. Also in some power systems, there are buses with deficiency in communication facility. So there is a great need for PMUs' optimum allocation. There are different methods for determining the number and location of PMUs in the power system. One of these methods is the Tree Search Method (TSM). It gives near optimal solutions for different unobservability depth. However, TSM results depend on the way the power network is seen. So there is a need for other optimization techniques to determine the optimal PMUs' allocation.

Particle Swarm Optimization (PSO) is a stochastic, population-based evolutionary algorithm for problem solving [7]. It is a kind of swarm intelligence that is based on social-psychological principles. It provides insights into social behavior, as well as contributing to engineering applications. It is a powerful method to find the minimum of a numerical function, on a continuous definition domain [8]. A successful adapted PSO algorithm is used for solving discrete optimization problems [9]. It requires only a small number of particles and the computation times required are also small compared with other algorithms.

In power system, state estimation plays a great and important role in the monitoring and control of power systems. State estimation is the process of determining a value to an unknown system state variable based on measurements from the system according to some criteria. Usually the process involves imperfect measurements that are redundant and the process of estimating the system states is based on a statistical criterion that estimates the true value of the state variables to minimize or maximize the selected criterion [10]. A commonly used and familiar criterion is to "fine-tune" state variables by minimizing the sum of the residual squares (the squares of the differences between the estimated and true values of a function) [10]–[12]. This is the well-known least squares (LS) method, which has become the cornerstone of classical statistics.

For power systems, as in case of load flow analysis, the aim of state estimation is to obtain the best possible values of the bus voltage magnitudes and relative phase angles at the system nodes by processing the available network data. Most state estimation programs in practical use are formulated as overdetermined systems of non linear equations and solved as Weighted Least Squares (WLS) problems [10], [12]. State estimators may be both static and dynamic. Both have been developed for power systems [12]. Some modifications are introduced now in order to achieve a higher degree of accuracy of the solution at the cost of some additional computations. One of these modifications is to utilize the new available techniques or devices in state estimations, such as PMUs.

In this paper, a modified Discrete (Integer) Particle Swarm Optimization (DPSO) technique is developed to determine the optimal number and locations for PMUs in power system network for different depth of unobservability. The developed technique is applied on a large system depicted from the Egyptian unified electrical power network to determine the optimal PMUs' allocation for system complete observability. Tree Search Method (TSM) is used to assign a near optimal allocation to be considered as one of the initial conditions of DPSO. DPSO technique is used to give the optimal PMUs' allocation comparable to TSM. The effect of PMUs phasor measurements on the state estimation analysis accuracy is studied with a new methodology. A comparison between PMUs' optimal allocation for system topological observability, and PMUs' optimal number and locations for minimum state estimation residual error is also discussed.

II. COMPLETE AND INCOMPLETE OBSERVABILITY

A. Observability Definition

Observability means the ability of measuring bus voltage phasor or directly calculating it using the PMU voltage and line current of the nearest connected bus [5], [6]. On the other side, unobservability depth (incomplete observability) means the number of buses which its voltages can't be directly calculated in same zone. Fig. 2 illustrates the idea of observability and unobservability depth. Bus 2 is measured by the PMU. Bus 1 and 3 can be directly calculated, while bus 4 and 5 are unobserved. Bus 4 and 5 are in the same zone which leads to a two unobservability depth system.

B. Tree Search Method (TSM – Initial Condition) [5], [6]

TSM is a method for PMUs' allocation according to the desired observability. In this method, the electrical power network loops are cut and opened to be converted into a radial system. This radial system looks like the tree with its branches, sub-branches and nodes. System buses are equivalent to the tree nodes. PMUs are distributed over the nodes applying the concept that the distance between every two successive PMUs should fulfill (3 + ע). Where ע refers to unobservability depth (e.g. ע = 1 for one unobservability depth). In some cases this distance concept is avoid due to the network connection or deficiency in communication facilities.

TSM is an effective method, although PMUs' allocation using TSM depends on the way of converting the power network into radial system.

A Matlab program is developed for distributing PMUs on any network using TSM. The TSM program result is taken to be considered as one of the DPSO initial conditions.

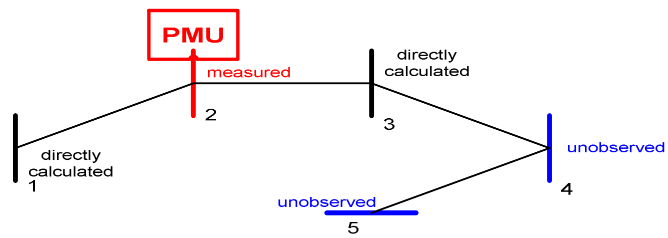

Fig. 2. Two unobservability depth system

III. DISCRETE (INTEGER) PARTICLE SWARM OPTIMIZATION TECHNIQUE

Particle Swarm Optimization (PSO) is an optimization technique which imitates the swarms food searching, especially birds. PSO starts with some randomly distributed particles in searching space. Each particle moves with certain velocity (V) to certain position (X) trying to find a better position. After each movement, each particle compares its new and old position to find what is called its personal best (P_{best}). A comparison is held for all particles P_{best} to find the global best (G_{best}). Each particle determines its velocity and position according to:

$$V_{new} = w*V_{old} + c_1*rand*(P_{best} - X) + c_2*rand*(G_{best} - X) \quad (1)$$

$$X_{new} = X_{old} + V_{new} \quad (2)$$

In the previous equations, w, c_1 and c_2 are weights. It is found that c_1 and c_2 is preferred to be equal to 2. P_{best} and G_{best} are determined according to the problem Cost Function (CF). In some problems there are constraints which should be considered in accepting or refusing the new particle position [7]–[9].

PMUs' allocation is a discrete problem with two main constraints. The constraints are:

1. The PMU locations should be inside the searching space which is the N-buses of the power network.

2. No duplication in PMUs' locations. Some modifications are introduced on the classical PSO to be compatible with discrete problems.

This modified technique is utilized in solving PMU's allocation problem with its mentioned constraints and its cost function. The CF consists of three parts

$$CF = CF_1 + CF_2 + CF_3 \quad (3)$$

First part (CF_1) is for the observability depth, second part (CF_2) is for the bus communication facility and third part (CF_3) is for the number of PMUs used.

$$CF_1 = K_1 * \sum \left[10^{[\ (\text{1st min(PMUD)} - \text{round}\ ((\text{ע}+3)/2)\ ) + (\text{2nd min(PMUD)} - \text{ceil}\ ((\text{ע}+3)/2)\ )\ ]} - 1\right] \quad (4)$$

$$CF_2 = K_2 * \sum |\text{PMU's connected bus communicat}^{\underline{n}}\ \text{facility} - 1| \quad (5)$$

$$CF_3 = K_3 * N_{PMUs} \quad (6)$$

K_1, K_2 and K_3 are three different constant, set as $K_1 = K_3 = 1$ and $K_2 = 100$. PMUD is a matrix; with its rows representing the distances of certain PMU's connected bus to all system buses, and its columns show the distance of certain system bus to each PMU's connected bus. N_{PMUs} is the total number of PMUs used by the swarm particle.

A Matlab program is developed to solve the PMUs' allocation problem using DPSO technique. The program algorithm is illustrated in the flowchart shown in Fig. 3. The program initial positions are the TSM result in addition to generated random position.

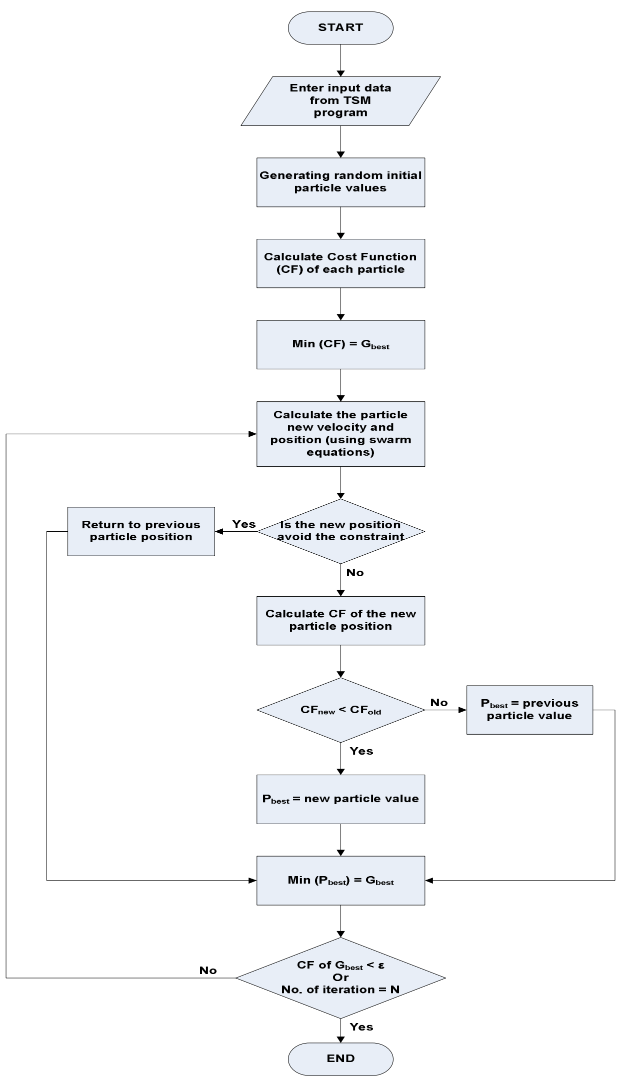

Fig. 3. The flowchart of Discrete Particle Swarm Optimization (DPSO) technique

IV. A STUDY SYSTEM DEPICTED FROM THE EGYPTIAN UNIFIED ELECTRICAL POWER NETWORK

TSM is applied to a large system depicted from the Unified Electrical Power Network (UEPN) of Egypt to determine the optimal PMUs' allocation for complete observability. The studied system consists of 81 buses depicted from the 220 kV network of the UEPN of Egypt. TSM gives near optimal PMUs' allocation, as 24 PMUs are connected to the system buses as shown in Fig. 4.

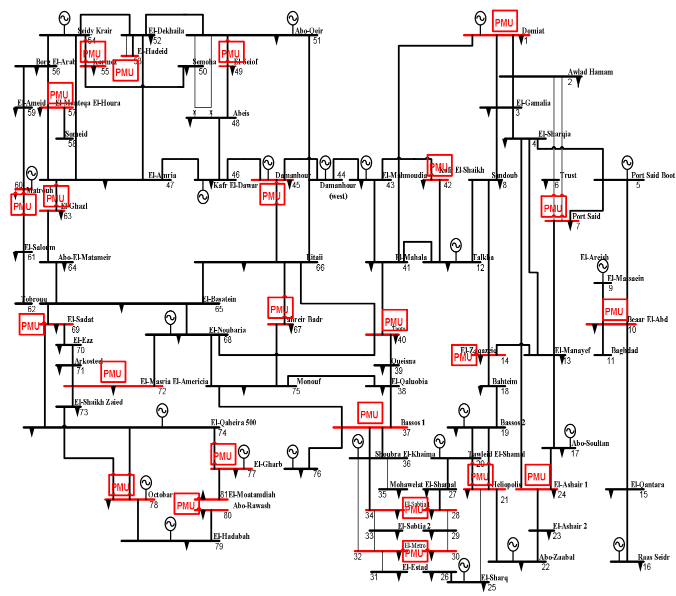

Fig. 4. PMUs' allocation on the Unified Electrical Power Network (UEPN) of Egypt for complete observability using TSM

DPSO technique is then applied to the studied system, with the TSM solution as one of its initial conditions. It gives 22 PMUs distributed as shown in Fig. 5, instead of the 24 PMUs (from TSM), which means that DPSO gives almost better locations and less number of PMUs than TSM.

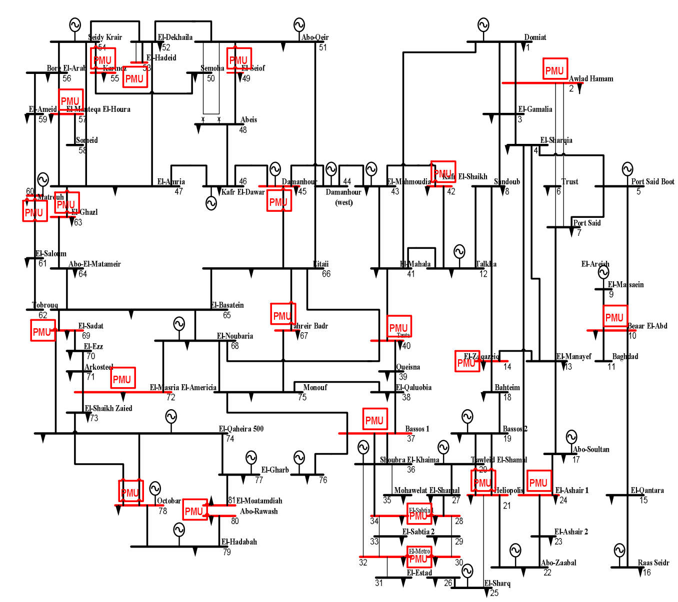

Fig. 5. PMUs' allocation on the Unified Electrical Power Network (UEPN) of Egypt for complete observability using DPSO technique

V. TRADITIONAL STATE ESTIMATION ALGORITHMS

A. Least Square (LS) State Estimation

The problem of power system state estimation is a special case of the more general problem of estimation of a random vector *x* from the numerical values of another related random vector *z* with relatively little statistical information being available for both *x* and *z* [12]. In such cases, the method of LS error estimation may be utilized with good results and has accordingly been widely employed.

$$z = Hx + r \tag{7}$$

z: is a vector of m random measured quantities.
x: is another vector of n random estimated variables, (n < m).
H: is a known matrix of dimension m * n (correlated *z* and *x*).
r: is a zero mean random variable of the same dimension as *z.*

The estimate x is defined to be the LSE if it is computed by minimizing the estimation index *J*.

$$J = z'z - z'Hx - x'H'z - x'H'Hx \tag{8}$$

For minimizing $J = f(x)$, the following condition must be satisfied.

$$\text{grad}_x\, J = 0 \tag{9}$$

It leads to

$$H'Hx - H'z = 0 \tag{10}$$

Equation (10) is called the normal equation and may be solved explicitly for the LSE of the vector x as

$$x = (H'H)^{-1}\, H'\, z \tag{11}$$

H': is the transpose matrix of *H.*

B. Weighted Least Square (WLS) State Estimation

The estimated given by (11) is often referred to as ordinary LS state estimation and is obtained by minimizing the index function that puts equal weights to the errors of estimation of all components of vector *z.* It is often desirable to put different weights *W* on the different measured values of *z* since some of the measurements may be more reliable and accurate than the others and these should be given more importance. It is achieved by

$$H'WHx - H'Wz = 0 \tag{12}$$

This leads to the desired WLS state estimation

$$x = (H'WH)^{-1}\, H'W\, z \tag{13}$$

VI. HYBRID PARAMETERS STATE ESTIMATION

For power system estimation analysis, the traditional WLS state estimation measured quantities z_{meas} are the real and reactive power injected to each bus (P_i, Q_i), the real and reactive power flow from bus i to bus j (P_{ij}, Q_{ij}) and bus voltage magnitudes (V), or some of them according to the available system measurements. The calculated functions for these measured quantities f(x) are given by

$$P_i = \Sigma\,(V_i\, V_j\, Y_{ij} \cos(\delta_i - \delta_j + \theta_{ij})), \qquad \text{for } j = 1, 2, \ldots, N \tag{14}$$

$$Q_i = \Sigma\,(V_i\, V_j\, Y_{ij} \sin(\delta_i - \delta_j + \theta_{ij})), \qquad \text{for } j = 1, 2, \ldots, N \tag{15}$$

$$P_{ij} = V_i\, V_j\, Y_{ij} \cos(\delta_i - \delta_j + \theta_{ij}) - V_i^2\, Y_{ij} \cos(\theta_{ij}) \tag{16}$$

$$Q_{ij} = V_i\, V_j\, Y_{ij} \sin(\delta_i - \delta_j + \theta_{ij}) - V_i^2\, (Y_{ij} \sin(\theta_{ij}) + B_{capij}) \tag{17}$$

where
Y_{ij}: is the admittance magnitude of the line connected bus i and bus j.
θ_{ij}: is the admittance angle of the line connected bus i and bus j.
δ: is the angle of the bus voltage.
B_{capij}: is the total line charging susceptance.
N: is the total of the network buses.

And the estimated states x are the bus voltages magnitudes and angles, which are calculated using

$$x_n = x_{n-1} + \Delta x \tag{18}$$

$$\Delta x = (H'\, W\, H)^{-1}\, H'\, W\, (\Delta z) \tag{19}$$

where
Δx: is the variation in the estimated states.
H: is the Jacobian of the calculated functions f(x).
W: is the weight of the measured quantities.
Δz: is the difference between the measured and calculated quantities ($z_{meas} - f(x)$).

Synchronized phasor measurement unit is connected to the power system buses as shown in Fig. 6 to measure the bus voltage and current phasor synchronized with the rest of the power network. It gives the opportunity to modify the traditional state estimation by concerning the voltage measured angles δ as a measured quantity in addition to the previous mentioned ones.

So the available z_{meas} is P_i, Q_i, P_{ij}, Q_{ij}, V, and δ. And the Jacobian H is extended with the identity matrix "the first derivative of the new added quantity".

$$H = \begin{bmatrix} H_{Pi\delta} & H_{Piv} \\ H_{Qi\delta} & H_{Qiv} \\ H_{Pij\delta} & H_{Pijv} \\ H_{Qij\delta} & H_{Qijv} \\ 0 & I_N \\ I_{N-1} & 0 \end{bmatrix} \tag{20}$$

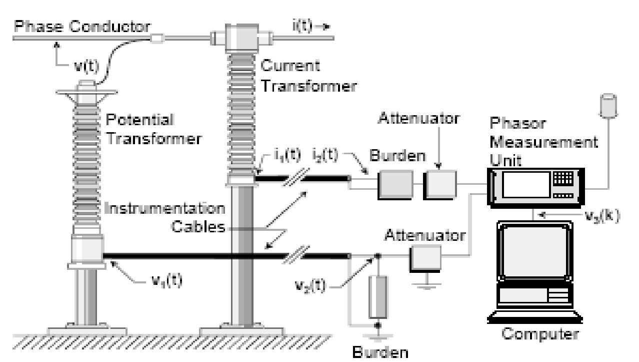

Fig. 6. Components of typical voltage and current instrumentation channel

where

$H_{Pi\delta} \equiv \partial P_i / \partial \delta$

$H_{Piv} \equiv \partial P_i / \partial V$

$H_{Qi\delta} \equiv \partial Q_i / \partial \delta$

$H_{Qiv} \equiv \partial Q_i / \partial V$

$H_{Pij\delta} \equiv \partial P_{ij} / \partial \delta$

$H_{Pijv} \equiv \partial P_{ij} / \partial V$

$H_{Qij\delta} \equiv \partial Q_{ij} / \partial \delta$

$H_{Qijv} \equiv \partial Q_{ij} / \partial V$

A Matlab program for state estimation modified technique is built. The program flow chart is shown in Fig. 7

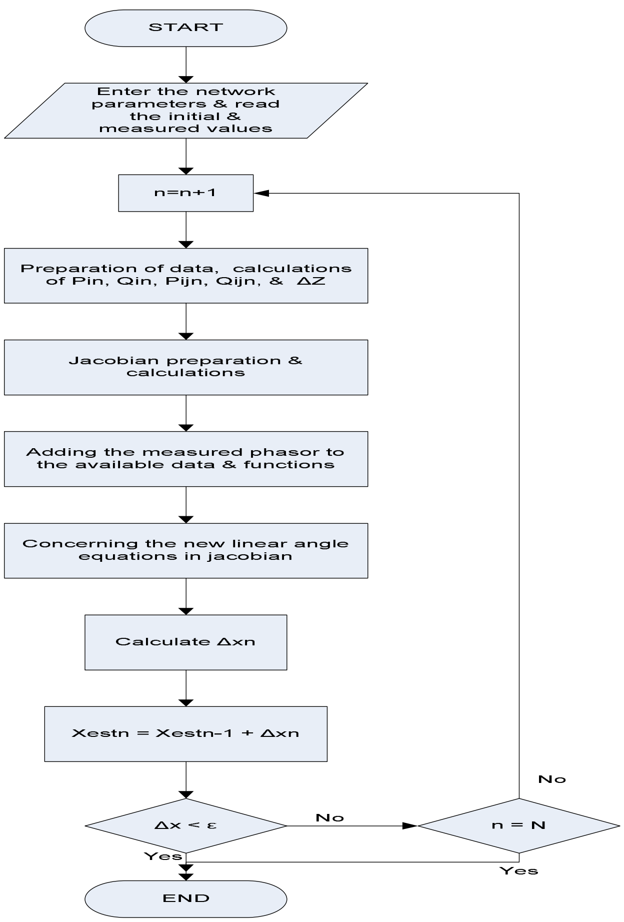

Fig. 7. The flow chart of the hybrid parameters state estimation program

VII. TESTED SYSTEM

The new hybrid parameters state estimation technique is applied to the system depicted from UEPN of Egypt. The effect of changing the locations and numbers of PMUs through the buses of the power network on the system state estimation is studied with a new methodology. The new method is achieved by moving one PMU over all the network buses till fixing the location of PMU on the bus with the minimum state estimation residual error. Another PMU is moved, by the same way, over the remaining network buses till the optimal number and locations of PMUs with least minimum residual error is found.

The PMUs are connected in each analysis to different bus locations, also the analysis is repeated for different numbers of PMUs to find the most optimal numbers and locations of PMUs which gives the most improvement to the accuracy of the state estimation. The built program is applied for each PMUs connection and it is found that the voltage magnitudes and angles begin to have a negligible variation after the 5th iteration. So, all the presented and discussed results are of the 5th iteration.

When the computer program for solving the new state estimation technique is applied to the studied system, PMUs' outputs affect the state estimation analysis in a precious way. It improves the response and the output of the traditional state estimation. The optimal locations of the twenty seven PMUs, which are determined to be the optimal connected PMUs' numbers, according to the least minimum residual error are shown in Fig. 8. The least minimum residual error (e_R) for the 27 connected PMUs is equal to 1.3 x 10^{-13}.

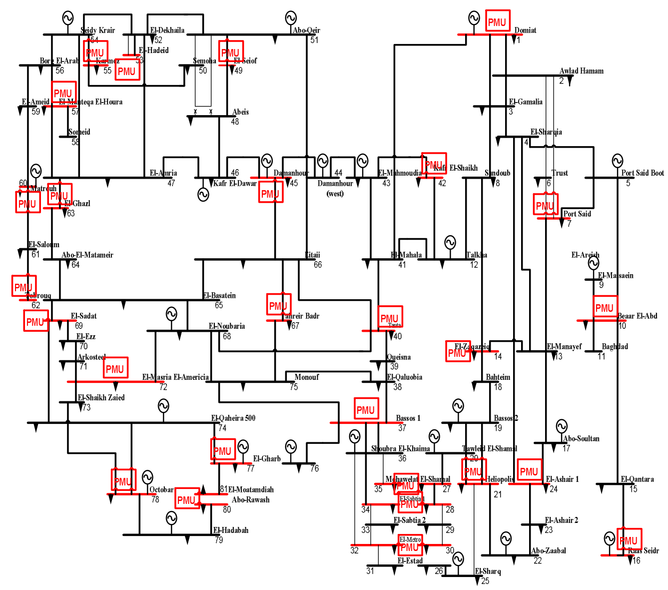

Fig. 8. A large system depicted from the UEPN of Egypt with PMUs' allocation for least minimum residual error of state estimation

The variation of the residual error with different PMUs' numbers and locations are presented in Fig. 9. It starts from the right side by the residual error of the conventional WLSE, which is found to be – 4.9 x 10^{-9}. The residual error decreases with the connection of more PMUs in the optimal locations of the system, until it reaches 1.3 x 10^{-13} for the optimal distributed twenty seven PMUs (shown in Fig. 8). The residual error continues in increasing with positive sign, as the numbers of the connected PMUs increase over the optimally located twenty seven PMUs.

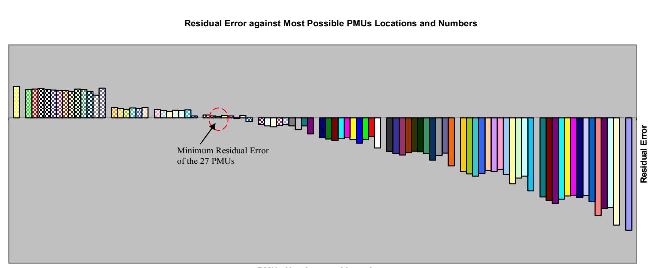

Fig. 9. Residual errors against different numbers of PMUs with different locations on the UEPN of Egypt

VIII. CONCLUSION

Discrete (Integer) Particle Swarm Optimization (DPSO) technique is developed to determine the optimal number and locations for PMUs in power system network for different depth of unobservability. Tree Search Method (TSM) is used to assign a near optimal allocation to be considered as one of the initial conditions of DPSO. DPSO technique is applied on a large system depicted from the Egyptian Unified Electrical Power Network (UEPN) to determine the optimal PMUs' allocation for system complete observability.

Correlating PMUs location to state estimation is very important for power system monitoring and control. PMUs' outputs affect the state estimation analysis in a precious way. It improves the response and the output of the traditional state estimation.

A comparison between PMUs' optimal allocation for system topological observability, and PMUs' optimal number and locations for minimum state estimation residual error is held, using the system depicted from the Egyptian UEPN. It is observed that the locations of PMUs according to state estimation improvement do not need to be similar to these locations according to observability depth. The number of PMUs, used for improving the accuracy of state estimation results is between N/2 and N/3 (N is the number of the system buses), while the number of PMUs used for complete observability is varied between N/3 and N/4. The system parameters, system layout and power flow affect the PMUs' positioning for optimal state estimation. It is seen that for each system there is certain number of PMUs with certain connection that reduces the estimation error significantly. As the number of PMUs' increases over the optimal solution, the estimation analysis begins to magnify the measurements error of the other devices.

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X. BIOGRAPHIES

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He has more than 90 published papers in journals and conferences. He supervised about 35 M.Sc. and Ph.D. thesis (granted), mainly in protection and control on power system and/or machine. His major research interest is in power system protection, measurement and control.

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