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Faults Analysis Theory and Schemes of Four-Phase Power Systems

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***Abstract*—In this paper the analysis of unsymmetrical transversal faults that can occur on a four-phase networks is presented by means of Fortescue Symmetrical Component Transformation (SCT). The various types of fault are analyzed and the connections between sequence networks and the related sequence equations are used to evaluate the phase fault currents.**

The obtained results are compared with those obtained for three-phase and six-phase systems. The related considerations are also extended to the generic *n*-phase systems.

***Index Terms*— Circuit topology, fault currents, Fortescue poly-phase systems.**

I. Introduction

THE analysis of poly-phase networks gives, nowadays, a particularly importance to the four-phase case [1]-[6]. In fact, for a long time, the knowledge of all the advantages coming increasing the number of phases n - related to land occupation, the energetic aspect, the power limit, and reliability - has started a series of theoretical and experimental researches with the aim of quantifying the advantages derived from an increase to n phases compared to the classical three-phase system. These researches have initially investigated, with reference to the transmission issue, the twelve- and six-phase configurations thanks to their easy applicability, by simple modification of the connections of the windings of a three-phase traditional transformer [4]-[5]. In these cases the transpositions introduce high complexity as well as high costs. In an analogous way, the high number of fault configurations has highlighted the difficulty in developing fault analysis and protection design. Under these circumstances, the twelve- and six-phase transmissions did not evolve towards concrete application, except in a few particular cases.

Successively, the researchers have considered the four phase system and studied the advantages deriving from it compared to the classical three-phase one [4]-[6].

Investigators [6] showed that the four wire system is a multi-phase system, similar to the three-phase one, which has the advantages of a multi-phase system but overcome the disadvantages of the six- and twelve-phase systems. In fact, it has a transmitted power 33% higher than the three-phase that leads to an increase of 1.333 times of the power limit and to an improvement in the stability at the same conditions. Furthermore, the change of the ratio V/V_f from $\sqrt{3}$ to $\sqrt{2}$ leads to a corresponding reduction of the electric field; these aspects make feasible a high reduction of the occupied line land and of the environmental impact. The fault configurations for the four-phase systems are much less than that for six-phase systems and other multi-phase systems. Taking into account the achieved results, it is possible to foresee an increasing diffusion of the four-phase solutions in the power systems field, but also in the electromechanical conversion and the power electronic drive related topics. Finally, some recent papers [7], [8] presents a detailed theorical and applicative method to analyze the four-phase systems but give only some notes about fault analysis.

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Since fault study constitutes an important step in the design of protective relaying schemes, in this paper the symmetrical and unsymmetrical faults analysis for four-phase systems is developed. These studies are made by means of Fortescue Symmetrical Component Transformation (SCT) [9], [10] applied to the four-phase case and the related equivalent sequence networks.

In particular the various types of transversal fault that can occur in a four-phase system - the four-phase-to-ground fault, the four-phase fault, the single-phase-to-ground fault, the two-phase-to-ground fault, the two-phase fault, the three-phase-to-ground fault, and the three-phase fault – are analyzed.

So much attention will be debit to the methodological aspects and to the comparison with three-phase and six-phase cases. A generalization of the developed approach to the multiphase case will be also presented.

II. Four Phase Symmetrical Component Transformation

The general Fortescue SCT [9], [10] for n=4 can be expressed by the following matrix equation [11]:

$$\begin{bmatrix} \overline{X}_a \\ \overline{X}_b \\ \overline{X}_c \\ \overline{X}_d \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} \overline{X}^{0} \\ \overline{X}^{+} \\ \overline{X}^{00} \\ \overline{X}^{-} \end{bmatrix} = \left[\overline{A}\right] \cdot \begin{bmatrix} \overline{X}^{0} \\ \overline{X}^{+} \\ \overline{X}^{00} \\ \overline{X}^{-} \end{bmatrix} \tag{1}$$

in which there are, respectively, the zero-, positive-, pseudozero- and negative sequence components.

The inverse relationship, taking into account that:

$$\left[\overline{A}\right]^{-1} = \left[\overline{A}\,*\right]^{T} \tag{2}$$

corresponds to the following relationship:

$$\begin{bmatrix}\overline{X}^{0}\\ \overline{X}^{+}\\ \overline{X}^{00}\\ \overline{X}^{-}\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}\cdot\begin{bmatrix}\overline{X}_a\\ \overline{X}_b\\ \overline{X}_c\\ \overline{X}_d\end{bmatrix}=\left[\overline{A}\right]^{-1}\cdot\begin{bmatrix}\overline{X}_a\\ \overline{X}_b\\ \overline{X}_c\\ \overline{X}_d\end{bmatrix} \qquad (3)$$

III. Analysis of Transversal Faults

In a four-phase system, the possible types of transversal faults are in number of seven, compared with only five in the case of three-phase systems but eleven of six-phase systems. Four-phase fault is the most severe and the least probable one, whereas single-line-to-ground faults are the least severe but the most probable type of fault to occur on a power system. In addition, faults involving two phases may be more frequent in four-phase systems than in three-phase systems but less than in six-phase systems.

In the present investigation, all possible cases of transversal faults are considered using SCT. Sequence networks connection and expression for fault currents and voltages in each case are presented. The various types of fault that can occur in a four-phase system are:

- Four-phase-to-ground fault
- Four-phase fault
- Single-phase-to-ground fault
- Two-phase-to-ground fault
- Two-phase fault
- Three-phase-to-ground fault
- Three-phase fault

In the following, the four-phase system is assumed to be balanced and all faults are assumed to be bolted.

A. Preliminary remarks

In this case the network under analysis – supplied by four symmetric voltages – is shown in Fig. 1a. Generalizing the topological approach – based on substitution theorem already applied in three-phase systems [12] – the general transversal unbalanced fault is represented by four unknown voltage generators presenting the same voltages acting on the fault itself (Fig. 1b). The four generators can be split following the SCT as shown in Fig. 1c. The superposition principle leads to obtain four different, disconnected, balanced and symmetrical networks.

Starting from those, the equivalent one-mesh sequence networks can be deduced.

The sequence equations can be written in the following matrix form:

$$\begin{bmatrix}\overline{V}^{0}\\ \overline{V}^{+}\\ \overline{V}^{00}\\ \overline{V}^{-}\end{bmatrix}+\begin{bmatrix}\overline{Z}^{0} & 0 & 0 & 0\\ 0 & \overline{Z}^{+} & 0 & 0\\ 0 & 0 & \overline{Z}^{00} & 0\\ 0 & 0 & 0 & \overline{Z}^{-}\end{bmatrix}\cdot\begin{bmatrix}\overline{I}^{0}\\ \overline{I}^{+}\\ \overline{I}^{00}\\ \overline{I}^{-}\end{bmatrix}=\begin{bmatrix}0\\ \overline{E}^{+}\\ 0\\ 0\end{bmatrix} \qquad (4)$$

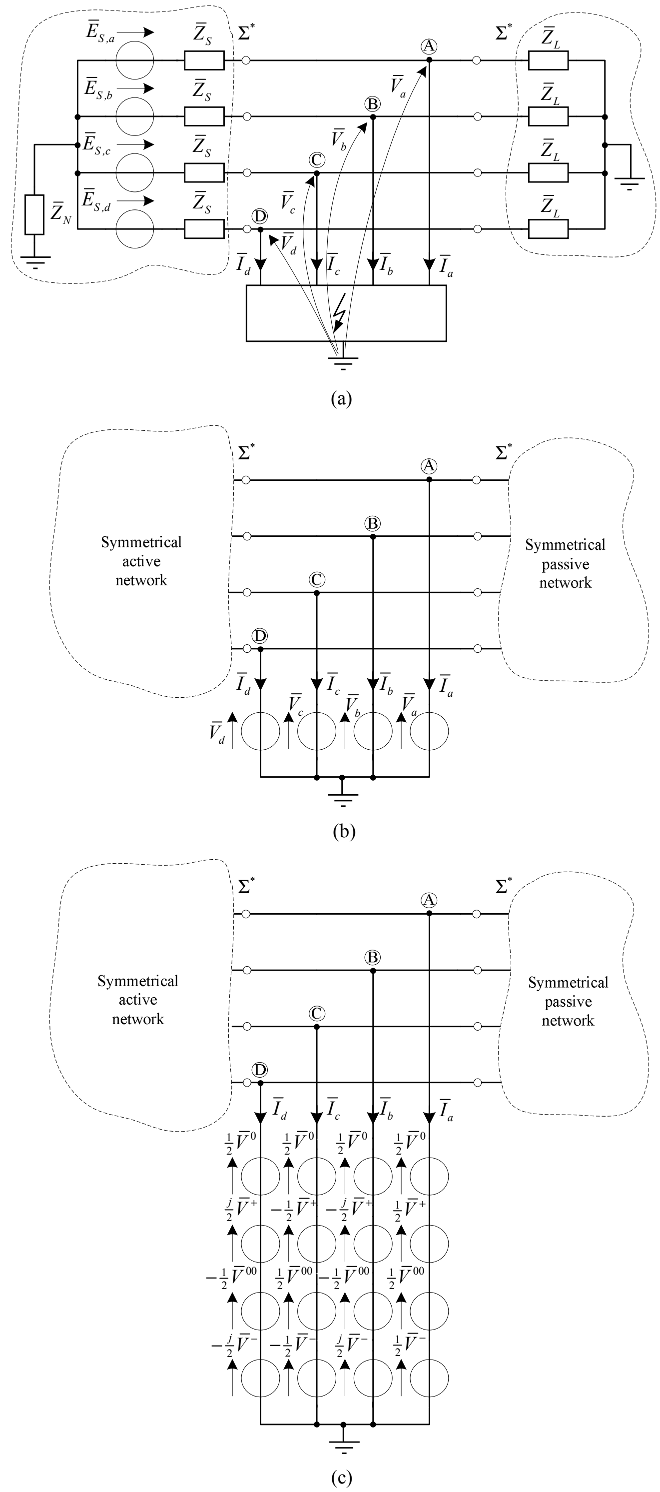

Fig. 1. (a) The four-phase network considered for the transversal fault analysis. (b) Representation of general transversal fault in term of unknown voltages. (c) Analysis using the Fortescue SCT.

Concerning on the sequence impedances formulation, taking into account an uncoupled symmetrical four-phase network, it has [6]-[8]:

$$\begin{cases}\overline{Z}^{k}=\overline{Z} \quad \text{with} \quad k=+,00,-\\ \overline{Z}^{0}=\overline{Z}+4\overline{Z}_N\end{cases} \qquad (5)$$

The pseudozero-sequence impedance does not depend on the neutral impedance $\overline{Z}_N$. On the contrary, the zero-sequence impedance depends on the neutral impedance - like on the three-phase case - but with a multiplying factor 4 instead of 3. Moreover, it is possible to note that there are four relations with four unknown voltage sequence components and four unknown current sequence components. The remaining equations necessary to solve the problem are obtained transforming - by the Fortescue SCT - the phase circuit connections corresponding to the analysed fault configuration. The equivalent sequence network are then obtained for each fault type.

In the case of unsymmetrical faults, for the sake of simplicity, the different type of faults are assumed to occur on particular phases, as in the three-phase case [9]. This, of course, introduces no limitation on the analysis since the system is assumed to be symmetrical up to the point of the faults. An exception is represented by the two-phase and two-phase-to-ground faults. In these cases in fact there is a difference if the fault takes in to account two adjacent phases or not. In the follow the analysis will be performed considering the most onerous condition of not closest phases [13].

Faults sometimes occur simultaneously at two different points which may be widely separated [15]. The general process of analysis is similar to that used in case of single fault, but the symmetrical components of the network must adjust themselves to meet the conditions imposed by the faults at two points instead of at one point. This kind of faults will be analyzed in further works.

B. Four-phase-to-ground and four-phase faults

The four-phase-to-ground fault is the least probable one to occur on power systems. In this case the configuration, shown in Fig. 2a, is symmetrical. The terminal conditions are:

$$\overline{V}_a = \overline{V}_b = \overline{V}_c = \overline{V}_d = 0 \tag{6}$$

The sequence voltages calculated from Fortescue SCT (3) are:

$$\overline{V}^0 = \overline{V}^+ = \overline{V}^{00} = \overline{V}^- = 0 \tag{7}$$

The corresponding sequence networks connection is shown in Fig. 2g. According to the symmetrical conditions, there is not connection between the four sequence networks. Furthermore, the positive-sequence network is the only active one. Basing on these considerations, the sequence currents are:

$$\begin{cases} \overline{I}^+ = \overline{E}^+ / \overline{Z}^+ \\ \overline{I}^{00} = \overline{I}^- = \overline{I}^- = 0 \end{cases} \tag{8}$$

Applying the SCT (1) the following phase currents are obtained:

$$\overline{I}_a = \overline{I}_b = \overline{I}_c = \overline{I}_d = \overline{I}^+ / 2 = \overline{E}^+ \Big/ \left(2 \cdot \overline{Z}^+ \right) \tag{9}$$

Concerning on the four-phase fault, it can be analyzed following the same approach previously developed. The obtained results are expressed by (6) and (9) yet.

C. Single-phase-to-ground fault

This fault, depicted in Fig. 2b, is unsymmetrical like all the following other faults. It is the most probable type of fault that can occur on power systems. The terminal conditions are:

$$\overline{V}_a = 0 \quad , \quad \overline{I}_b = \overline{I}_c = \overline{I}_d = 0 \tag{10}$$

The corresponding sequence voltages and currents are:

$$\begin{cases} \overline{V}^0 + \overline{V}^+ + \overline{V}^{00} + \overline{V}^- = 0 \\ \overline{I}^0 = \overline{I}^+ = \overline{I}^{00} = \overline{I}^- = \overline{I}_a / 2 \end{cases} \tag{11}$$

The sequence network connection is shown in Fig. 2h: the networks are connected in series. This result recalls the one obtained for each multi-phase system when a single-phase-to-ground fault occurs on.

The following sequence currents are obtained:

$$\overline{I}^+ = \overline{E}^+ \Big/ \left(\overline{Z}^+ + \overline{Z}^0 + \overline{Z}^{00} + \overline{Z}^- \right) \tag{12}$$

By the SCT, the phase-voltages are:

$$\begin{cases} \overline{V}_a = 0 \\ \overline{V}_b = \dfrac{-\overline{Z}^0 + \overline{Z}^{00} - j\left(\overline{Z}^+ + \overline{Z}^0 + \overline{Z}^{00} + 2\overline{Z}^-\right)}{\overline{Z}^+ + \overline{Z}^0 + \overline{Z}^{00} + \overline{Z}^-} \cdot \dfrac{\overline{E}^+}{2} \\ \overline{V}_c = \dfrac{-\overline{Z}^0 - \overline{Z}^+ - \overline{Z}^{00} - \overline{Z}^-}{\overline{Z}^+ + \overline{Z}^0 + \overline{Z}^{00} + \overline{Z}^-} \cdot \dfrac{\overline{E}^+}{2} \\ \overline{V}_d = \dfrac{-\overline{Z}^0 + \overline{Z}^{00} + j\left(\overline{Z}^+ + \overline{Z}^0 + \overline{Z}^{00} + 2\overline{Z}^-\right)}{\overline{Z}^+ + \overline{Z}^0 + \overline{Z}^{00} + \overline{Z}^-} \cdot \dfrac{\overline{E}^+}{2} \end{cases} \tag{13}$$

and the phase currents are:

$$\overline{I}_a = \overline{I}^+ / 2 = \overline{E}^+ \Big/ \left[2 \cdot \left(\overline{Z}^+ + \overline{Z}^0 + \overline{Z}^{00} + \overline{Z}^- \right) \right] \tag{14}$$

Comparing (9) and (14), it is possible to note that the four-phase fault is more severe than the single-phase-to-ground one.

D. Two-phase-to-ground and two-phase faults

A two-phase-to-ground fault is shown schematically in Fig. 2c. The fault is arbitrarily assumed between phases *a* and *c*. The terminal conditions are:

$$\overline{I}_b = \overline{I}_d = 0, \qquad \overline{V}_a = \overline{V}_c = 0 \tag{15}$$

The sequence currents calculated from (3) are obtained as:

$$\overline{I}^0 = \overline{I}^{00}, \qquad \overline{I}^+ = \overline{I}^- \tag{16}$$

The sequence voltage components are obtained from (3) as follows:

$$\begin{cases} \overline{V}^0 = -\overline{V}^{00} = \left(\overline{V}_b + \overline{V}_d \right) / 2 \\ \overline{V}^+ = -\overline{V}^- = j\left(\overline{V}_b - \overline{V}_d \right) / 2 \end{cases} \tag{17}$$

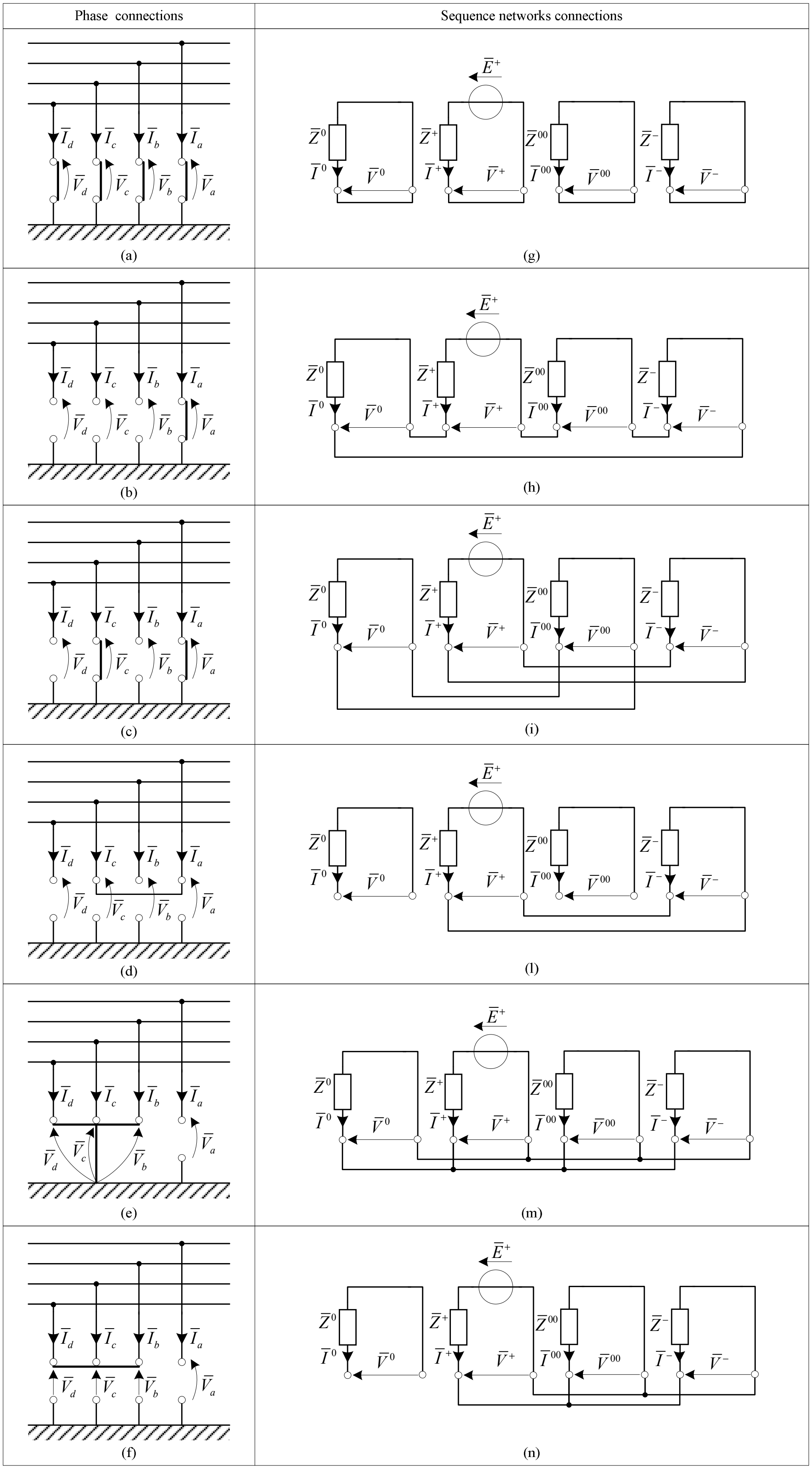

Fig. 2. Summary of transversal fault: phase and sequence networks fault connection.

From (17) it can be easily shown that:

$$\overline{V}^0+\overline{V}^{00}=0,\quad \overline{V}^{+}+\overline{V}^{-}=0 \tag{18}$$

The sequence networks connection is shown in Fig. 2i. Solving the network, we have:

$$\begin{cases}\overline{I}^{+}=\overline{I}^{-}=\overline{E}^{+}\big/\left(\overline{Z}^{+}+\overline{Z}^{-}\right)\\ \overline{I}^{0}=\overline{I}^{00}=0\end{cases} \tag{19}$$

The phase currents obtained from (1) are:

$$\overline{I}_a=-\overline{I}_c=\left(\overline{I}^{+}+\overline{I}^{-}\right)\big/2=\overline{E}^{+}\big/\left(\overline{Z}^{+}+\overline{Z}^{-}\right) \tag{20}$$

It is possible to observe that the voltages of the non faulted phases will remain at their previous values. Moreover, taking into account the (5), the phase currents are equal to the one calculate in the four-phase fault cases.

It is interesting to note that positive- and negative-sequence networks and zero- and pseudozero-sequence networks are connected in series between them. Furthermore, although this is an unbalanced fault involving ground, the zero-sequence currents is zero. The same result can be obtained studying a $n/2$ phase-to-ground fault in n-phase systems, with n even number (i.e. the three-phase-to-ground fault in six-phase systems [13]-[15]). This fault does not correspond any fault in three-phase systems or other odd poly-phase systems.

Concerning the two-phase fault (see Fig. 2d), involving phases a and c again, the terminal conditions are:

$$\overline{I}_a=-\overline{I}_c,\quad \overline{I}_b=\overline{I}_d=0,\quad \overline{V}_a=\overline{V}_c \tag{21}$$

The sequence currents and voltages calculated from (3) are obtained as:

$$\begin{cases}\overline{I}^{+}=\overline{I}^{-}\\ \overline{V}^{+}+\overline{V}^{-}=0\end{cases} \tag{22}$$

These suggest the connection of sequence networks as depicted in Fig. 2l. It is to be noted that zero- and pseudozero-sequence networks are open whereas the positive- and negative- are connected in series.

E. Three-phase-to-ground fault

The considered three-phase-to-ground fault configuration is shown in Fig. 2e. The corresponding terminal conditions are:

$$\overline{I}_a=0\quad,\quad \overline{V}_b=\overline{V}_c=\overline{V}_d=0 \tag{23}$$

by which it is possible to obtain the following sequence voltages and currents:

$$\begin{cases}\overline{I}^{0}+\overline{I}^{+}+\overline{I}^{00}+\overline{I}^{-}=0\\ \overline{V}^{0}=\overline{V}^{+}=\overline{V}^{00}=\overline{V}^{-}\end{cases} \tag{24}$$

that lead to the equivalent network shown in Fig. 4m. The four networks are connected in parallel: this result can be obtained for each n-phase system when a (n-1)-phase-to-ground fault occurs on.

Introducing the following voltage:

$$\overline{U}=\overline{V}^{+}=\overline{V}^{00}=\overline{V}^{-}=\overline{V}^{0}=\frac{\overline{E}^{+}/\overline{Z}^{+}}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}} \tag{25}$$

the sequence currents are:

$$\begin{cases}\overline{I}^{+}=\dfrac{\overline{E}^{+}-\overline{U}}{\overline{Z}^{+}}=\dfrac{\overline{E}^{+}}{\overline{Z}^{+}}\left(\dfrac{1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}\right)\\ \overline{I}^{00}=-\dfrac{\overline{U}}{\overline{Z}^{00}}=-\dfrac{\overline{E}^{+}/\left(\overline{Z}^{+}\overline{Z}^{00}\right)}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}\\ \overline{I}^{-}=-\dfrac{\overline{U}}{\overline{Z}^{-}}=-\dfrac{\overline{E}^{+}/\left(\overline{Z}^{+}\overline{Z}^{-}\right)}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}\\ \overline{I}^{0}=-\dfrac{\overline{U}}{\overline{Z}^{0}}=-\dfrac{\overline{E}^{+}/\left(\overline{Z}^{+}\overline{Z}^{0}\right)}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}\end{cases} \tag{26}$$

Finally, the phase voltages are the following:

$$\begin{aligned}\overline{V}_a&=\left(\overline{V}^{0}+\overline{V}^{+}+\overline{V}^{00}+\overline{V}^{-}\right)\big/2=2\cdot\overline{U}=\\ &=2\left(\overline{E}^{+}/\overline{Z}^{+}\right)\Big/\left(1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}\right)\end{aligned} \tag{27}$$

and the phase currents are:

$$\begin{cases}\overline{I}_b=\dfrac{\overline{E}^{+}}{2\overline{Z}^{+}}\left(\dfrac{1}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}\right)\left(\dfrac{-1-j}{\overline{Z}^{0}}-\dfrac{2j}{\overline{Z}^{-}}+\dfrac{1-j}{\overline{Z}^{00}}\right)\\ \overline{I}_c=-\dfrac{\overline{E}^{+}}{\overline{Z}^{+}}\left(\dfrac{1}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}\right)\left(\dfrac{1}{\overline{Z}^{00}}+\dfrac{1}{\overline{Z}^{0}}\right)\\ \overline{I}_d=\dfrac{\overline{E}^{+}}{2\overline{Z}^{+}}\left(\dfrac{1}{1/\overline{Z}^{+}+1/\overline{Z}^{00}+1/\overline{Z}^{-}+1/\overline{Z}^{0}}\right)\left(\dfrac{-1+j}{\overline{Z}^{0}}+\dfrac{2j}{\overline{Z}^{-}}+\dfrac{1+j}{\overline{Z}^{00}}\right)\end{cases} \tag{28}$$

Taking into account the (5), we obtain:

$$\left|\overline{I}_c\right|=\left|\frac{\overline{E}^{+}}{2\overline{Z}^{+}}\left(\frac{\overline{Z}^{+}+2\overline{Z}_N}{\overline{Z}^{+}+3\overline{Z}_N}\right)\right|\le\left|\frac{\overline{E}^{+}}{2\overline{Z}^{+}}\right| \tag{29}$$

by which it is possible to conclude that the phase currents are lesser than or equal to the phase currents in the four-phase fault.

F. Three-phase fault

The considered three-phase fault configuration is shown in Fig. 2f. The corresponding terminal conditions are:

$$\overline{I}_a=0,\quad \overline{I}_b+\overline{I}_c+\overline{I}_d=0,\quad \overline{V}_b=\overline{V}_c=\overline{V}_d \tag{30}$$

These can be put into sequence components form as follows:

$$\begin{cases}\overline{I}^{0}+\overline{I}^{+}+\overline{I}^{00}+\overline{I}^{-}=0\\ 3\cdot\overline{I}^{0}-\overline{I}^{+}-\overline{I}^{00}-\overline{I}^{-}=0\\ \left(1-j\right)\cdot\overline{V}^{+}-2\overline{V}^{00}+\left(1+j\right)\cdot\overline{V}^{-}=0\\ \overline{V}^{+}-\overline{V}^{-}=0\end{cases} \tag{31}$$

The sequence current and voltage conditions are:

$$\overline{I}^0 = 0, \quad \overline{I}^+ + \overline{I}^{00} + \overline{I}^- = 0, \quad \overline{V}^+ = \overline{V}^{00} = \overline{V}^- \tag{32}$$

These relationships recall the sequence-networks connection shown in Fig. 2n. It is important to note that this connection corresponds to the two-phase fault in three-phase systems and five-phase fault in six-phase ones [13]-[15]: this result can be obtained for each *n*-phase system when a (*n*-1)-phase fault occurs on.

Solving the network, we have:

$$\begin{cases} \overline{I}^+ = \dfrac{\overline{E}^+}{\overline{Z}^+}\left(\dfrac{1/\overline{Z}^{00} + 1/\overline{Z}^-}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-}\right) \\ \overline{I}^{00} = -\dfrac{\overline{E}^+/\left(\overline{Z}^+\overline{Z}^{00}\right)}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-} \\ \overline{I}^- = -\dfrac{\overline{E}^+/\left(\overline{Z}^+\overline{Z}^-\right)}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-} \end{cases} \tag{33}$$

Applying the SCT, the following phase voltages are obtained:

$$\begin{cases} \overline{V}_b = \overline{V}_c = \overline{V}_d = \dfrac{1}{2}\dfrac{-\overline{E}^+/\overline{Z}^+}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-} \\ \overline{V}_a = \dfrac{3}{4}\dfrac{\overline{E}^+/\overline{Z}^+}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-} \end{cases} \tag{34}$$

and the phase currents are:

$$\begin{cases} \overline{I}_b = \dfrac{\overline{E}^+}{2\overline{Z}^+}\left(\dfrac{1}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-}\right)\left(\dfrac{1-j}{\overline{Z}^{00}} - \dfrac{2j}{\overline{Z}^-}\right) \\ \overline{I}_c = -\dfrac{\overline{E}^+}{\overline{Z}^+}\left(\dfrac{1}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-}\right)\dfrac{1}{\overline{Z}^{00}} \\ \overline{I}_d = \dfrac{\overline{E}^+}{2\overline{Z}^+}\left(\dfrac{1}{1/\overline{Z}^+ + 1/\overline{Z}^{00} + 1/\overline{Z}^-}\right)\left(\dfrac{1+j}{\overline{Z}^{00}} + \dfrac{2j}{\overline{Z}^-}\right) \end{cases} \tag{35}$$

Taking into account the (5), we have:

$$\left|\overline{I}_c\right| = \left|\frac{1}{3}\frac{\overline{E}^+}{\overline{Z}^+}\right| \tag{36}$$

so the phase currents are lesser than in the four-phase fault.

IV. Conclusions

The general theory and the methodological approach to study unsymmetrical faults in four-phase symmetrical systems are presented. The obtained results have been compared to the ones obtained in three-phase and six-phase cases.

Furthermore, thanks to the general proprieties of the Fortescue transformation applied to multi-phase networks, the general rules for the sequence networks connection, for every types of fault that can occur on a multi-phase system, are presented.

Further research work on the subject could concern on the generalization to the dynamical condition and the analysis of simultaneous faults.

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VI. Biographies

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