# A Practical Probabilistic Design Procedure for LV Residential Distribution Systems

R. Herman, Senior Member, IEEE & C.T. Gaunt

described in the published paper was then developed.

#### I. DECLARATION

This paper was accepted for publication in the *IEEE TRANSACTIONS ON POWER DELIVERY* (Digital Object Identifier 10.1109/TPWRD.2008.919041) and is presented at the 2009 IEEE Bucharest Power Tech Conference with the authorization of the IEEE Power & Energy Society.

# $\mathit{Index Terms}{-\!\!\!-\!\!\!Probabilistic methods, (LV)}$ distribution, load modeling.

# II. BACKGROUND

THERE are two basic styles in low voltage distribution – the American system and the Continental system. Apart from the differences in voltage levels and in operating frequencies there are also important differences in application. The American system usually employs numerous MV:LV transformers with short "lateral" LV feeders. In the Continental system there are fewer transformers and longer LV feeders. This is the practice in mostly used in Africa. The reason for this is chiefly historical, since the continent was colonized chiefly by European powers and were present at the advent of electricity.

Being supplied with electrical power is regarded as a symbol of progress in most developing countries. In recent years a drive has been launched in South Africa to connect millions of new and existing homes. This development emphasized the need to revisit residential load models and their application in the planning and design of LV distribution networks.

Traditional design procedures are deterministic and use average values for the uncertain loads. This average representative load value is often referred to as the diversity maximum demand (ADMD) and is based on the mean demand at system peak for a large number (typically about 1000) of customers belonging to a particular customer class. When considering the collective loading of a smaller group of customers, the loss of diversity is usually accounted for by applying a correction factor (diversity factor or the inverse, coincidence factor). Concern at the discrepancy of design calculations led to a comparative study of various deterministic design procedures in South Africa, revealing a large variation in results. The probabilistic design procedure

# III. LOAD DATA COLLECTION AND MODELING

The most important requisite in the design of LV distribution networks is an accurate estimate of the design loads. Residential loads are subject to considerable statistical variation and therefore both their electrical and statistical characteristics must be modeled. This problem was addressed by an extensive load research program in South Africa, commencing in 1988 and continued for 8 years [1].

# A. The electrical parameters of a load

In an electrical sense, loads may be represented as resistance, current or power. The form in which the load is modelled has implications for both the gathering of data and the analysis of the network.

The representation of loads as current sinks provides an acceptable and convenient model for the following reasons:

- It is the best representation of the mixed loads typical for appliances of residential customers.
- The magnitude of the current-modelled load is independent of the voltage drop along a feeder or the distance from the source.
- The measurement of loads as currents can be carried out accurately and inexpensively.
- The representation of loads as power requires an iterative volt-drop calculation method.
- The current model is consistent with the observed behaviour of real loads. The load reduces with reduced voltage, but to a lesser extent than would be expected of a pure resistance load.
- In traditional methods of volt-drop calculations in LV feeders, loads specified as powers are usually converted to equivalent load current at rated voltage.

The averaging or integrating period used to measure loads will affect slightly the validity of the current representation of loads.

#### B. Typical Load Current Histogram

By collecting a statistically sufficient number of load current samples it is possible to identify the common interval in which the maximum demand for the group occurs. It is also possible then to determine the distribution of currents at that interval. A typical histogram of load currents at the time of maximum demand is shown in Fig. 1. Extensive work was done to identify the most appropriate probability density

R Herman and C.T. Gaunt are with the University of Cape Town, South Africa, (e-mails: ronald.herman@uct.ac.za and CT.Gaunt@uct.ac.za)

function (pdf) to represent the load current data [2]. For a variety of reasons the Beta pdf was found to be the most suitable.



Fig. 1 The Beta probability density function fitted to load current data

# IV. DEVELOPMENT OF A PROBABILISTIC VOLTAGE DROP CALCULATION METHOD

When designing the LV networks in the European style of distribution (or when the laterals have appreciable length with several connected consumers) it is found that voltage drop is the major constraint in sizing conductors. A probabilistic method was developed to calculate voltage drops in LV feeders using load currents that are distributed according to a given Beta probability density function with parameters  $\alpha$ ,  $\beta$  and scaling value C. Theoretical development of the various Beta parameter algorithms for three-phase and dual-phase networks are described in several published articles [3]-[4]. Essentially, the analysis derives the Beta parameters of the resultant voltage drop using two statistical moments that are expressed in terms of the network configuration and the load

#### V. TYPICAL SPREADSHEET INPUTS AND OUTPUTS

While the analytical process appears to be complex the resultant equations are all linear and may easily be incorporated into a computer program, such as a typical spreadsheet. The inputs required are:

- $\alpha$ ,  $\beta$  and C parameters of the loads at each node
- number of consumers on each phase at each node
- length and conductor size for each section
- operating temperature for the correction of the conductor resistance
- specified level of confidence (conversely, level of risk)

Probabilistic calculations of voltage drops may thus be performed by most designers. An extensive knowledge of probability theory is not essential. Fixed rather than probabilistically distributed loads (such as pumps for example) may be represented in the calculation procedure by using the equivalent Beta parameters as reported in [5].

# VI. GENERAL LOAD PARAMETER FORECASTING

Residential load research may not be a priority in many countries. The work presented in this section is intended to facilitate the use of the Beta pdf in the absence of detailed load data.

The design of LV distribution systems in most countries is based on an estimated average diversified demand (ADMD). Based on the large amount of load data collected in South Africa we have examined, for various residential customer classes, the relationship between d and the coefficient of variation,  $\gamma$ , where d is demand in kVA.

Our research has shown that for all the customer residential classes in South Africa the  $\gamma$  vs. d curve may be approximated by

$$\gamma = 1.1427 d^{-0.412} \qquad (1)$$

Fig. 2 shows the correlation between the curve (1) and values predicted by a forecasting tool that uses collected load data. An  $R^2$  correlation coefficient of 96.07% is indicated.



Fig. 2 Coefficient of Variation vs. ADMD [kVA]

In the absence of extensive load data this approximation should yield acceptable results in most developing countries. Once the value of  $\gamma$  has been determined the  $\alpha$ - and  $\beta$ -parameters of the load current may be calculated and thence the calculation of the probabilistic voltage drops.

### VII. CONCLUSIONS

The South African Electricity Distribution Industry has had more than 10 years experience with the algorithm and has accepted it as the prescribed design procedure [6]. The validation procedure involved Monte Carlo simulation using actual measured sample load currents (sample size greater than 70).

#### VIII. REFERENCES

- R Herman & C T Gaunt, "Measurement and representation of maximum demand for individual and grouped domestic consumers including constraints", *Proc. CIRED-1991 Conf*, Liege., Belgium, April 1991.
- [2] R. Herman & J.J. Kritzinger, "The statistical description of grouped domestic electrical load currents", *Elect. Power Syst. Res.*, vol 27 pp 43 – 48, 1993.
- [3] R. Herman, J.S. Maritz & J.H.R Enslin, "The analysis of voltage regulation using the beta distribution model", *Elect. Power Syst. Res.*, vol 29 pp213 – 216, 1993.
- [4] R. Herman & J.S. Maritz, "Voltage regulation algorithm for a bi-phase distribution system feeding residential customers using a Beta pdf. load model", *Elect. Power Syst. Res.*, vol 43, no. 2, pp 77 – 80, 1997.
- [5] R. Herman, S.W. Heunis, "Load Models for Mixed-class Domestic and Fixed, Constant Power Loads for use in Probabilistic LV Feeder analysis", *Elect. Power Syst. Res.*, vol 66, no. 2, pp 149-153, 2003
- [6] NRS 034-1: Electricity Distribution Guidelines For The Provision Of Electrical Distribution Networks In Residential Areas, Part 1: Planning and design of distribution systems, Standards South Africa, Pretoria, South Africa.



Ron Herman obtained a BSc (Eng.) degree from the University of Cape Town, Rondebosch, Western Cape, South Africa, and the MSc and PhD. degrees in Electrical Engineering from the University of Stellenbosch, Stellenbosch, South Africa.

After working with ESKOM he joined the staff of the University of Stellenbosch. He developed the Herman-Beta method that is used in South African low voltage distribution design. He is currently a parttime Research Officer at the University of Cape Town, Rondebosch, Western Cape, South Africa.

Trevor Gaunt is a Professor in the Department of Electrical Engineering, University of Cape Town, Rondebosch, Western Cape, South Africa.