Paper accepted for presentation at 2009 IEEE Bucharest Power Tech Conference, June 28th - July 2nd, Bucharest, Romania

Optimal Placement of Voltage Regulators in Distribution Systems

C.A.N. Pereira and C.A. Castro, *Senior member, IEEE*

Resumo **– A two-step algorithm for the optimal placement of voltage regulators in distribution systems is presented in this paper. In the first step, voltage regulators are placed (and the tap position is determined) at candidate buses, aiming at minimizing voltage drops and real power losses. In the second step, an attempt to reduce the number of voltage regulators is made, taking into consideration economical aspects (minimization of installation and maintenance costs). It is shown that the proposed algorithm is fast, efficient and provides reliable results.**

Palavras Chaves **– Voltage regulators, distribution systems, voltage control, loss minimization, cost minimization.**

I. INTRODUCTION

THE voltage drop along primary distribution systems has been a crucial operating problem, especially for lengthy feeders, with an significant load concentration at their ends, and particularly in radial rural feeders. So, utilities look for solutions for this problem, from both technical and economical standpoints. The main goal is to keep high service quality indicators, according to the consumers needs and the requirements of the regulatory agencies of the electric sector.

Many papers can be found in the literature where capacitor banks are used for minimizing power losses and improving voltage levels [1], [2], [3], [4]. However, papers about voltage regulator (VR) placement are few, and there is plenty of room for new and/or improved models and algorithms.

Among the papers about the optimal placement of VR we point out [5] and [6], since they inspired the development of the algorithm proposed in this paper. In [5] VRs are placed in the system with the main objective of minimizing voltage deviations from a nominal value (reference). In [6], the algorithm comprises two steps. In the first one, a preliminary VR placement is defined so as to meet technical criteria. In the second step, an objective function based on economical aspects is defined and the algorithm aims to reducing the number of VRs.

In this paper a two-step procedure is also used. The basic ideas of [6] are used for the process of reducing the number of VRs (economical analysis), however, the initial placement of VRs based on technical aspects is original, so that the placement as well as the VRs' tap positions is defined by using a modified version of the objective function of [5], to meet voltage drop and power losses criteria.

The method proposed in this paper showed to be efficient computationally, and the results are reliable and coherent for practical applications.

II. SOME BASIC IDEAS

A. Load flow

The load flow method used in this paper is the back-forward sweep, described in detail in [7]. It presents excellent convergence characteristics and can be applied to radial as well as weakly meshed systems. It also allows incorporating limits (of transformer taps, for instance), and controls (of distributed generator buses, for instance). In the back sweep step, currents are acumulated starting from the end buses towards the substation. In the forward sweep step, bus voltages are updated starting from the substation towards the end buses.

B. Determination of critical paths

Prior to running the VR placement algorithm, the operating state of the system is determined by load flow calculation. The buses with the lowest voltage magnitudes are stored, along with the buses that form the paths towards the substation. These set of buses are hereafter called the critical paths.

C. Objective function considering technical aspects

The constraints regarding voltage drops and power losses are represented by percentual factors that are computed for each system configuration during the VR placement procedure. These factors are used as figures of merit to determine the best configurations. Note that each configuration has VRs placed at different positions, with different tap positions.

The voltage drop percentual factor $Fat_V\%$ indicated the quality of a certain configuration in terms of voltage profile. It is based on [5] and defined as

$$Fat_V\,\% = \frac{\sum_{i=1}^{N}(V_{nom} - V_i^f)^2}{\sum_{i=1}^{N}(V_{nom} - V_i^0)^2} \cdot 100, \qquad (1)$$

where N is the number of buses, V_{nom} is the system's nominal voltage, and V_i^0 and V_i^f are the voltage magnitudes at bus i respectively at the initial configuration and at some tentative

---

C.A.N. Pereira is with Rede Energia, Brazil, and C.A. Castro is with the University of Campinas (UNICAMP), Brazil (e-mails: carlos.nogueira@redecaiua.com.br; ccastro@ieee.org).

978-1-4244-2235-7/09/$25.00 ©2009 IEEE

configuration involving the placement of VRs along the critical path.

Likewise, the power losses percentual factor $Fat_P\%$ is defined as

$$Fat_P\% = \frac{\sum_{j=1}^{M} PL_j^f}{\sum_{j=1}^{M} PL_j^0} \cdot 100, \qquad (2)$$

where M is the number of branches, and PL_j^0 and PL_j^f are the power losses at branch j respectively at the initial configuration and at some tentative configuration involving the placement of VRs along the critical path.

From (1) and (2) it is possible to define an objective function that takes into account the technical aspects described above as

$$FT = pv \cdot Fat_V\% + pp \cdot Fat_P\%, \qquad (3)$$

where pv and pp are weights that can be defined as:

- $pv = 1$ and $pp = 0$ - whenever voltage drop only is to be considered,
- $pv = 0$ and $pp = 1$ - whenever power losses only are to be considered, or
- $pv = 0.5$ and $pp = 0.5$ - whenever both aspects are to be considered.

FT is computed for each system configuration during the VR placement procedure. Smaller FTs indicate the best configurations regarding VR placements from the technical standpoint. Of course, all constraints (voltage magnitudes, current flows, tap position limits, etc.) are met.

D. Objective function considering economical aspects

The annual investment and maintenance costs of all VRs connected to the system must also be considered, as proposed in [6].

The cost associated to real power losses without VRs is given by

$$Fc_{\max} = kp \cdot \sum_{j=1}^{M} PL_j^{\max} + ke \cdot \sum_{\tau} \left\{ \sum_{j=1}^{M} PL_j(\tau) \cdot \tau \right\}, \qquad (4)$$

where kp is the annual demand cost [U\$/kW.yr], $PL_j^{\max}$ are the power losses at branch j for heavy load conditions [kW], ke is the energy cost [U\$/kWh], τ is the time period (where the total time period is 8760 hours) for which the power losses are constant, and $PL_j(\tau)$ are the power losses at branch losses during time period τ [kW].

The cost associated to real power losses after the pleacement of VRs are similar to (4), and is given by

$$Fc,r = kp.\sum_{j=1}^{M} PL_{j\max}^r + ke.\sum_{\tau} \left\{ \sum_{j=1}^{M} PL_j^r(\tau).\tau \right\}, \qquad (5)$$

where superscript r stand for *regulator*.

From (4) and (5) it is possible to define an objective function that takes into account the economical aspects described above as

$$FE = \left(Fc_{\max} - Fc,r\right) - C_{reg}, \qquad (6)$$

where C_{reg} is the annual investment and maintenance cost given by

$$\begin{aligned} C_{reg} &= \sum_{i=1}^{N} r_i \cdot C_{reg,i} \\ &= \sum_{i=1}^{N} r_i \cdot \left[Ca,r_i \cdot A^{-1}(i_a,T) + Cs,r_i \right] \end{aligned}, \qquad (7)$$

where $r_i = 1$ when a VR is placed at bus i, otherwise $r_i = 0$. Ca,r_i is the VR investment cost [U\$], Cs,r_i is the VR annual maintenance cost [U\$/yr], and $A^{-1}(i_a,T)$ is the capital recovery factor, given by

$$A^{-1}(i_a,T) = i_a.\frac{(1+i_a)^T}{(1+i_a)^T - 1}, \qquad (8)$$

where i_a is annual interest rate (no inflation) and T is the expected life for the VR.

Objective function (6) is computed after the VR placement algorithm is carried out considering technical aspects only. Whenever more than one VR are placed in the system, the method proposed in [6] is carried out to reduce the number of VRs, thus maximizing objective function (6).

III. VR PLACEMENT ALGORITHM

Part I: Selection, placement and control of VRs

(A) Compute the operating point through a load flow calculation. Identify the critical paths.

(B) For each critical path do:
- **(1)** Place a VR at the end node of the critical path
- **(2)** Run a load flow, setting the VR's tap position so as to eliminate the voltage violation at this node.
- **(3)** Compute FT for this configuration.
- **(4)** Move the VR upstream to the next bus of the critical path and go back to step **(2)**.
- **(5)** The procedure is interrupted when the substation bus is reached.

(C) The VR is placed at the bus which resulted in the best *FT.*

Each load flow is run taking the last solution as a new starting point, thus reducing the number of load flow iterations. Note that Part I seeks the best alternative based on technical aspects.

Part II: Reduction of the number of VRs

The method proposed in [6] determines possible paths for reallocating VRs in order to minimize the number of VRs. The procedure is described below.

(A) The VRs are moved upstream (towards the substation) as far as no voltage violations are detected. Therefore, each

VR will have a path, for which voltage regulation is possible.

(B) Path pairs (for each combination of two VRs) are defined.

(C) If there are common buses in the path pairs, one only VR will be placed at the common bus, replacing both VRs initially placed.

(D) Compute objective function *FE*.

The goal of the procedure of Part II is to obtain the best alternative according to economical aspects, by reducing the number of VRs.

IV. SIMULATION RESULTS

Fig. 1 shows the one-line diagram of a 12.66kV, 70-bus distribution system [2], corresponding to a portion of the PG&E distribution system. The thicker lines represent the areas with voltage violations. Voltage magnitudes are considered as acceptable if they fall within a +5/-7% range (from the nominal voltage). The original data from [2] were modified to simulate the system under heavy load condition. In this case, the power supplied by the substation is 20.7MVA. The worst voltage magnitude violation occurs at critical bus 36 (9.55%).

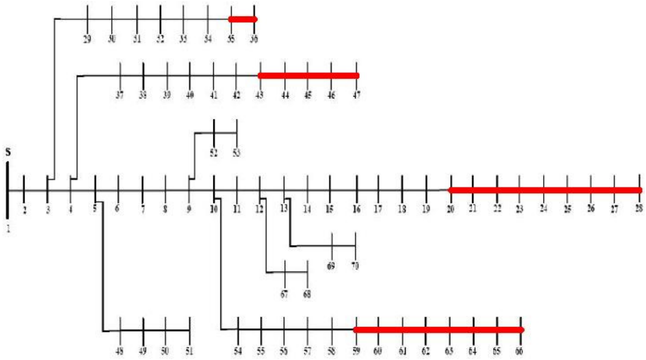

Fig. 1. 70-bus distribution system of [2].

Table I shows the system's data. The basic configuration, without VRs, is shown in the first seven columns. The voltage violations are also highlighted.

Fig. 2 shows the system's voltage profiles. The dashed line shows the initial voltage profile, without any VRs. Note the voltage violations (below the 0.93pu horizontal line).

The proposed method was implemented using Turbo-Pascal (Delphi). The CPU time for the simulation that will be described below was 1s with an AMD - Athlon 64 Mobile, 1.8 GHz. The simulation was carried out considering *pv=1* and *pp=0* in (3), so that the results of the proposed method could be compared to those of [6].

Table II shows the results of Part I for the proposed method and the method of [6]. At this point, the latter showed to be simpler and faster, resulting in the placement of four VRs. The proposed method reached three VRs, which is certainly a better solution from the economical point of view.

TABLE I
BASIC DATA FOR THE 70-BUS SYSTEM (COLUMNS 1 TO 7) AND RESULTS AFTER VR PLACEMENT (COLUMNS 8 AND 9)

Branch		Impedance [Ω]		To-bus power		To-bus voltage [pu]		
From	To	R	X	P [kW]	Q [kVAr]	Without VRs	Proposed	Ref. [6]
1	2	0.0005	0.0012	0.00	0.00	0.9999	0.9999	0.9999
2	3	0.0005	0.0012	0.00	0.00	0.9997	0.9997	0.9997
3	4	0.0000	0.0000	0.00	0.00	0.9997	0.9997	0.9997
4	5	0.0015	0.0036	0.00	0.00	0.9996	0.9996	0.9996
5	6	0.0251	0.0294	0.00	0.00	0.9987	0.9987	0.9987
6	7	0.3660	0.1864	2.63	2.16	0.9883	0.9886	0.9886
7	8	0.3811	0.1941	40.37	2.16	0.9775	0.9781	0.9781
8	9	0.0922	0.0470	74.66	53.43	0.9749	0.9756	0.9756
9	10	0.0493	0.0251	30.00	21.62	0.9736	0.9987	0.9743
10	11	0.8190	0.2707	28.00	20.00	0.9651	0.9905	0.9904
11	12	0.1872	0.0619	145.50	103.83	0.9632	0.9886	0.9886
12	13	0.7114	0.2351	145.50	103.83	0.9571	0.9826	0.9826
13	14	1.0300	0.3400	8.13	5.46	0.9498	0.9756	0.9755
14	15	1.0440	0.3450	8.13	4.56	0.9426	0.9685	0.9685
15	16	1.0580	0.3496	0.00	0.00	0.9353	0.9614	0.9614
16	17	0.1966	0.0650	45.53	30.59	0.9339	0.9601	0.9601
17	18	0.3744	0.1238	49.50	35.33	0.9315	0.9577	0.9577
18	19	0.0047	0.0016	49.50	35.33	0.9315	0.9577	0.9577
19	20	0.3276	0.1083	0.00	0.00	0.9296	0.9559	0.9559
20	21	0.2106	0.0696	50.95	40.64	0.9284	0.9547	0.9547
21	22	0.3416	0.1129	113.95	81.30	0.9266	0.9530	0.9529
22	23	0.0140	0.0046	95.29	73.55	0.9265	0.9529	0.9529
23	24	0.1591	0.0526	0.00	0.00	0.9260	0.9524	0.9523
24	25	0.3463	0.1145	88.17	60.01	0.9248	0.9512	0.9512
25	26	0.7488	0.2475	0.00	0.00	0.9227	0.9492	0.9491
26	27	0.3089	0.1021	164.00	89.99	0.9218	0.9484	0.9483
27	28	0.1732	0.0572	184.00	99.99	0.9216	0.9481	0.9481
3	29	0.0044	0.0108	860.01	685.55	0.9993	0.9993	0.9993
29	30	0.0640	0.1565	460.10	385.55	0.9946	0.9947	0.9947
30	31	0.3978	0.1315	0.00	0.00	0.9824	0.9830	0.9830
31	32	0.0702	0.0232	0.00	0.00	0.9803	0.9809	0.9809
32	33	0.3510	0.1160	0.00	0.00	0.9695	0.9705	0.9705
33	34	0.8390	0.2816	1887.40	897.80	0.9438	0.9456	0.9456
34	35	1.7080	0.5646	890.50	800.67	0.9165	0.9731	0.9731
35	36	1.4740	0.4873	915.76	783.87	0.9045	0.9618	0.9618
4	37	0.0044	0.0108	660.01	485.55	0.9991	0.9991	0.9992
37	38	0.0640	0.1565	626.01	485.55	0.9913	0.9916	0.9917
38	39	0.1053	0.1230	0.00	0.00	0.9837	0.9843	0.9845
39	40	0.0304	0.0355	824.00	787.12	0.9816	0.9822	0.9824
40	41	0.0018	0.0021	524.00	487.12	0.9814	0.9821	0.9823
41	42	0.7283	0.8509	717.60	597.50	0.9421	0.9446	0.9454
42	43	0.3100	0.3623	0.00	0.00	0.9282	0.9847	0.9326
43	44	0.0410	0.0478	860.00	680.28	0.9264	0.9830	1.0008
44	45	0.0092	0.0116	0.00	0.00	0.9261	0.9827	1.0005
45	46	0.1089	0.1373	892.28	863.61	0.9223	0.9791	0.9970
46	47	0.0009	0.0012	1899.20	976.36	0.9223	0.9791	0.9970
5	48	0.0034	0.0084	0.00	0.00	0.9996	0.9996	0.9996
48	49	0.0851	0.2083	79.05	56.40	0.9985	0.9985	0.9985
49	50	0.2898	0.7091	84.68	274.48	0.9952	0.9952	0.9952
50	51	0.0822	0.2011	384.69	274.48	0.9946	0.9947	0.9947
9	52	0.0928	0.0473	40.54	28.33	0.9749	1.0000	0.9756
52	53	0.3319	0.1114	3.61	2.68	0.9749	0.9999	0.9755
10	54	0.1740	0.0886	4.35	3.49	0.9709	0.9961	0.9961
54	55	0.2030	0.1034	26.36	18.97	0.9678	0.9931	0.9930
55	56	0.2842	0.1447	24.00	17.12	0.9635	0.9889	0.9889
56	57	0.2813	0.1433	0.00	0.00	0.9593	0.9848	0.9848
57	58	1.5900	0.5337	0.00	0.00	0.9378	0.9640	0.9639
58	59	0.7837	0.2630	0.00	0.00	0.9273	0.9537	0.9537
59	60	0.3042	0.1006	2.00	72.08	0.9232	0.9497	0.9497
60	61	0.3861	0.1172	0.00	0.00	0.9182	0.9449	0.9448
61	62	0.5075	0.2555	1244.00	887.73	0.9108	0.9377	0.9376
62	63	0.9740	0.0496	32.00	22.84	0.9086	0.9355	0.9355
63	64	0.1450	0.0738	0.00	0.00	0.9082	0.9352	0.9351
64	65	0.7105	0.3619	227.01	161.62	0.9063	0.9333	0.9333
65	66	1.0410	0.5302	59.01	41.74	0.9057	0.9327	0.9327
12	67	0.2012	0.0611	18.00	12.85	0.9632	0.9886	0.9885
67	68	0.0047	0.0014	18.00	12.85	0.9632	0.9886	0.9885
13	69	0.7394	0.2444	28.00	19.98	0.9567	0.9823	0.9823
69	70	0.0047	0.0016	28.00	19.98	0.9567	0.9823	0.9823

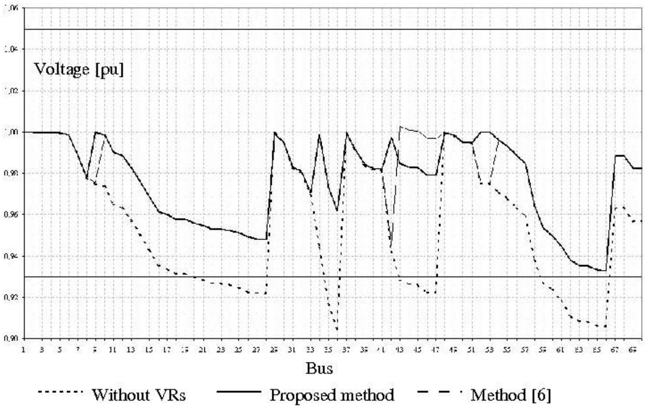

Fig. 2. Voltage profiles for the 70-bus system.

TABLE II
COMPARISON OF RESULTS FOR THE 70-BUS SYSTEM – PART I

70-bus system	Initial	Proposed Method	Method of [6]
Total real losses [kW]	926.79	857.46	828.89
Total reactive losses [kVAr]	656.10	604.69	586.25
Worst violation [%] / Critical bus	9.55 / 36	6.73 / 66	6.76 / 59
Number of load flow iterations	4	106	20
VR bus / Tap position	without VRs	9 / +4 34 / +9 42 / +9	34 / +9 20 / +12 59 / +12 43 / +12

Table III shows the main final results for the 70-bus system, including those provided by the proposed method and for the method of [6]. The final solution provided by the proposed method was the one obtained after Part I, whereas the method of [6] was able to reduce the number of VRs to three. Fig. 2 shows the voltage profiles after the VR placement procedures (proposed and of [6]).

TABLE III
COMPARISON OF RESULTS FOR THE 70-BUS SYSTEM - FINAL

70-bus system	Initial	Proposed Method	Method of [6]
Total real losses [kW]	926.79	857.46	850.00
Total reactive losses [kVAr]	656.10	604.69	595.10
Worst violation [%] / Critical bus	9.55 / 36	6.73 / 66	6.74 / 43
Number of load flow iterations	4	138	172
VR bus / Tap position	without VRs	9 / +4 34 / +9 42 / +9	10 / +4 34 / +9 43 / +12

By comparing the results, it is clear that the overall number of iterations provided by the proposed method is smaller. Moreover, the proposed method tends to place VRs with smaller tap positions. This detail can be an important advantage in the long run, since the same VR can be used for in a larger period of time in a steadily increasing load scenario, as it is the case nowadays.

Tables IV and V show the results provided by the proposed method for different weights in the objective function.

TABLE IV
PROPOSED METHOD – PART I

70-bus system	pv = 1.0 pp = 0.0	pv = 0.5 pp = 0.5	pv = 0.0 pp = 1.0
Total real losses [kW]	857.46	857.46	842.43
Total reactive losses [kVAr]	604.69	604.69	598.45
Worst violation [%] / Critical bus	6.73 / 66	6.73 / 66	6.13 / 16
Number of load flow iterations	106	106	451
VR bus / Tap position	9 / +4 34 / +9 42 / +9	9 / +4 34 / +9 42 / +9	34 / +9 58 / +9 42 / +9 16 / +10

TABLE V
PROPOSED METHOD – FINAL RESULTS

70-bus system	pv = 1.0 pp = 0.0	pv = 0.5 pp = 0.5	pv = 0.0 pp = 1.0
Total real losses [kW]	857.46	857.46	857.59
Total reactive losses [kVAr]	604.69	604.69	604.76
Worst violation [%] / Critical bus	6.73 / 66	6.73 / 66	6.73 / 36
Number of load flow iterations	138	138	864
VR bus / Tap position	9 / +4 34 / +9 42 / +9	9 / +4 34 / +9 42 / +9	34 / +9 10 / +4 42 / +9

After Part I, the power losses are indeed smaller for $pv = 0$ and $pp = 1$. However, economical factors influence the final results for this system, which are similar to the other cases.

V. CONCLUSION

The method proposed in this paper showed to be efficient, providing adequate alternatives for meeting both technical and economical constraints. It is important to point out that in an ever increasing demand scenario, the appropriate placement of VRs with smaller tap positions, while abiding by all electrical and economical constraints, is an important feature, since the VRs can be kept in operation for a longer period of time.

VI. REFERENCES

[1] M. E. Baran and F. F. Wu, "Optimal Sizing of Capacitors Placed on a Radial Distribution System," *IEEE Trans. on Power Delivery*, vol. 4, n. 1, pp. 735–743, Jan. 1989.

[2] M. E. Baran and F. F. Wu, "Optimal Capacitor Placement on Radial Distribution System," *IEEE Trans. on Power Delivery*, vol. 4, n. 1, pp. 725–734, Jan. 1989.

[3] H. D. Chiang, I. C. Wang, and G. Darling, "Optimal Capacitor Placement, Replacement and Control in Large-Scale Unbalanced Distribution Systems: System Solution Algorithms A Numerical Studies," *IEEE Trans. on Power Systems*, vol. 10, n. 1, pp. 363–369, Feb. 1995.

[4] H. D. Chiang, I. C. Wang, and G. Darling, "Optimal Capacitor Placement, Replacement and Control in Large-Scale Unbalanced Distribution Systems: System Modeling and a New Formulation," *IEEE Trans. on Power Systems*, vol. 10, n. 1, pp. 356–362, Feb. 1995.

[5] A. S. Safigianni and G. J. Salis, "Optimum Voltage Regulator Placement in a Radial Power Distribution Network", *IEEE Trans. on Power Systems*, vol. 15, n. 2, pp. 879-886, May 2000.

[6] M. F. Medeiros Jr. and M. C. Pimentel Filho, "Optimal placement of three-phase voltage regulator banks in radial distribution feeders", *in Proc. XIV Brazilian Congress on Automatics,* 2002.

[7] D. Shirmohammadi, H. W. Hong, A. Semlyen, and G. X. Luo, “A Compensation-Based Power Flow Method For Weakly Meshed Distribution and Transmission Networks,” *IEEE Trans. on Power Delivery*, vol. 3, n. 2, pp. 753–762, May 1988.

VII. BIOGRAPHIES

Carlos A. N. Pereira was born in Três Lagoas, Brazil, in 1978. He received the B.S. degree from Paulista State University (UNESP), Brazil, in 2000. He is with Rede Energia, Brazil, since 2005, where he is an operation engineer. He is also working towards his M.S. degree at UNICAMP. His main interests are in the development of computational applications for planning and optimization of distribution systems.

Carlos A. Castro (S'90, M'94, SM'00) received the B.S. and M.S. degrees from UNICAMP, in 1982 and 1985 respectively, and the Ph.D. degree from Arizona State University, Tempe, AZ, in 1993. He has been with UNICAMP since 1983, where he is currently an Associate Professor.