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Line Indices for Voltage Stability Assessment

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Abstract— **Voltage stability has become one of the most important issues in electrical power systems. The power system ability to maintain acceptable bus voltage is a very important characteristic of a network.**

This paper analyzes the performance of line stability indices. These indices were tested in IEEE 14 and IEEE 57 busbar test systems, with satisfactory results. Simulation results show that using line stability indices the most critical line and the weakest bus of the system can be correctly identified.

Index Terms—Voltage Stability, Voltage Collapse, Maximal Load, Voltage Stability Index.

I. Introduction

In recent years, the increase in peak load demand has been leading systems to operate close to their limits, with reduced stability margins. Thus, is very important to know how far the current system's operating point is from the voltage stability limit [1], [2].

Problems associated with the voltage instability have attracted more attention of the utilities and researchers due to several major system collapses incidences observed around the world [3].

A system is voltage stable at a given operating condition if for every bus in the system, bus voltage magnitude increases as reactive power injection at the same bus is increased. A system is voltage unstable if, for at least one bus in the system, bus voltage magnitude decreases as the reactive power injection at the same bus is increased [4]. Therefore, a power system is said to have a situation of voltage instability when a disturbance causes a progressive and uncontrollable decrease in voltage level.

The study of voltage stability has been analyzed under different approaches that can be basically classified into dynamic and static analysis.

Dynamic analysis uses time-domain simulations to solve nonlinear system differential and algebraic equations, which include generators dynamics, tap changing transformers, etc.

Static analysis is based on the solution of conventional or modified powerflow equations. Static analysis involves only the solution of algebraic equations and therefore is computationally much more efficient than dynamic analysis. Static analysis is ideal for the bulk of studies in which voltage stability limits for many pre-contingency and post-contingency cases must be determined [5].

Although, a large number of voltage collapse indices have been proposed in the literature, using static and dynamic techniques, in this paper, only static tools were used.

The ability of line stability indices to provide reliable indication about the proximity of voltage instability in a power system makes these indices an excellent tool to planning power systems and prevent voltage collapse in the system. These indices are also easy to compute. Usually, their values changes between 0 (no load) and 1 (voltage collapse).

The voltage stability analysis, using different line indices, will be highlighted in this paper and the results obtained from simulating on IEEE 14 and IEEE 57 busbar test system will be discussed.

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II. Indices Formulation

In order to reveal the critical bus of an electrical power system and the stability of each line connected between two bus in an interconnected network, several line stability indices have been proposed.

Some of them are briefly discussed below.

A. Line stability Index L_{mn}

M. Moghavemmi *et al.* [6] established a criterion of stability which shows the proximity to voltage collapse of each line of a network. The L_{mn} index can have a maximum value of 1 if the system is about to suffer a voltage collapse and a minimum value of 0 when there is no load on the system. This index is based on the power transmission concept in a single line, in which discriminant of the voltage quadratic equation is set to be greater or equal than zero to achieve stability. If the discriminant is smaller than zero, the roots will be imaginary, which means that cause instability in the system.

Figure 1 ilustrates a single line of an interconnected network where the L_{mn} is derived from.

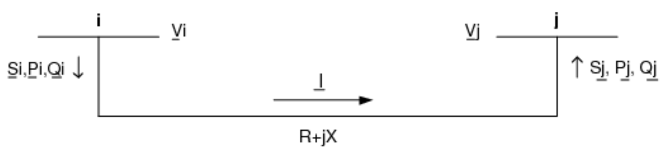

Fig. 1 Typical one-line diagram of transmission line

The line stability index, for this model, can be defined as:

$$L_{mn} = \frac{4XQ_j}{[V_i sen(\theta - \delta)]^2} \quad (1)$$

where θ is the line impedance angle and δ is the angle difference between the supply voltage and the receiving end voltage, X is the line reactance, Q_j is the reactive power flow at the receiving bus and V_i is the voltage on sending bus.

Lines that presents values of L_{mn} close to 1, indicates that those lines are closer to theirs instability points. To maintain a secure condition, the L_{mn} index should be less than 1.

B. Line Stability Index FVSI

The line stability index $FVSI_{ij}$ proposed by I. Musirin *et al.* [7] is based on a concept of power flow through a single line. For a typical transmission line, the stability index is calculated by:

$$FVSI_{ij} = \frac{4Z^2 Q_j}{V_i^2 X} \quad (2)$$

where Z is the line impedance, X is the line reactance, Q_j is the reactive power flow at the receiving end and V_i is the sending end voltage.

The $FVSI_{ij}$ index can be calculated for any of the lines of the network and depends, essentially, of the reactive power.

The line that gives index value closest to 1 will be the most critical line of the bus and may lead to the whole system instability.

The calculated *FVSI* can also be used to determine the weakest bus on the system [8]. The determination of the weakest bus is based on the maximum load allowed on a load bus. The most vulnerable bus in the system corresponds to the bus with the smallest maximum permissible load.

C. Line Stability Index LQP

The *LQP* index derived by A. Mohamed *et al.* [9] is obtained using the same concept as [6] and [7], in which the discriminant of the power quadratic equation is set to be greater or equal than zero.

The *LQP* is obtained as follows:

$$LQP = 4\left(\frac{X}{V_i^2}\right)\left(\frac{X}{V_i^2}P_i^2 + Q_j\right) \quad (3)$$

where X is the line reactance, Q_j is the reactive power flow at the receiving bus, V_i is the voltage on sending bus and P_i is the active power flow at the sending bus.

To maintain a secure condition, the value of *LQP* index should be maintained less than 1.

D. Line Stability Indices VCPI:

The *VCPI* indices proposed by M. Moghavvemi *et al.* [10] investigates the stability of each line of the system and they are based on the concept of maximum power transfered through a line.

The stability indices of the line i-j, from figure 1, are defined as:

$$VCPI(Power) = \frac{P_R}{P_{R(max)}} \quad (4)$$

$$VCPI(Losses) = \frac{P_{Losses}}{P_{Losses(max)}} \quad (5)$$

where the values of P_R and P_{Losses} are obtained from conventional power flows calculations, and $P_{R(max)}$ and $P_{Losses(max)}$ are the maximum active power and maximum active power losses that can be transferred through a line.

One of the main causes of the occurrence of voltage collapse due to the excess of power transferred through a line or the excessive absorption of power by the line.

With the increasing power flow transferred by transmission lines, the values of *VCPI* (power) and *VCPI* (losses) increase gradually, and when they reach to 1, the voltage collapse occurs. So if any line of network reach that value, it is possible to predict the voltage collapse. Therefore, the *VCPI* indices varies from 0 (no load condition) to 1 (voltage collapse).

III. Test Results And Discussion

In order to verify the effectiveness of the line stability indices, numerical studies have been made in 2 IEEE test systems: IEEE 14 busbar and IEEE 57 busbar.

In all IEEE test systems used, the reactive power was increased only in one bus at a time, while the loads on the other nodes remained constant.

A program to calculate the stability indices for each line was developed, and the following steps have been implemented:

1) Run the load flow program to the base case, using the Newton-Raphson method.

2) Calculate the value of the line stability index, for the base

case, in all the lines of the IEEE test systems used.

3) Gradually, increase the reactive power in a given bus, keeping the loads on the other nodes constant until power flow solution stop converge.

4) Calculate the value of the line stability index for each variation of the load.

5) Calculate which line of the bus presents the greatest value. This line is called the most critical line of the bus.

6) Select another bus PQ and repeat steps 1 to 5.

The test was carried out for several buses but only the cases of buses 10, 11 and 14 from IEEE 14 busbar test system are presented. The same study was made for IEEE 57 busbar test system, but we only show the simulations for buses 27, 31 and 57.

The IEEE 14 busbar test system has 5 generator busbars, 9 load busbars and 20 interconnected branches.

The reactive load was gradually increased, only in one bus of the IEEE 14 busbar test system at a time, from the base case until its maximum allowable load, keeping the load at the other busbars fixed at base load.

The charts presented in figures 2 and 3 show the value of the line stability index L_{mn}, in each variation of the load for buses 14 and 11, respectively.

The line that presents the largest index with respect to a bus is considered the most critical line of that bus.

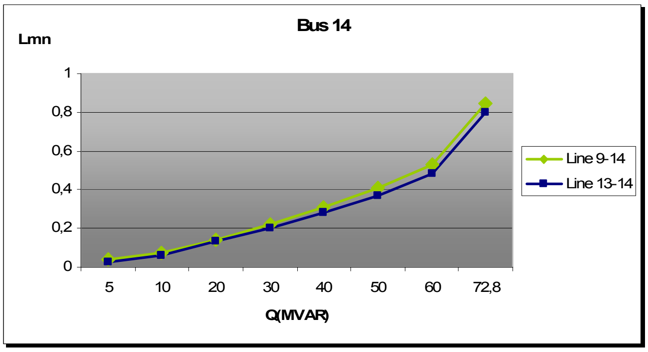

Fig. 2 L_{mn} vs. reactive load variation for bus 14 of IEEE 14 busbar test system

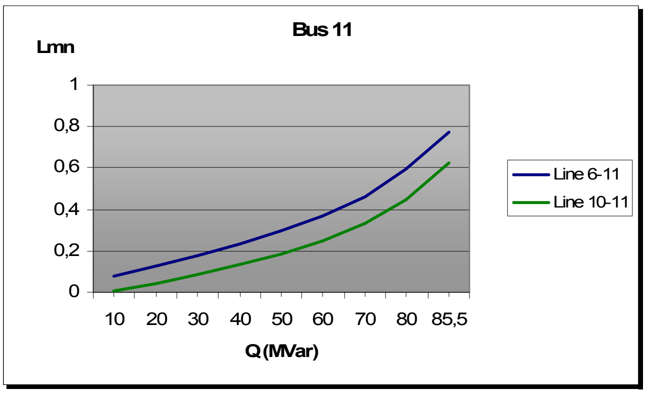

Fig. 3 L_{mn} vs Q for bus 11 of IEEE 14 busbar test system

Figure 2 shows that the line 9-14 is the critical line referred to bus 14, and figure 3 shows that the line 6-11 is the critical line referred to bus 11.

Figure 4 illustrates the response of L_{mn} index with the reactive load variation.

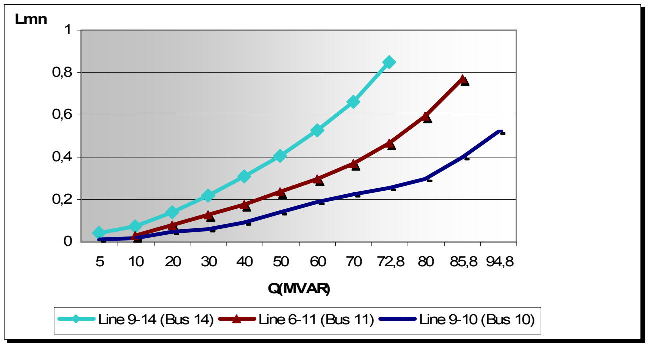

Fig. 4 L_{mn} vs. reactive load variation for IEEE 14 busbar test system

The individual L_{mn} curve presents in figure 4 is the most critical line referred to a bus. For instance, the L_{mn} curve of the line that connect bus 9 to bus 10 is the most critical line referred to bus 10. The line 6-11 and line 9-14 are the most critical lines of the bus 11 and bus 14, respectively.

The charts presented in figures 5 and 6 show the value of the line stability index *VCPI(Power),* in each variation of the load for buses 14 and 11, respectively.

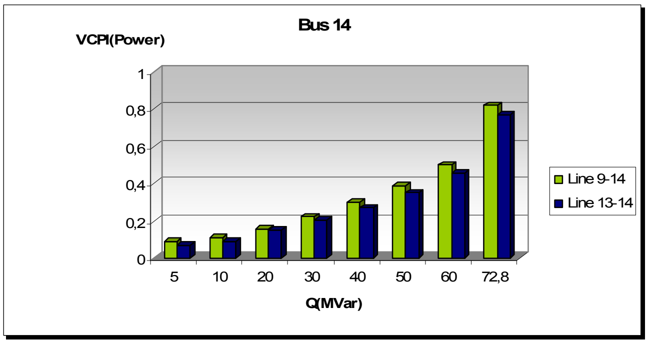

Fig. 5 VCPI(Power) vs Q for bus 14 of IEEE 14 busbar test system

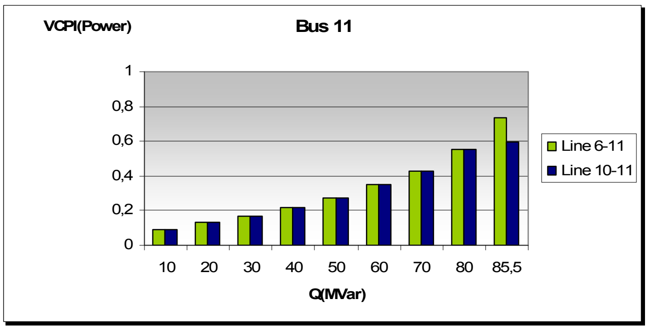

Fig.6 VCPI(Power) vs Q for bus 11 of IEEE 14 busbar test system

Figures 5 and 6 demonstrate again that the line 9-14 is the most critical line referred to bus 14 and line 6-11 is the critical line referred to bus 11.

Table I shows the stressed conditions of the lines of IEEE 14 busbar system for the maximum loadability of the buses. The values of the line stability indices are maximum at the maximum loadability of each individual load bus and the line that presents the largest index with respect to a bus is considered the most critical line of that bus.

Table I - Line Stability Indices for IEEE 14 Busbar Test System

Load (p.u.)	Line	L_{mn}	*FVSI*	*LQP*	*VCPI(P)*	*VCPI(L)*
Q_{10}=0,948	9-10	0,52	0,541	0,474	0,495	0,084
	10-11	0,52	0,535	0,456	0,4947	0,088
Q_{11}=0,855	6-11	0,77	0,827	0,681	0,7311	0,29
	10-11	0,63	0,667	0,569	0,5969	0,13
Q_{14}=0,728	9-14	0,85	0,899	0,756	0,8184	0,33
	13-14	0,8	0,849	0,7	0,7668	0,26

Table I shows an agreement between the different line stability indices.

From Table I it is observed that the line that connect bus 6 to bus 11 is the most critical line referred to bus 11 because it presents the highest indices values for the maximum loadability of the bus.

Similar meaning, line 9-10 is the most critical line with respect to bus 10 and line 9-14 is the most critical line of bus 14.

Line stability indices can also determine the weakest bus in the system and it is based on the maximum permissible load.

It is observed, in figure 7, that buses 10, 11 and 14 indicates 0,948 p.u., 0,855 p.u. and 0,728 p.u. as the maximum permissible reactive load, respectively.

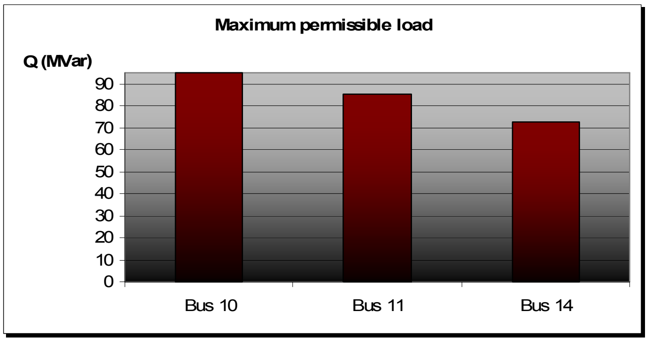

Fig.7 Maximum permissible reactive load on IEEE 14 Busbar Test System

Since bus 14 has the smallest maximum loadability, it is considered the most critical bus, so this bus sustains the lowest load of the IEEE 14 busbar test system.

The simulation was carried out for IEEE 57 busbar test system also.

The IEEE 57 busbar test system has 7 generator busbars, 50 load busbars and 80 interconnected branches.

The reactive load was gradually increased, only in one bus of the IEEE 57 busbar test system at a time, from the base case until its maximum allowable load, keeping the load at the other busbars fixed at base load.

The charts presented in figures 8 and 9 show the value of the line stability index *FVSI*, in each variation of the load for buses 27 and 31, respectively.

Lines that presents the highest values of *FVSI* with respect to a bus is considered the most critical line of that bus.

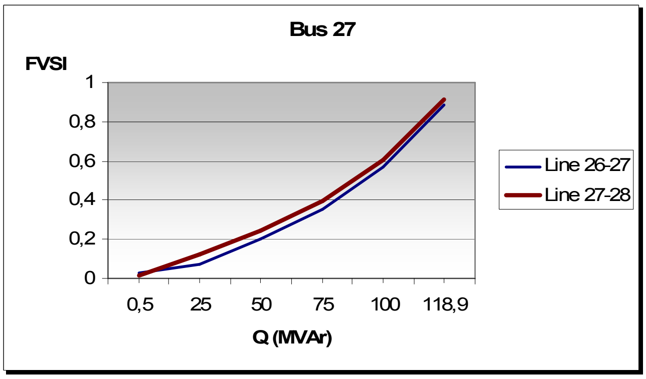

Fig. 8 FVSI Index vs reactive load variation for bus 27 of IEEE 57 busbar test system

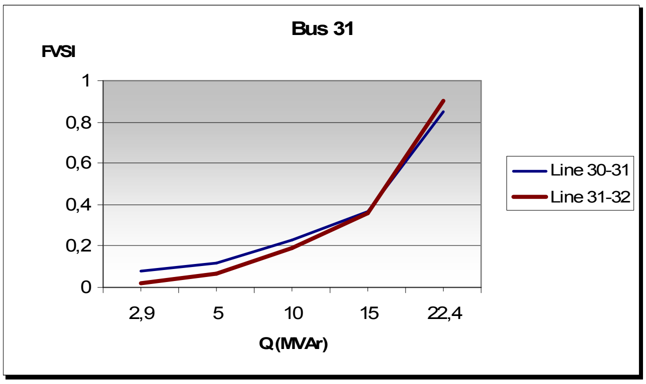

Fig. 9 FVSI Index vs reactive load variation for bus 31 of IEEE 57 busbar test system

Figure 8 compares branches that are connected to the bus 27 when it reaches the maximum power transmissible. There are two lines connected to this bus: the line that connects the bus 26 to bus 27 and the line between the bus 27 and bus 28. Analyzing this figure, we concluded that the most critical line referred to bus 27 is line 27-28.

In Figure 9 we can see that the line 31-32 presents the highest *FVSI* value, so line 31-32 is the critical line referred to bus 31.

The charts presented in figures 10 and 11 show the value of the line stability index *LQP*, in each variation of the load for buses 27 and 31, respectively.

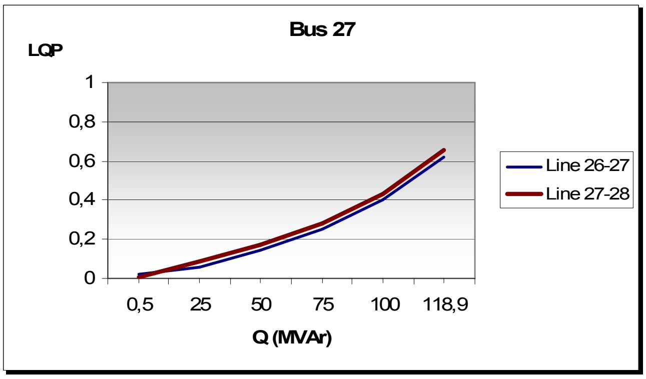

Fig. 10 LQP Index Vs reactive load variation for bus 27 of IEEE 57 busbar test system

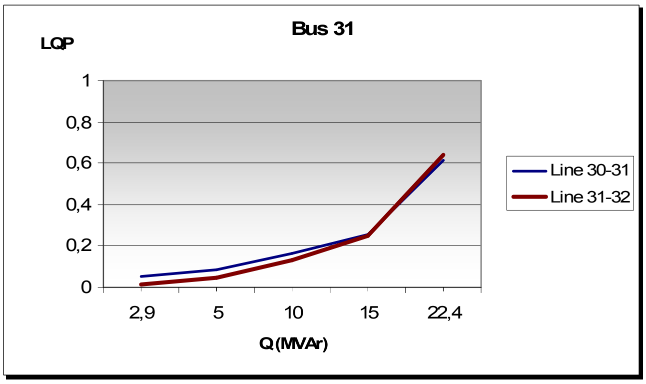

Fig. 11 LQP Index Vs reactive load variation for bus 31 of IEEE 57 busbar test system

If line stability index *LQP* remains less than 1, the system is stable, but when this index exceeds the value 1, the system loses its stability and voltage collapse occurs.

Figure 10 shows that the line 27-28 is the critical line referred to bus 27, and figure 11 shows that, although both lines have very similar values, the line 31-32 is the critical line referred to bus 31.

Table II shows the stressed conditions of the lines of IEEE 57 busbar system for the maximum loadability of the buses.

Line stability indices starts increasing with the increase of the reactive load on the bus and reaches to 1 at the point of bifurcation.

For the same loading, the IEEE 57 busbar test system were analysed using different line stability indices as we can see in table II.

It is observed that all line stability indices have the highest value at the maximum loadability of each load bus and when they are close to one the system reached its stability limit.

Table II - Line Stability Indices for IEEE 57 Busbar Test System

Load (p.u.)	Line	L_{mn}	FVSI	LQP	VCPI(P)	VCPI(L)
Q_{27}=1,189	26-27	0,75	0,88	0,62	0,7	0,19
	27-28	0,81	0,91	0,65	0,73	0,24
Q_{31}=0,224	30-31	0,78	0,85	0,61	0,72	0,22
	31-32	0,8	0,91	0,64	0,74	0,24
Q_{57}=0,404	39-57	0,993	1	0,989	1	0,79
	56-57	0,8	0,88	0,627	0,73	0,23

Table II shows that the performance of the line stability indices studied has high degree of accuracy, reliability and the results are very closed in agreement.

From Table II we concluded that the line that connect bus 27 to bus 28 is the most critical line referred to bus 27 because presents the largest index value for the maximum loadability of the bus, the line 31-32 is the most critical line with respect to bus 31 and line 39-57 is the most critical line of bus 57.

As we can see in figure 7, line stability indices can also determine the weakest bus in the system and it is based on the maximum permissible load.

In figure 12, buses 27, 31 and 57 indicates 1,189 p.u., 0,224 p.u. and 0,404 p.u. as the maximum permissible reactive load, respectively.

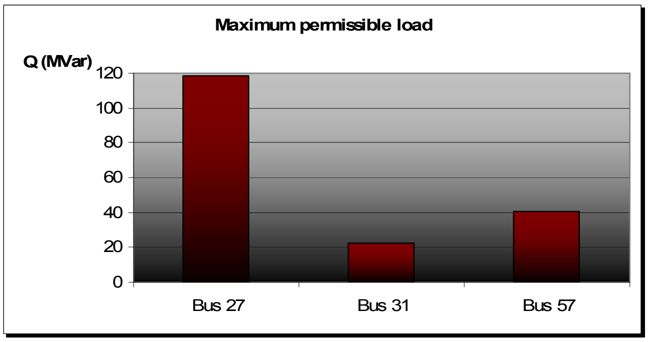

Fig.12 Maximum permissible reactive load on IEEE 57 Busbar Test System

Bus 31 has the smallest maximum loadability, therefore it is considered the most critical bus, so this bus sustains the lowest load of the IEEE 57 busbar test system.

IV. Conclusion

This paper presents a comparative study and analysis of the performance of line static voltage collapse indices. The application of those indices on IEEE 14 and on IEEE 57 bus testing system gave accurate results.

Voltage collapse occurs when a system is heavily loaded, which causes a sudden decline of bus voltage magnitude as the reactive power injection at the same bus is increased.

These indices can be used to identify the critical line referred to a bus and reveal the weakest bus of a power system. A line is considered to be critical if the line stability indices are close to 1.

The shown simulations indicate that the bus 14 of IEEE 14 busbar test system is considered the weakest bus in the system, on the other hand, bus 31 of IEEE 57 busbar test system is considered the weakest bus in the system.

Line indices provide an accurate information with regard to the stability condition of the lines. The research shows an agreement between the different line stability indices.

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