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Neutral-Point Shifting and Voltage Unbalance due to Single-Phase DG Units in Low Voltage Distribution Networks

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***Abstract*—When distributed generation (DG) units are connected to a low voltage (LV) single-phase distribution network the voltage profile will change. The connection of single-phase DG units to three-phase distribution networks will not only alter the voltage profile in the connected phase, but also the two other phase voltages will be influenced due to neutral-point shifting and voltage unbalance. The purpose of this paper is to examine the influence of single-phase DG units on the phase voltages, in different circumstances. This will be investigated by performing several simulations. In order to make these detailed simulations, an unbalanced multiphase harmonic power flow method, considering the neutral wire, will be proposed. The presented model uses the iterative forward/backward method. Furthermore, the network is solved by using symmetrical components and the prevalent convergence problems will be tackled in this paper.**

***Index Terms*—Neutral-point shifting, Voltage unbalance, Radial distribution networks, Distributed generation (DG), Symmetrical components, Harmonic power flow method**

I. Introduction

SINGLE-PHASE distributed generation units, such as PV-units, are becoming more widely spread in low voltage distribution networks. In an already inherently unbalanced distribution network, the increasing presence of these single-phase DG units may result in violations of the voltage constraints. In order to qualify these problems, there is a need for accurate power flow methods. Next to the commonly known voltage drop, this power flow method has to take into account the two main contributions to the altered phase voltage along the feeder, namely, the neutral-point shifting and the voltage unbalance.

The neutral-point shifting origins from the return current through the neutral conductor. This means that the power flow method has to consider the neutral conductor and also the harmonic currents which can contribute largely to the total neutral current [1], [2].

Furthermore, to study the voltage unbalance factor (VUF), defined as the ratio of the inverse voltage to the direct voltage component, the power flow method has to be capable to simulate single-phase, two-phase and three-phase wire sections plus neutral conductor. Also balanced and unbalanced loads between phases or between the phases and neutral must be included into the load flow program. These loads can be modeled as constant power, constant impedance or constant current loads.

In the next section, the power flow method and the main equivalent circuits of the network elements will be discussed. Because an iterative harmonic load flow method is used to solve the network, convergence problems can occur, which will be explained and solved. Following this section, an analytical deduction of the phase voltage variations caused by a single-phase DG unit will be analysed. Then, by means of simulations, this will be elaborated. The influence of increased DG output power, its point of connection (POC), and the connected phase will be investigated. Furthermore, a comparison of the influence in rural and urban networks will be made.

II. Unbalanced Multiphase Harmonic Power Flow Method

Next to the commercial available software packages numerous power flow methods have been proposed in literature [3]–[11]. These models all have their own innovative contribution to the development of fast, accurate and robust power flow algorithms. Due to the topology and properties of the distribution grid, it was made clear that the forward/backward method is the most adequate to use [8]. It also allows the introduction of meshes into the grid without large computational burden [3], [5]. The presence of the neutral and ground wire has been introduced in [9] and the introduction of harmonics in the power flow algorithm has been presented in [10]. At last, a lot of equivalent circuits of network elements have been presented [4], [7] and the inherently present unbalance in distribution networks has been addressed in [10], [11].

The contribution of the presented power flow algorithm lies in the accuracy of the equivalent circuits and the use of symmetrical components. The latter leads to a faster algorithm, especially in the cases with a low degree of unbalance. The difference with the research presented in [11] is in the removal of the multi- to single-phase converting algorithm. As compared to [9] this method has the advantage that a 3x3 impedance matrix is used, without the loss of the explicit information about the neutral currents and voltages. Furthermore, the line impedance matrix in symmetrical components is diagonal.

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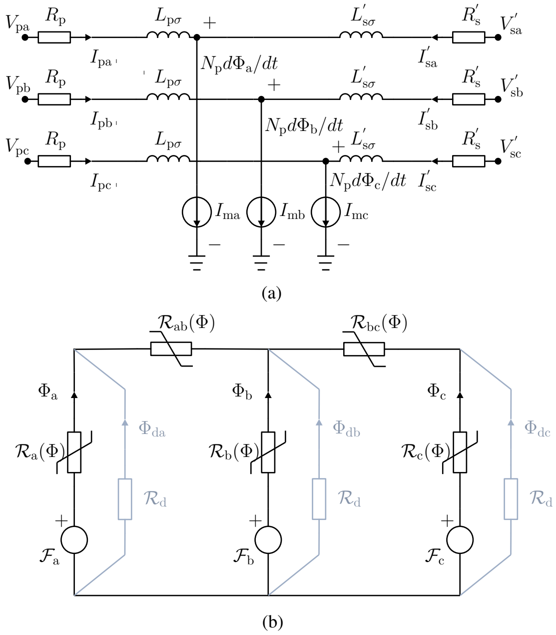


Fig. 1. Equivalent electric (a) and magnetic circuit (b) of a three-phase three-legged transformer.

Consequently, where in [6] approximations were needed to obtain such an impedance matrix, this is not needed in the current approach. The possibility of the load flow algorithm to consider harmonic components is another advantage. The additional problems introduced by the harmonic components regarding to convergence criteria will be discussed and solved.

A. Load Flow Algorithm

The forward/backward method is used to analyse the radial distribution system. This iterative method can be devided in two steps. The first step is the backward sweep: based on a known harmonic voltage profile, the currents in every node are calculated. Then, the currents that flow in all network branches can be calculated, starting from the receiving end of the feeder and going towards the sending end.

The second step is the forward sweep: starting from the source node and going towards the terminal nodes the voltage at every node is calculated.

When these two steps are performed, the voltage spectrum in every node is compared with that of the preceding iteration. If the difference in every node lies beneath a given tolerance, the iteration process stops, otherwise a new iteration is performed.

The branch current consists of the different currents injected in the nodes and the magnetizing currents of the transformers (§ II-B). The node currents originate from the linear and nonlinear loads in the nodes or from the capacitance in case of a cable segment (§ II-C). Converter-connected DG units are an additional source of injected currents (§ II-D).

B. Transformer Model

The transformer model developed by the authors and presented in [12] is based on the harmonic balance method. The

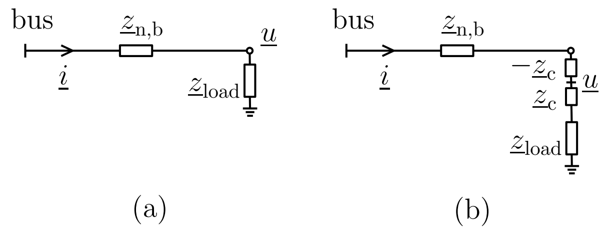


Fig. 2. Single node network without reactance pair (a) and with reactance pair (b).

electric and magnetic equations of the transformer are derived from their equivalent circuit (Fig. 1). The transformer model is implemented in symmetrical components and can be used with asymmetrical supply voltages. Also the saturation of the yokes, the coupling between the phases and the interaction between the different harmonics have been taken into account.

In every backward current sweep the magnetizing current is injected between the primary and the secondary impedance of the transformers (Fig. 1(a)).

C. Load and Cable Model

1) Linear Loads: Typical load models known as constant current, constant impedance and constant power can be included in the methodology of this paper.

The constant power loads are defined by their apparent power and their displacement power factor (dPF), which is used to describe the power factor (PF) using the fundamental frequency components only [13]. These loads are represented by a voltage-dependent impedance analogous to [11]. This means that the impedance value of these loads is updated based on the direct voltage component after every iteration.

Because an iterative harmonic analysis (IHA) is used numerical instability problems could occur. These convergence problems have already been tackled in the domain of the AC-DC power systems [14]–[18]. A first conclusion drawn is the divergence of the IHA in cases where the system impedance is large. Another reason can be found in the reaction of the devices to changes in the harmonic voltage level, as large sensitivity could result in instabilities. The first conclusion can be elucidated by considering the simple network of Fig. 2 (a). The relation between the current and the voltage in the k^{th} and the $(k+1)^{\text{th}}$ iteration can be expressed as:

$$\begin{bmatrix} \underline{i}_{k+1} \\ \underline{u}_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{\underline{z}_{\text{load}}} \\ \underline{z}_{\text{n,b}} & \dfrac{\underline{z}_{\text{load}} - \underline{z}_{\text{n,b}}}{\underline{z}_{\text{load}}} \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_k \\ \underline{u}_k \end{bmatrix} \quad (1)$$

This gives the following eigenvalues:

$$\begin{aligned} \lambda_1 &= 1 \\ \lambda_2 &= \frac{\underline{z}_{\text{n,b}}}{\underline{z}_{\text{load}}} \end{aligned} \quad (2)$$

To avoid the convergence problem the eigenvalues of the matrix describing the relationship between the currents and the node voltages in iteration k and $k+1$ have to be located in the unity circle. So, the magnitude of the load impedance and the impedance between the node and the sending bus of the feeder determines the magnitude of the eigenvalues. A higher load impedance has a positive effect on the stability, where a higher impedance between the node and the sending bus of the feeder has a negative effect. Therefore, analogously to the solution in AC-DC systems a reactance pair is inserted [15]. The voltage used to calculate the current drawn by the linear load is not the node voltage, but the voltage of the fictitious node (Fig. 2 (b)). Physically, the network does not change, but from a mathematical point of view, the load impedance is higher and the line impedance has a lower absolute value. In this way, the forward/backward method can be used for the harmonic analysis of distribution networks.

2) Nonlinear Loads: The model allows to simulate every kind of nonlinear load. In this paper however, the nonlinear loads are based upon the research presented in [19]. In this paper the fourier series of three kinds of rectifiers are given taking into account the phase cancellation by using a statistical summation. The $\mathrm{THD_I}$ of the used rectifier is 54.8%.

3) Cable Model: The currents due to the capacity of the cable are calculated analogously to the linear loads. The main problem, from a mathematical point of view, for the capacitive current calculation is the decrease of the impedance of the capacitive element for higher frequencies while the line impedance increases for these higher order harmonics. Therefore, the currents absorbed by these capacitors cannot be calculated directly [10]. In this paper, the problem is solved as follows. At first, a reactance pair has been inserted between the capacitor and the line impedance. This does not solve the problem completely. Therefore, the increase of the capacitive current between two iterations has been limited. The relaxation of the capacitive current results in an injected current that lies between the current of the previous iteration and the calculated current. Consequently, the overall iterative procedure does not end when the voltage has converged, also the currents drawn by the capacitors need to have converged.

D. *Converter-Connected DG model*

A photovoltaic (PV) inverter is used as a model for the converter-connected DG unit. The harmonic current spectrum of a PV inverter is dependent on the voltage profile at the point of connection. In order to take this voltage dependency into account the harmonic fingerprint of a commonly used inverter has been used in the simulations [20]. The harmonic fingerprint of a device describes the interaction between the harmonic voltages and the harmonic currents.

III. Analytical deduction of the voltage variations

In this section the influence of a single-phase DG unit on the voltage variations will be analysed. Next to the voltage drop along the feeder, this voltage variation can be subdivided into two parts. First there is the neutral-point shifting which originates from the voltage drop along the neutral conductor and secondly the inverse voltage component gives rise to voltage variations. The latter will be referred to in this paper as the voltage unbalance.

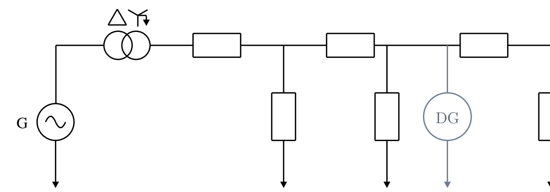


Fig. 3. Simple radial network with a single phase DG unit.

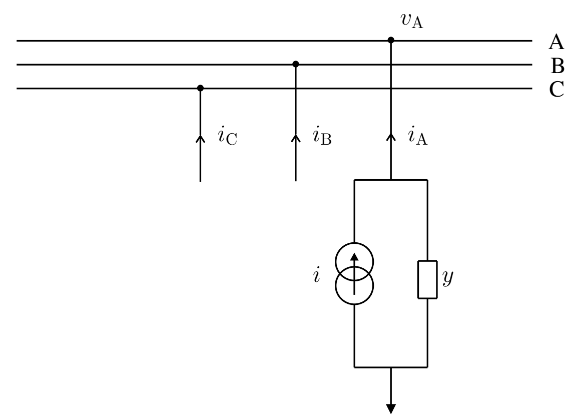


Fig. 4. Connection of the single-phase DG unit.

For the deduction of the voltage variation, the simple radial network of Fig. 3 is considered. The network consists of one three-phase (black lines) feeder with three nodes. The feeder is connected through a delta-grounded wye transformer with a perfectly balanced network, here represented as a generator. In every node there is a three-phase star-connected constant impedance load and in the second node a single-phase (grey lines) DG unit, connected to phase A, is placed.

The influence of the single-phase DG unit is found by considering Fig. 4. The DG unit is represented by its Norton-equivalent which results in the following equations for the line currents:

$$
\begin{aligned}
i_\mathrm{B} &= i_\mathrm{C} = 0 \\
i_\mathrm{A} &= i - v_\mathrm{A} y
\end{aligned}
$$

Transforming these equations in symmetrical components gives:

$$
\begin{aligned}
i_0 &= i_1 = i_2 \\
i_0 &= \frac{i}{3} - (v_0 + v_1 + v_2)\frac{y}{3}
\end{aligned}
$$

The first equation results in the series connection of the zero sequence, the direct and the inverse networks, while the second one allows to connect the three networks with the Norton equivalent of the DG unit. The equivalent scheme in symmetrical components is depicted in Fig. 5. The homopolar (v_0), the direct (v_1) and the inverse (v_2) voltage drop caused by the DG unit are indicated on this figure.

In what follows, a numerical example will be given in order to point out the influence of a single-phase DG unit. In this example the main properties of the low voltage distribution

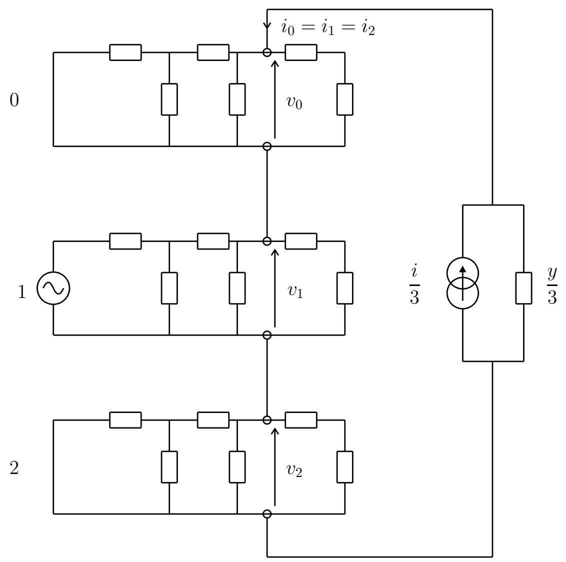


Fig. 5. Equivalent circuit in symmetrical components.

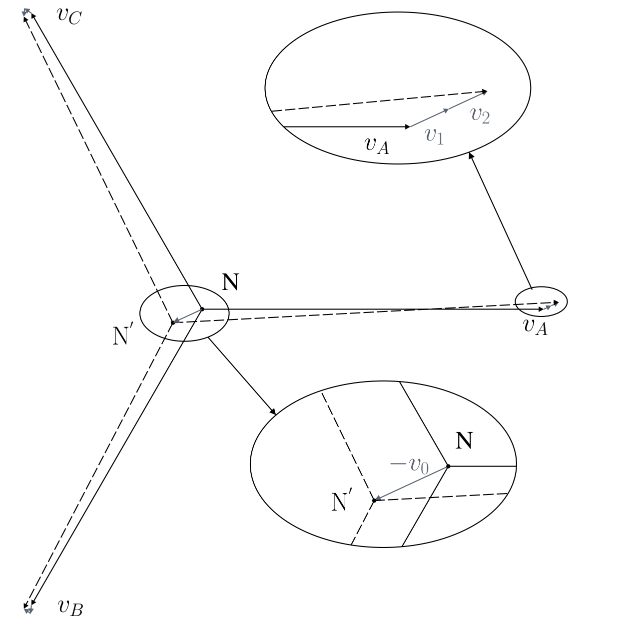


Fig. 6. Influence of DG on the neutral-point shifting. Full: Phase voltage before the connection of the DG unit. Dashed: Phase voltage after the connection of the DG unit.

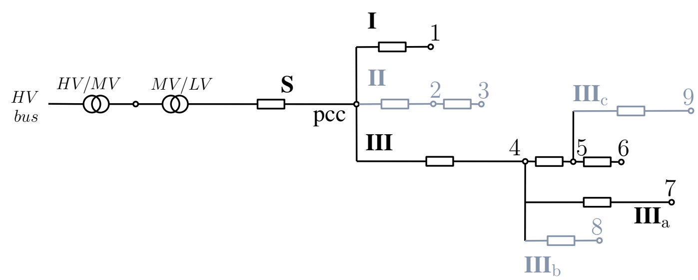


Fig. 7. Topology of the considered radial distribution network.

grid are considered. The line and the load impedances are mainly resistive-inductive of nature, which results in resistive-inductive equivalent sequence impedances.

The voltage drops caused by a single-phase DG unit, are quantified by using the following values for the network depicted in Fig 3. The line impedance is $0.059 + j0.027\Omega$ for the direct and the inverse component, where for the zero sequence components four times this value has been used. The impedance of the load is $32.137 + j15.565$ which corresponds to a 4kVA load with a dPF (displacement power factor) of 0.9 and a phase voltage of 230V. The voltages are considered to be balanced before the connection of the DG unit (full black lines in Fig. 6).

The influence of a 10kW single-phase DG unit is obtained by short-circuiting the voltage source in Fig. 5 and calculating the symmetrical voltage components in the node where the DG unit is connected. The resulting phase voltage (dashed lines in Fig. 6) is obtained by summing these direct and inverse voltage components to the phase voltages, taking into account the phase angles, and by shifting the neutral point with the opposite of the zero sequence voltage.

From Fig. 6 can be concluded that the main voltage variation results from the neutral-point shifting while the inverse voltage drop has a smaller influence on the resulting phase voltage. A second conclusion results from the direction of the neutral-point shifting. Due to the resistive-inductive nature of the distribution grid it is directed between the phase voltages v_{B} and v_{C}, but considerably more towards v_{B}. So, when a single-phase DG unit is connected to phase A, the voltage in phase A enlarges while the voltage in phase B and C decrease with a larger decrease in phase B.

IV. Influence of single-phase DG units in distribution networks

The above mentioned load flow algorithm will first be applied on the network model presented in Fig. 7. This network is based on the IEEE 13 node test feeder [21] and has, just like the majority of the low voltage (LV) public distribution networks, a radial layout, with a number of LV feeders starting from the point of common coupling (PCC). Each feeder may include one or more subfeeders and the loads and DG units can be connected anywhere along the feeder or its subfeeders.

The LV network lines are either underground cable lines, encountered mainly in urban areas, or overhead lines. In the network used for the simulations both are present. The cables are represented by grey lines and the overhead lines are represented by black lines (Fig. 7). The properties of the line segments have been altered in order to meet the characteristics of an LV network. Every line section consists of four wires (three phase and neutral) and the properties of the lines are given in Table I and II. The properties listed in Table II are valid for the fundamental component, for the higher order harmonics the resistance has been adjusted according to [22].

Starting from a base case, where no DG units are connected to the grid, several scenarios will be presented in order to study the influence of single-phase DG units to the phase voltages. The load distribution for the base case is given in

Table III. Due to the nonlinear asymmetrical load in node 5, the distribution network is in an unbalanced state.

The absolute value of the direct voltage component and the $\mathrm{THD_V}$ for the base case are given in Table IV. The influence of the asymmetrical nonlinear load in node 5 can be seen in this table.

A. *Output Power*

The influence of increased output power of a DG unit on the voltage profile is investigated by connecting a DG unit to phase A in node 8. The output power of the DG unit is increased from 0 kW to 30 kW in steps of 5 kW. The results of the simulations can be found in Fig. 8. The case of 0 kW represents the base case (where no DG unit is connected to the grid). On the abscissa the place is given, starting with the HV bus, followed by the voltage at the secondary of the high voltage transformer and the distribution transformer, then the voltage in the (PCC) and the voltages in the main nodes of Feeder III. The phenomena pointed out in Section III can be clearly found in these figures. The voltage in phase A increases with enlarged output power (Fig. 8(a)), while the voltage in phase B encounters a large decrease (Fig. 8(b)). The voltage in phase C however only experiences a small decrease (Fig. 8(c)). This means that when several small DG units are connected to the grid not only the voltage rise in the connected phase has to be considered, but also the voltage decrease in the other phases deserves some attention.

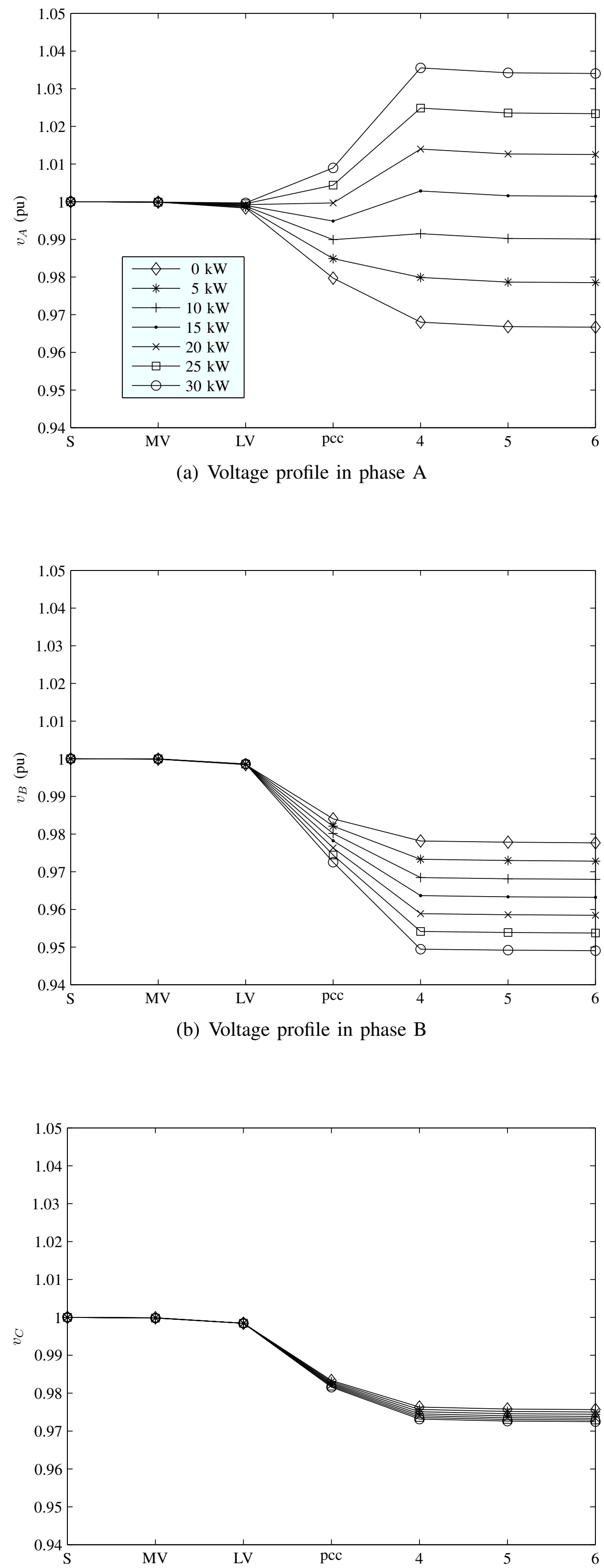
The influence noticed in Fig. 8 is mainly caused by the neutral-point shifting. In what follows, the influence of the point of connection and the phase of connection will be discussed. The conclusions drawn about the absolute value of the inverse voltage drop are also applicable for the homopolar voltage drop (and thus for the neutral-point shifting).

TABLE I
FEEDER LENGTH AND NOMINAL POWER

Feeder	length (m)	S_{nom} (kVA)
S	620	90
I	155	38
II	248	36
III	806	40
III$_\mathrm{a}$	310	9
III$_\mathrm{b}$	155	25
III$_\mathrm{c}$	248	18

TABLE II
THE LINE PROPERTIES

Feeder	R (Ω/km)	L (mH/km)	C (μF/km)
S	0.0439	0.0573	/
I	0.351	0.344	/
II	0.226	0.223	0.29
III	0.0585	0.086	/
III$_\mathrm{a}$	0.351	0.344	/
III$_\mathrm{b}$	0.226	0.223	0.29
III$_\mathrm{c}$	0.226	0.223	0.29



(a) Voltage profile in phase A

(b) Voltage profile in phase B

(c) Voltage profile in phase C

Fig. 8. Influence of increased output power on the voltage profile

TABLE III
THE LOAD DISTRIBUTION

node	RL-load (kVA)	dPF	nonlinear load (kVA)	phase
1	20	0.9	6	A-B-C
2	15	0.95	5	A-B-C
3	7	0.9	/	/
4	4	0.9	/	/
5	7	0.95	3	A
6	3	0.85	1	A-B-C
7	2	0.9	2	A-B-C
8	6	0.9	/	/
9	2	0.9	3	A-B-C

TABLE IV
THE BASE CASE

node	v_1 (pu)	$\mathrm{THD}_{\mathrm{V_A}}$ (%)	$\mathrm{THD}_{\mathrm{V_B}}$ (%)	$\mathrm{THD}_{\mathrm{V_C}}$ (%)
1	0.973	2.480	2.246	2.266
2	0.976	2.080	1.865	1.867
3	0.975	2.073	1.858	1.860
4	0.974	2.833	2.379	2.324
5	0.973	2.981	2.420	2.426
6	0.973	2.997	2.435	2.442

B. Point of Connection

The influence of the location of the DG unit on the network can be discussed by considering four different cases. In these cases the POC of a 3 kW DG unit is altered between node 1, 4, 6 and 8. In addition, also the combination of these cases has been considered (POC 1, 4, 6 and 8), where a 3 kW DG unit is connected in the nodes 1, 4, 6 and 8.

In Fig. 9 the relative unbalance factor with respect to the base case is given for the different cases. There can be seen that when a single DG unit is connected to the network the voltage unbalance decreases, where for the combined case the unbalance increases. The voltage unbalance in the PCC is independent of the place of the DG unit. When the DG unit is connected to Feeder I (POC 1), the decrease of the unbalance along Feeder III remains the same as in the PCC. This means that the VUF in a feeder, caused by a single-phase DG unit in another feeder, is equal to the VUF in the first mutual point of these feeders.

When the DG unit is connected along Feeder III there is a larger decrease of the unbalance than in the PCC.

C. Phase of Connection

Due to the unbalanced load in node 5, connected to phase A, there is a voltage unbalance along the network. In Fig. 10 a comparison is made between the cases where a DG unit is connected to node 6, but to phase A, B and C respectively. There can be concluded that the voltage unbalance decreases when the DG unit is connected to the most loaded phase (POC 6,A). In the other cases the unbalance increases. However, the increase is not exactly the same for the cases POC 6,B and POC 6,C. This is caused by the already present unbalance in node 5, which leads to different phase voltages when no DG units are connected to the grid. Consequently, the currents injected by the converter-connected DG unit are not exactly the same, resulting in this different increase of the voltage unbalance factor.

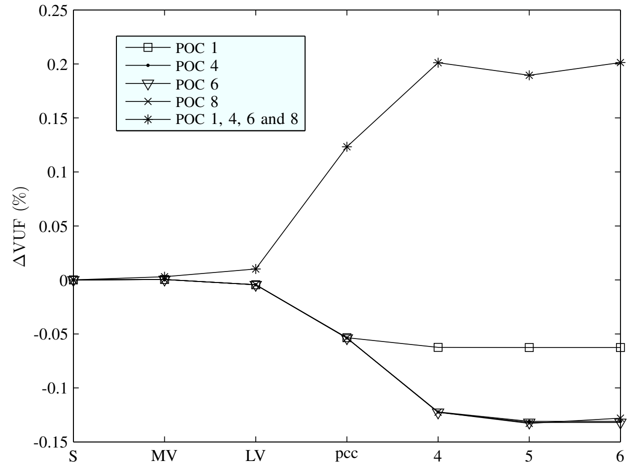


Fig. 9. Influence of DG placement on the VUF

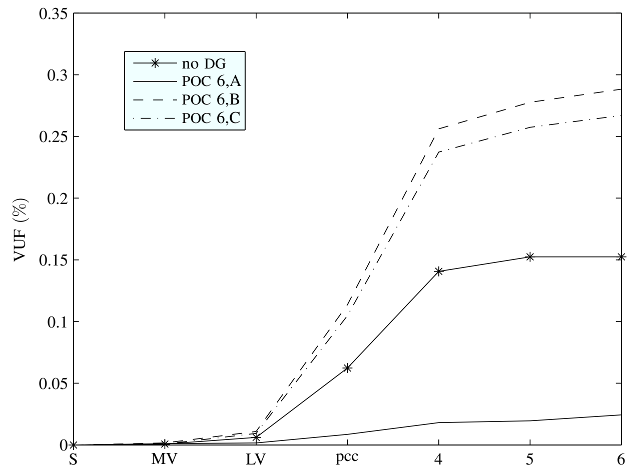


Fig. 10. Influence of the load unbalance on the VUF

D. Rural Versus Urban Networks

In order to make the comparison of the influence in rural and urban networks the following typical Belgian network cables are used (Table V). On the rural network five identical three-phase customers are connected equally spread along the feeder. The customers consist of a 2 kVA linear load with a dPF of 0.9, and a 1 kW nonlinear load. On the urban network 33 single-phase customers are connected. They are also equally

TABLE V
RURAL VS URBAN LINES

	Rural	Urban
	3x95+54.6	4x150
R_p (Ω/km)	0.410	0.265
R_n (Ω/km)	0.713	0.265
L (mH/km)	0.243	0.248
length (m)	800	400

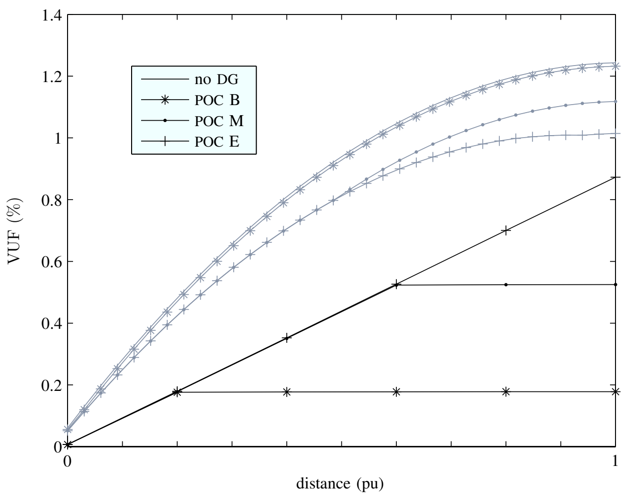


Fig. 11. Influence on the VUF for the inverse component

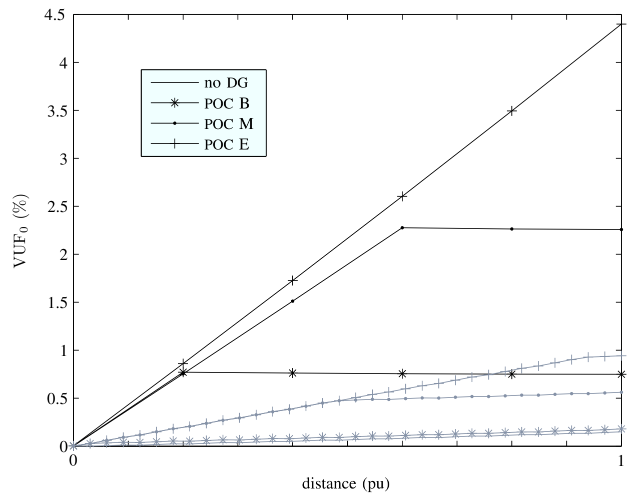


Fig. 12. Influence on the neutral-point shifting, described as homopolar voltage unbalance (VUF_0)

spread and they consist of a 1 kVA linear load with a dPF of 0.9 and a 100 W nonlinear load. The customers connected to the urban network are alternated between phase A, phase B and phase C.

First the feeder will be simulated when no DG units are connected along the feeder (no DG). Then, the influence is tested by connecting a 4 kW DG unit at the beginning of the feeder (POC B), at the middle of the feeder (POC M, where the middle is node 3 and node 16, for the rural and urban network respectively) and at the end of the feeder (POC E, node 5 and node 31).

The VUF for the inverse component (v_2/v_1) is given in Fig. 11. The abscissa gives the place along the feeder, starting from the PCC (here the secondary of the MV/LV transformer) and referred to the total length of the feeder. The black lines give the VUF along the rural network and the grey for the urban network. At first, it can be noticed that the VUF for the rural network is almost zero when no DG units are connected along the feeder, while for the urban network, an inherent unbalance exists caused by the single-phase loads. Starting from this, there can be seen that the further the DG unit is connected along the grid, the more the VUF is influenced. With respect to the rural network, the VUF increases, and for the urban network the VUF decreases. Secondly, there can be seen that for the rural network the VUF remains almost constant in the nodes after the DG unit. At last, there can be concluded that a single DG unit will not violate the constraints of voltage unbalance, but when on another feeder an unbalance would be present, this could contribute (as concluded out of POC 1 in Fig. 9) to unbalance problems, especially in rural networks.

In Fig. 12 the neutral-point shifting is quantified as a homopolar voltage unbalance factor ($\mathrm{VUF}_0 = v_0/v_1$). There can be seen that for both type of networks the VUF_0 is low when no DG units are connected. When a DG unit is connected along the network, the largest influence is found when it is connected further along the feeder. For both the rural and the urban network, the VUF_0 increases, but this phenomenon is much more apparent in rural networks where the neutral resistance (R_n) is large. Thus, the rms value of the voltage will be influenced more in rural networks as compared to urban networks. This is not only the case in the phase where the DG unit is connected but also in the other phases, as has been mentioned in § IV-A.

V. Conclusions

This paper presents a multiphase harmonic load flow program to take the unbalance into account. The method uses symmetrical components wich leads to an accurate, fast and robust power flow method. Also the neutral conductor has been taken into account. The most important convergence problems have been discussed and solved in this paper.

Furthermore, the model is used to study the influence of single-phase DG units on the phase voltage in low voltage distribution networks. The influence on the phase voltage is subdivided in the neutral-point shifting and voltage unbalance. The neutral-point shifting also results in altered phase voltages in the phases where no DG unit is connected. However, due to the direction of neutral-point shifting in resistive-inductive networks, these phase voltages are not altered in exactly the same way. Furthermore, the single-phase DG units do not always alter the VUF in the same way. So, in some cases the connection of a DG unit has a positive effect on the VUF (especially in urban networks). At last, one can conclude that a single DG unit will not violate the voltage constraints, but in a network with large penetration of DG units some attention is needed.

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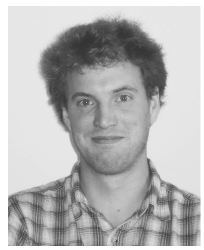

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