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Optimal Bidding Strategy of Generating Companies in Imperfect Electricity Markets

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***Abstract*—This work presents a new method for calculating the optimal bidding strategies among Generating Companies (GENCOs) in the electricity markets with imperfect competition and complete information. In this paper, the parameterized Supply Function Equilibrium (SFE) is employed for modeling the imperfect competition among GENCOs. A pay-as-LMP pricing mechanism is assumed for settling the market and calculating the GENCOs' profits. In this paper, the GENCO's profit expression is calculated in terms of its strategy and then the optimal strategy is determined analytically. An example and a case study are used to illustrate the properties of this market and the efficiency of the proposed method.**

***Index Terms*—Imperfect electricity market, Nash equilibrium, optimal bidding strategy.**

I. Introduction

In recent decades, the electricity supply industry throughout the world has been moved from nationalized monopolies into competitive markets. Electricity is evolving into a distributed commodity in which market forces are bound to drive the price of it and reduce the net cost through increased competition. In such a market, the existence of an independent entity called Independent System Operator (ISO) is necessary. The ISO is a regulatory organization and one of its responsibilities is to balance the network in a manner that maximizes the welfare of the industry as a whole [1], [2].

In a restructured electricity market, each Generating Company (GENCO) submits bids to the ISO with the goal of maximizing its own benefits. So each GENCO tries to establish a suitable bidding strategy to maximize its potential profit [4]. Finding the optimal bidding strategy of GENCOs depends on the type of competition. In the perfect competition all of the market participants are called price takers and don't have ability to influence the market price through their individual actions. Developing bidding strategy in perfect competition is based on price forecasting. Forecasted price will be used in a Price-Based Unit Commitment (PBUC) program for determining the bid that maximizes profit [5]-[7].

There are several approaches to analyze the problem of developing optimal bidding strategy in electricity markets with imperfect competition that could be categorized into non-equilibrium and equilibrium models [8]. The basic idea in non-equilibrium models is to use an approximate model for analyzing the impact of a GENCO's bidding strategies on market clearing price. For example, an ordinal optimization method was used in [9] to find the "good enough" bidding strategy for power suppliers. In equilibrium models, game theory concepts are utilized to simulate bidding behaviors of GENCOs. The solution of this game, if it exists, is the optimal bidding strategy of each GENCO and represents a market Nash Equilibrium (NE) which means that each GENCO's profit will reduce if it unilaterally changes its bidding strategy while the other GENCO's bidding strategy remain fixed. If there is no collusion and each player's payoff are known to all players, then the optimal bidding strategy problem could be considered as a noncooperative game with complete information.

In recent works, the strategic bidding problem is formulated as a bilevel optimization problem using the supply function equilibrium (SFE) model for modeling the imperfect competition among GENCOs. In this bilevel optimization problem, the upper level subproblem maximizes the individual GENCOs' payoffs and the lower subproblem (convex quadratic programming) solves the ISO's market clearing problem. Weber in [10] represented the problem as a bilevel optimization problem and utilized price and dispatch sensitivity information, available from the OPF solution, to determine how a market participant should vary its bid in order to increase its profit. Via a bilevel optimization technique and Karush-Kuhn-Tucker (KKT) complementary conditions, Hobbs in [11] transformed the strategic bidding problem to a nonlinear programming model or, more specifically, to a mathematical program involving linear complementary constraints. Also Li in [12] utilized the primal-dual interior point method and sensitivity functions to solve this bilevel problem.

The existence and uniqueness property of NE is a very important issue that considered in many literatures. For example, Couchman in [13] has proved that with the assumption of pay-as-bid pricing mechanism, there is a unique NE under supply capacity constraint, however, transmission constraints are ignored in their analysis. The issue of finding multiple NE is investigated in [14]-[15] when network constraints are taken into account.

In this paper, the bilevel optimization model, applied by [12], or equivalently, the mathematical problem with equilibrium constraints (MPEC) model, applied by [11], is employed for developing optimal bidding strategy for competitor suppliers participating in the day-ahead (DA) energy market. In this market, it's supposed that the ISO uses a DC optimal power flow (DC OPF) to clear the market after the market collecting bids and pays the suppliers under pay-as-LMP pricing. Suppliers are assumed to bid affine nondecreasing supply curve. Strategic behavior is represented

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via a parameterized SFE model and the $x\alpha y$ parameterization technique is considered for the SFE model in which the suppliers can manipulate the slope and the intercept proportionally.

This paper is organized as follows: section II introduces the notation used in this paper. Section III presents the problem formulation including the ISO problem, the supplier problem, and the solution technique. The properties of imperfect market are demonstrated for an illustrative example in section IV. Section V gives results from a case study based on the modified 30-Bus IEEE test system. Concluding remarks are provided in section VI.

II. NOTATION

The notations used in this paper are introduced as follows. For a dummy variable x, the notation x_i for $i=1\ldots n$, is used to refer to each element of vector x. The lower and upper bound on the value of x_i is represented by $x_i^{\min}$ and $x_i^{\max}$, respectively. $u \perp v$ is used to represent the relation $u^T \cdot v = 0$. $diag(\omega)$ is used for the square matrix whose diagonal elements are the elements of ω and whose other elements are zero.

The electrical network is composed of N nodes with G GENCOs, indexed by i or j. A demand at node i is represented by Q_{D_i}. Total demand of this network is represented by P_{Load}. The set of arcs is shown by A and, if $ij \in A$ there is an arc between i and j. The power flow between nodes i and j is represented by T_{ij}. $T_{ij}^{\max}$ is the capacity limit of the line connecting nodes i and j. L is the set of Kirchhoff loops in the network, indexed by m such that L_m is the ordered set of arcs associated with Kirchhoff loop m. z_{ij} is the reactance on arc $ij \in L$ and $s_{ijm} = \pm 1$, depending on the orientation of arc ij in loop m. R denotes the (arc, loop) incidence matrix which is equal to $s_{ijm} z_{ij}$ if $ij \in L$ and is zero otherwise. Δ denotes the (node, arc) incidence matrix of the electrical network whose entries Δ_{il} are $+1$ if $l = ij$ and -1 if $l = ji$, and are zero otherwise ($ij \in$ set of arcs and $j \in$ set of network nodes).

cap^+ and cap^- are the sets of generating companies that should produce their maximum and minimum capacity, respectively. cap is the union of these two sets.

The marginal cost curve of every single supplier is assumed to be affine in the form $MC_i = 2 \cdot a_i \cdot P_i + b_i$, where a_i and b_i are the coefficients of cost function that is in the form $(\cos t)_i = a_i \cdot P_i^2 + b_i \cdot P_i + c_i$. For each player i in the game, k_i and $\Pi_i(k_i)$ denote its strategy and payoff, respectively and k_{-i} denotes its rivals strategies.

III. PROBLEM FORMULATION

A. Market Assumption

Here, supply curves of the energy are restricted to be affine and nondecreasing. An SFE model is adopted to represent the strategic behavior of the suppliers. If an SFE model is chosen, then the supply function is in the form of $x + y \cdot P$. Each GENCO can choose different values for x and y which are referred to as strategic variables. Four parameterization techniques for strategic variables are considered, including x parameterization, y parameterization, $x\alpha y$ parameterization, and x, y parameterization. Baldick [16] showed that the parameterization effect on the market results is significant. For simplicity, it's supposed that each GENCO has an equivalent cost function of its own generators [3] and will submit a bid to the ISO according to the following linear supply function.

$$Bid(P_i) = k_i \cdot MC_i = k_i \cdot (2 \cdot a_i \cdot P_i + b_i) \tag{1}$$

where $Bid(P_i)$ is bidding price of GENCO i for producing the power of P_i and k_i is bidding strategy of GENCO i (a real number).

Also the k_i value is close to 1 for price takers in equilibrium points.

B. Market Clearing Mechanism (Lower Level Subproblem)

The market clearing mechanism is based on the maximization of the declared social welfare, or equivalently, the minimization of the consumer payments subject to transmission and suppliers physical constraints. Accordingly, locational marginal prices (LMPs) are calculated as:

$$\min_{P_i} \sum_{i \in G} k_i \cdot (2 \cdot a_i \cdot P_i + b_i) \cdot P_i \tag{2}$$

s.t.

$$Q_{D_i} - P_i + \sum_{j:ij \in A} T_{ij} - \sum_{j:ji \in A} T_{ji} = 0 \qquad \forall i \in N \tag{3}$$

$$\sum_{ij \in l_m} s_{ijm} \cdot z_{ij} \cdot T_{ij} = 0 \qquad \forall m \in L \tag{4}$$

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad \forall i \in G \tag{5}$$

$$0 \le T_{ij} \le T_{ij}^{\max} \qquad \forall ij \in A \tag{6}$$

where (2) represents the total cost of providing energy which depends on the bids submitted by the suppliers. Equations (3)-(4) state the Kirchhoff's current and voltage laws, respectively (based on DC load flow formulation). Equations (5)-(6) specify the supplier capacity and the line limits, respectively. To avoid nonconvexities in (2)-(6) due to 0-1 unit commitment (UC) decisions, the suppliers are assumed to take the 0-1 states as given based on the UC results [17].

Forming the KKT conditions [3] for the primal problem (2)-(6), and using dual variables μ, π, λ, and γ, the following nonlinear complementary formulation of the primal problem (by dropping indices) is obtained:

$$\begin{array}{lll} 0 \le P^{\max} - P & \perp & \mu \ge 0 \\ 0 \le P - P^{\min} & \perp & -\lambda + \mu + diag(k)\left(diag(4 \cdot a)\,P + b\right) \ge 0 \\ 0 \le T^{\max} - T & \perp & \pi \ge 0 \\ 0 \le T & \perp & \Delta^T \lambda + \pi + R^T \gamma \ge 0 \\ \lambda \ free & & Q_D - P + \Delta T = 0 \\ \gamma \ free & & RT = 0 \end{array} \quad (7)$$

where matrices Δ and R were introduced in section II.

After solving (7) with respect to the strategy of each GENCO and calculating the dual variables, the following results will be obtained.

Lemma 1: The LMP and accepted power of each GENCO can be stated as:

$$LMP_i = \frac{1}{\dfrac{A_i}{4 \cdot a_i \cdot k_i} + B_i} \cdot LD_i \quad (8)$$

$$P_i = \frac{LMP_i}{4 \cdot a_i \cdot k_i} - \frac{b_i}{4 \cdot a_i} \quad (9)$$

where k_i is the strategy of ith GENCO and A_i, B_i, and LD_i, are parameters which depend on other GENCOs' strategies (k_{-i}) and have different values for different states of reaching a generation or transmission constraint.

Lemma 2: With the assumption of the network lines have large enough capacity, the expressions for A_i, B_i, and LD_i, would be as follows (These are proved in Appendix A):

$$\begin{aligned} &A_i = 1 \\ &B_i = \sum_{\substack{j \in G \\ j \notin cap, j \ne i}} \frac{1}{4 \cdot a_j \cdot k_j} \\ &LD_i = Q_d - \sum_{j \in cap^-} P_j^{\min} - \sum_{j \in cap^+} P_j^{\max} + \sum_{\substack{j \in G \\ j \notin cap}} \frac{b_j}{4 \cdot a_j} \end{aligned} \quad (10)$$

Lemma 3: If there is a congested line "*l*" in the network, then the expressions for A_i, B_i, and LD_i, would be as follows (These are proved in Appendix B):

$$A_i = \sum_{\substack{j \in G \\ j \notin cap}} \frac{\left(SF_{li} - SF_{lj}\right)^2}{\left(4 \cdot a_j \cdot k_j\right)}, \quad B_i = \sum_{\substack{r=1 \\ r \ne i \\ r \notin cap}}^{n_G - 1} \sum_{\substack{j=r+1 \\ j \ne i \\ j \notin cap}}^{n_G} \frac{\left(SF_{lr} - SF_{lj}\right)^2}{\left(4 \cdot a_r \cdot k_r\right) \cdot \left(4 \cdot a_j \cdot k_j\right)}$$

$$LD_i = \sum_{j \in G} \frac{SF_{1i} - SF_{1j}}{4 \cdot a_j \cdot k_j} \cdot \left(\begin{array}{l} SF_{lj} \cdot P_{Load} + (-1) \cdot F_{\max} - \\ (-1) \cdot \left(\begin{array}{l} \sum\limits_{n \in cap^-} \left(SF_{\ln} - SF_{lj}\right) \cdot P_{\min,n} + \\ \sum\limits_{n \in cap^+} \left(SF_{\ln} - SF_{lj}\right) \cdot P_{\max,n} \end{array} \right) + \\ (-1) \cdot \sum\limits_{\substack{n \in G \\ n \notin cap}} \left(SF_{\ln} - SF_{lj}\right) \cdot \dfrac{b_n}{4 \cdot a_n} \end{array} \right) \quad (11)$$

where SF_l is the shift factor of Line "*l*". Also all GENCOs in cap^+ and cap^- sets should produce their maximum and minimum capacity, respectively and according to (10) and (11), the strategy of these GENCOs don't have any effect on LMPs and accepted power of other GENCOs that aren't in cap set.

C. *Supplier Problem (Upper Level Subproblem)*

The problem faced by each player is the maximization of its profit where profit comprises the difference between revenue and production cost. The cost of producing energy is calculated as:

$$(\cos t)_i = a_i \cdot P_i^2 + b_i \cdot P_i + c_i \quad (12)$$

The revenue of the supplier is the revenue of selling energy in the market and can be calculated under a pay-as-LMP scheme as:

$$(revenue)_i = LMP_i \cdot P_i \quad (13)$$

The supplier payoff (profit) is revenue minus cost, namely

$$\Pi_i = LMP_i \cdot P_i - (a_i \cdot P_i^2 + b_i \cdot P_i + c_i) \quad (14)$$

Using the results of the market clearing problem and inserting the calculated LMP_i and P_i in (14), the supplier payoff can be written with respect to its strategy (k_i) as:

$$\begin{aligned} \Pi_i = &\left[\frac{-b_i \cdot (k_i + 0.5)}{A_i + B_i \cdot (4 \cdot a_i \cdot k_i)}\right] \cdot LD_i \\ &+ \left[\frac{4 \cdot a_i \cdot k_i - a_i}{\left(A_i + B_i \cdot (4 \cdot a_i \cdot k_i)\right)^2}\right] \cdot LD_i^2 \end{aligned} \quad (15)$$

After some manipulation, we have:

$$\begin{aligned} &\Pi_i = \frac{Q_i \cdot k_i^2 + R_i \cdot k_i + S_i}{\left(A_i + B_i' \cdot k_i\right)^2} = f(k_i) \\ &where \\ &Q_i = -4 \cdot a_i \cdot b_i \cdot B_i \cdot LD_i \\ &R_i = \left(-A_i \cdot b_i - b_i \cdot B_i \cdot (2 \cdot a_i)\right) \cdot LD_i + 4 \cdot a_i \cdot LD_i^2 \\ &S_i = -A_i \cdot b_i \cdot (0.5) \cdot LD_i - a_i \cdot LD_i^2 \\ &B_i' = B_i \cdot (4 \cdot a_i) \end{aligned} \quad (16)$$

Thus, the supplier problem is transformed from the bilevel optimization problem (or the MPEC problem) to the following simple optimization problem:

$$\max_{k_i} \quad \Pi_i = f(k_i) \quad (17)$$

where the impact of the ISO and the rivals' actions are observed through the A_i, B_i, and LD_i.

D. *Complete Information Gaming*

In an electricity market, each GENCO tries to maximize its own profit as shown in (17), thus by equating the first derivative of the profit with respect to its strategy (k_i) to zero (i.e., the necessary conditions for maximization), its optimal strategy will be calculated as:

$$\frac{\partial \Pi_i}{\partial k_i} = 0 \quad \Rightarrow$$

$$
\begin{aligned}
&B_i^2 \cdot \left[b_i \cdot (2 \cdot a_i) \cdot LD_i\right] + \\
&B_i \cdot \begin{bmatrix} \left(-b_i \cdot A_i \cdot LD_i - 4 \cdot a_i \cdot LD_i^2\right) + \\ x_i \cdot \left(A_i \cdot b_i \cdot (2 \cdot a_i) \cdot LD_i + 2 \cdot a_i \cdot (4 \cdot a_i) \cdot LD_i^2\right) \end{bmatrix} + \\
&x_i \cdot \left(-A_i^2 \cdot b_i \cdot LD_i + 4 \cdot a_i \cdot LD_i^2 \cdot A_i\right) = 0 \\
&, \quad x_i = \frac{1}{4 \cdot a_i \cdot k_i} \quad , \quad B_i = \sum_{\substack{j \in G \\ j \neq i}} \frac{1}{4 \cdot a_j \cdot k_j}
\end{aligned} \tag{18}
$$

The sufficient conditions for maximization will be reached, if the solution found in (18) satisfies the following inequality:

$$
\begin{aligned}
&\frac{\partial^2 \Pi_i}{\partial k_i^2} < 0 \quad \Rightarrow \\
&\frac{B_i' \cdot \left(2 \cdot B_i' \cdot S_i - R_i \cdot A_i\right) + \left(2 \cdot Q_i \cdot A_i - B_i' \cdot R_i\right) \cdot A_i}{\left(A_i + B_i' \cdot k_i\right)^4} < 0
\end{aligned} \tag{19}
$$

where Q_i, R_i, and S_i, are defined in (16).

Finally for computing the Nash equilibrium of the market through utilizing game theory technique, we can state the problem as an n-player game: There are n players in the game that they simultaneously play with their own bidding strategies. Therefore, a NE will be calculated from solving n equations similar to (18) for all players simultaneously and the solution of this set of equations means that no player will have incentive to unilaterally change its bidding strategy. After all, the nonnegativity of Lagrange multipliers of active inequality constraints must be checked. In lemma 4 and 5, the existence and uniqueness of Nash points are considered.

Lemma 4: There exists at most one Nash equilibrium point in the unconstrained game or perfect market (It is proved in Appendix B).

Lemma 5: The solution (Nash point) calculated in (18) in the constrained game or imperfect market may be had the following characteristics:

1. Nonexistence of Nash point.
2. Multiplicity of Nash point.
3. Nonoptimality of Nash point.

These characteristics are discussed in section IV.

IV. Illustrative Example

The six-bus power system depicted in Fig. 1 is considered in this section to show the characteristics of Nash point in imperfect markets. All transmission lines have zero resistances and equal reactances. The total demand in the following three cases (except Case C.2) is equal to 283.76 MW where demands in all buses are as ones in [17]. Transmission capacity limit of line 1 connecting buses 1 and 6 is set to 200 MW and leftover of the lines have large enough capacity. Now the cost coefficients of GENCOs are changed to show three possibilities mentioned in Lemma 5[1].

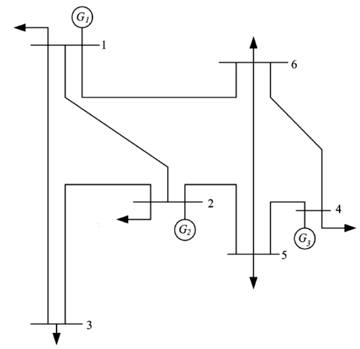

Fig. 1. Six-bus test system

TABLE I
GENCOs Cost Coefficients Data

NO.	a			b			P_{min} (MW)	P_{max} (MW)
	Case A	Case B	Case C.1	Case A	Case B	Case C.1		
G_1	0.031	0.016	0.028	12.26	17.97	13.47	0	150
G_2	0.023	0.011	0.043	15.8	21.95	14.74	0	150
G_3	0.027	0.046	0.031	17.6	36.46	14.55	0	100

A. Nonexistence of Nash Point

If the GENCOs' cost coefficients are according to Table I-Case A, then the market doesn't have any Nash point.

B. Multiplicity of Nash Point

If the GENCOs' cost coefficients are as in Table I-Case B, then the market has two Nash points (shown in Table II).

TABLE II
GENCOs' Parameters in Nash Point

GENCO No.	Nash Point 1		Nash Point 2	
	Strategy	Profit ($)	Strategy	Profit ($)
G_1	1.6754	2359	1.5378	3241.1
G_2	1.465	2675.7	1.604	1881
G_3	1.0074	94.19	0.9805	132.42

Table II shows that GENCO 2 uses the strategy of Nash point 1 and GENCO 1 and 3 use the strategies of Nash point 2 because the related profit is greater. In this situation, the profit of GENCO 1, 2 and 3 are 2625.6, 2161.6 and 60.31, respectively, which are less than the profits of Nash point.

C. Nonoptimality of Nash Point

In this section, the effect of two limitations, production and transmission capacity constraints, on the nonoptimality of Nash point will be considered.

1) *Production Capacity Constraint:*

If the GENCOs' cost coefficients are based on Table I-Case C.1, then the solution (Nash point) calculated in (18) is not an optimal point (as shown in Fig. 2).

The red line in Fig. 2 is the calculated strategy in (18) but is not a market Nash point because its corresponding profit is not maximum. The reason of nonoptimality of the calculated Nash point is reaching the GENCO 3 to its maximum capacity limitation when GENCO 1 increases its strategy (see fig.3).

[1] It is important to note that the constant coefficient (c) in GENCO's cost function does not any effect on the solution of the problem in (17).

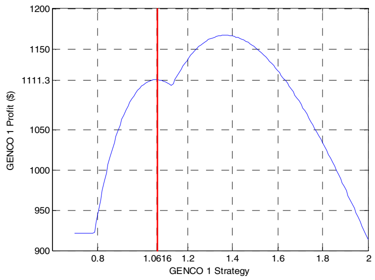

Fig. 2. The profit variation of GENCO 1 around its strategy

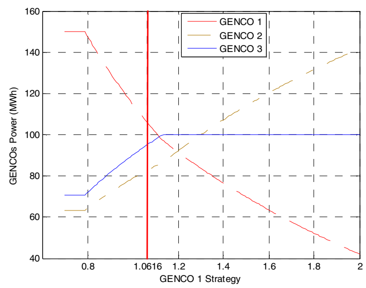

Fig. 3. The accepted power of three GENCOs in terms of GENCO 1 strategy

2) *Transmission Capacity Constraint:*

If the GENCOs' cost coefficients are as in Table III, then the calculated Nash point using (18) as shown in Fig. 4 is not an optimal Nash point.

As shown in Fig. 4, GENCO 3 can gain more benefit by changing its strategy. Thus the calculated Nash point (shown with red line in Fig. 4) is not optimal. Reaching the transmission line 3 to its maximum capacity is the reason of non-optimality of the calculated Nash point. The transmitted energy of line 3 in terms of GENCO 3 strategy is shown in Fig. 5.

TABLE III
GENCOs Cost Coefficients Data

NO.	**Cost function coefficients**		P_{min} (MW)	P_{max} (MW)
	a	b		
G_1	0.01527	16.389	0	1000
G_2	0.019202	11.737	0	1000
G_3	0.013791	17.858	0	1000

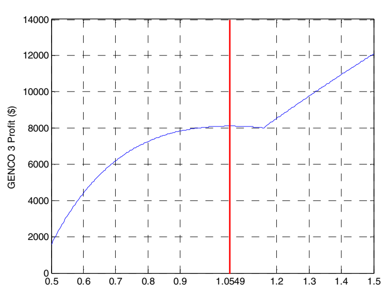

Fig. 4. The profit variation of GENCO 3 around its strategy

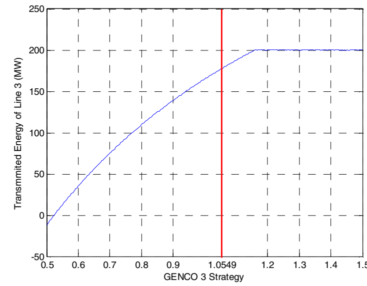

Fig. 5. The transmitted energy of line 3 in terms of GENCO 3 strategy

V. CASE STUDY

The 30-Bus test system depicted in Fig. 6 has been adopted from [2] with some modifications. GENCOs are denoted in Fig. 6 by G_i and their data is shown in Table IV. All transmission lines have zero resistances and their capacity limits are equal to 50 MW[2]. Other network data such as load and transmission reactances can be found in [2, Appendix D].

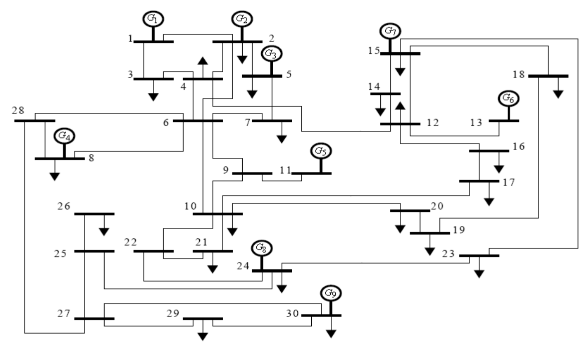

Fig. 6. Modified IEEE 30-Bus test system

TABLE IV
GENCOs Cost Coefficients Data

GENCO NO.	Cost Function Coefficients		P_{min} (MW)	P_{max} (MW)
	a	b		
G_1	0.0472	45.947	0	100
G_2	0.0399	41.336	0	100
G_3	0.0426	45.691	0	100
G_4	0.0476	40.748	0	100
G_5	0.0447	37.486	0	100
G_6	0.0487	45.579	0	100
G_7	0.0321	44.38	0	100
G_8	0.0297	47.096	0	100
G_9	0.0493	46.888	0	100

Three scenarios are used to illustrate the proposed method. In scenario A, there aren't any binding constraints on GENCOs' productions and transmission lines capacities in the calculated Nash point. In scenario B, the maximum production capacity of GENCO 1 is limited to 50 MW. In scenario C, the maximum transmission capacity of Line 14 connecting bus 4 and 10 is limited to 20 MW. The Nash point is calculated using (18) in scenarios B and C. Therefore in these scenarios, market power will be exist and the profit of GENCOs is more

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[2] Except Line 13 connecting bus 9 and 11 and Line 16 connecting bus 12 and 13 that their limits are set to maximum production capacity of GENCO 5 and 6, respectively.

than scenario A that is in perfect market. The most common index for assessing the market power is the Lerner Index (LI) [18] that compares the levels of prices under imperfect and perfect competition with calculating a value between 0 and 1 as:

$$LI = \frac{Price^{(PM)} - Price^{(PT)}}{Price^{(PM)}}$$
$$Price = \sum_{i \in G} LMP_i \cdot P_i \tag{20}$$

Where PM denotes that the situation of price maker of GENCOs or imperfect competition and PT denotes that the situation of price taker of GENCOs or perfect competition. Thus the LI is used to calculate the market power in the following three imperfect market scenarios.

A. Scenario A–Perfect Competition

In this case study, the market is perfect because the Nash point is calculated within the capacity limits. LMPs and GENCOs strategies, productions and profits are shown in Table V. This Nash point is optimal because if each GENCO unilaterally changes its strategy, its profit will be reduced. This concept has been shown in Fig. 7-A for GENCO 1 and also other GENCOs have similar figures as GENCO 1.

B. Scenario B – Imperfect Competition (Limitation on Supply Capacity)

In this case, it's supposed that there is a limitation on GENCO 5 production in 50 MW. The results are shown in Table VI and show the formation of market power in comparison to Table V. The LI is utilized for calculating the value of market power. The value of LI is equal to 1.21% in this case. This solution is a Nash point. The profit versus strategy curve is similar to fig. 7-A for all GENCOs (not shown here). The accepted power of GENCO 5 around its strategy is shown in Fig. 7-B.

TABLE V
GENCOs' Parameters in Nash Point

GENCO No.	Strategies	LMP ($/MWh)	Power (MW)	Profit ($)
G_1	0.9824	47.2327	11.3066	8.5079
G_2	0.9302	47.2327	59.1149	209.0707
G_3	0.98	47.2327	14.7084	13.4597
G_4	0.9169	47.2327	56.4956	214.3195
G_5	0.8836	47.2327	89.2647	513.6856
G_6	0.9771	47.2327	14.1617	13.6486
G_7	0.9697	47.2327	33.7169	59.7078
G_8	0.9986	47.2327	1.7091	0.1469
G_9	0.9951	47.2327	2.9222	0.586

C. Scenario C – Imperfect Competition (Limitation on Transmission capacity)

In this case, there is a limitation on line 14 capacity in 20 MW. The market power is formed in this situation too. The value of LI is equal to 2.25%. The profit versus strategy curve is similar to fig. 7-A for all GENCOs (not shown here). Also the transmitted energy of Line 14 in terms of GENCO 1 strategy is shown in Fig. 7-C. It's important to note that there are not any solutions (or Nash points) with consideration of binding constraints for other GENCOs' supply and transmission lines capacities.

TABLE VI
GENCOs' Parameters in Nash Point

GENCO No.	Strategies	LMP ($/MWh)	Power (MW)	Profit ($)
G_1	0.9763	47.8135	16.0434	17.8059
G_2	0.9297	47.8135	63.1871	249.8983
G_3	0.9747	47.8135	19.7421	25.2984
G_4	0.9158	47.8135	60.1664	252.6681
G_5	1.0298	47.8135	50	404.5707
G_6	0.9713	47.8135	18.7169	24.7555
G_7	0.9676	47.8135	39.2228	85.3085
G_8	0.9936	47.8135	8.645	3.9832
G_9	0.9878	47.8135	7.6764	4.1967

TABLE VII
GENCOs' Parameters in Nash Point

GENCO No.	Strategies	LMP ($/MWh)	Power (MW)	Profit ($)
G_1	0.9717	48.2738	19.789	27.5756
G_2	0.9287	48.2702	66.6252	284.7761
G_3	0.9714	48.2601	23.402	36.7914
G_4	0.9165	48.2539	62.4683	282.9922
G_5	1.1459	47.7157	23.2234	213.4486
G_6	0.9695	48.5629	23.1561	42.9707
G_7	0.9839	48.6143	39.2046	116.6873
G_8	1.0021	48.6888	12.5479	15.3095
G_9	0.9797	48.4485	12.9835	11.9418

TABLE VIII
GENCOs' Parameters in Nash Point

GENCO No.	Strategies	LMP ($/MWh)	Power (MW)	Profit ($)
G_1	0.97666	48.56	20	33.387
G_2	0.93355	48.559	66.878	304.53
G_3	0.97141	48.559	25.216	45.231
G_4	0.92074	48.559	62.93	302.89
G_5	1.1721	48.537	21.933	220.86
G_6	0.9704	48.571	22.958	43.021
G_7	0.98551	48.574	38.239	113.44
G_8	1.0023	48.577	11.509	13.106
G_9	0.97918	48.567	13.737	13.749

D. Scenario D – Imperfect Competition (Limitation on Supply and Transmission Capacity)

In this case, it's supposed that there are two limitations on minimum production of GENCO 1 in 20 MW and maximum transmission capacity of line 14 in 20 MW. The results are shown in Table VIII and show the formation of market power in comparison to Table I. The LI is utilized for calculating the value of market power. The value of LI is equal to 2.74% in this case that is more than case C. Also the solution is a Nash point and the accepted power of GENCO 1 around its strategy is shown in Fig. 7-D.

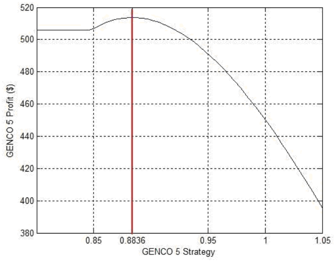

Fig. 7-A: GENCO 5 Profit variation around its strategy.

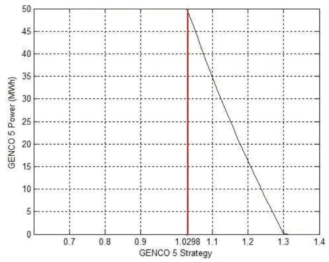

Fig. 7-B: GENCO 5 accepted power variation around its strategy.

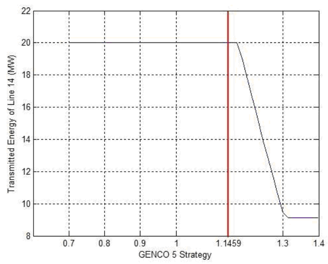

Fig 7-C: Transmitted energy variation of line 14 around GENCO 5 strategy.

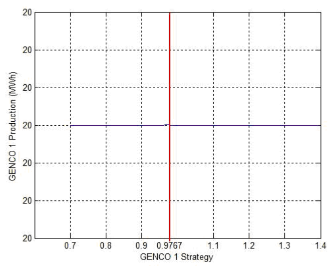

Fig 7-D: GENCO 1 accepted power variation around its strategy.

VI. CONCLUSION

In a fully competitive electricity market, each participant should bid at its marginal cost in order to maximize its revenue. However, some practical electricity markets such as Iran electricity market are not a perfectly competitive one because of the particular characteristics such as the severe generation and transmission constraints. So, it is critical for a GENCO to devise a good bidding strategy in order to maximize its potential profits.

It is proved in this paper that the bilevel optimization problem, or equivalently, the MPEC program, are used for determining the optimal bidding strategies of GENCOs, can be converted to an ordinal one-level optimization problem. Furthermore, the NE is calculated by solving simultaneously this optimization problem for all GENCOs. Also it is proved that if Nash equilibrium exists in the perfect competition conditions, it would be unique. But there may be multiple Nash equilibrium in the imperfect competition conditions.

VII. APPENDIX

A. LMP Expressions

Fu in [19] has shown that the LMP components in a lossless DC network model include marginal energy and congestion cost could be written as:

$$\mathrm{LMP} = \lambda - \mathrm{SF}^{\mathrm{T}} \cdot \pi \qquad (21)$$

where λ and π are the dual variables from expressions (7) and SF is the shift factor which is the sensitivity of a line flow to a bus generation increment (injection) [3].

Let us suppose the transmission line "l" is congested and further assume that all GENCOs in cap^+ and cap^- sets are limited in their maximum and minimum capacity, respectively. So the inequality constraint (5) is active for all GENCOs in cap set (that is the union of cap^+ and cap^- sets). Note that binding inequality constraints are considered equality constraints and nonbinding ones are disregards. Under these conditions the subproblem (2)-(6) is simplified to:

$$\begin{aligned}
&\min_{P_i} \sum_{i\in G} k_i \cdot (2\cdot a_i \cdot P_i + b_i)\cdot P_i \\
&\quad P_{Load} - \sum_{i\in G} P_i = 0 \qquad (\lambda) \\
&\quad SF_l \cdot (P - P_D) = T_l^{max} \;\equiv\; SF_l \cdot P = \underbrace{\left(T_l^{max} + SF_l \cdot P_D\right)}_{F_{max}} \qquad (\pi_l) \\
&\quad P_i^{min} = P_i \qquad \forall i \in cap^- \qquad (\mu_i^-) \\
&\quad P_i = P_i^{max} \qquad \forall i \in cap^+ \qquad (\mu_i^+)
\end{aligned} \qquad (22)$$

For simplicity and without loss of generality, we assume that only GENCO 1 supply capacity constraint is active. For calculating the dual variable, we have to setup the Lagrange equation for the problem as:

$$\begin{aligned}
L = &\sum_{i\in G} k_i \cdot (2\cdot a_i \cdot P_i + b_i)\cdot P_i + \lambda \cdot \left(P_{Load} - \sum_{i\in G} P_i\right) \\
&+ \pi \cdot (SF_l \cdot P - F_{max}) + \mu_1 \cdot (P_1 - P_1^{max})
\end{aligned} \qquad (23)$$

Using the KKT conditions and deriving the Lagrange equation with respect to the variables and the multipliers (dual variables), the following matrix will be yielded:

$$\underbrace{\begin{bmatrix}
\sum\limits_{\substack{i\in G\\ i\neq 1}} \dfrac{1}{4\cdot a_i \cdot k_i} & -\sum\limits_{\substack{i\in G\\ i\neq 1}} \dfrac{SF_{1i}}{4\cdot a_i \cdot k_i} & -\dfrac{1}{4\cdot a_1 \cdot k_1} \\
\sum\limits_{\substack{i\in G\\ i\neq 1}} \dfrac{SF_{1i}}{4\cdot a_i \cdot k_i} & -\sum\limits_{\substack{i\in G\\ i\neq 1}} \dfrac{SF_{1i}^2}{4\cdot a_i \cdot k_i} & -\dfrac{SF_{11}}{4\cdot a_1 \cdot k_1} \\
\dfrac{1}{4\cdot a_1 \cdot k_1} & -\dfrac{SF_{11}}{4\cdot a_1 \cdot k_1} & -\dfrac{1}{4\cdot a_1 \cdot k_1}
\end{bmatrix}}_{M} \cdot \begin{bmatrix} \lambda \\ \pi \\ \mu_1 \end{bmatrix} = \underbrace{\begin{bmatrix}
P_{Load} + \sum\limits_{i\in G} \dfrac{b_i}{4\cdot a_i} \\
F_{max} + \sum\limits_{i\in G} SF_{1i} \cdot \dfrac{b_i}{4\cdot a_i} \\
P_1^{max} + \dfrac{b_1}{4\cdot a_1}
\end{bmatrix}}_{N} \qquad (24)$$

After solving the equation (24) in terms of the multipliers and substituting them in (21), the LMP expressions will be calculated. In addition, if no transmission limit applies, the LMP expression can be calculated in the same way and would be in the form of (8).

B. Proof of Lemma 4

According to (18), there is a second-order equation w.r.t. B_i. After solving this equation and with some straightforward manipulations, two following solutions for B_i will be obtained as:

$$B_i = \frac{1}{2\cdot a_i} \qquad (25)$$

$$B_i = \frac{2\cdot LD_i}{b_i} - x_i \cdot \frac{b_i + 4\cdot a_i \cdot LD_i}{b_i} \qquad (26)$$

The first one isn't an acceptable solution for the GENCO's optimal strategy because for each player n and m, we have:

$$\left.\begin{aligned} B_n &= \sum_{\substack{j\in G\\ j\neq n}} \frac{1}{4\cdot a_j\cdot k_j} = \frac{1}{2\cdot a_n} \\ B_m &= \sum_{\substack{j\in G\\ j\neq m}} \frac{1}{4\cdot a_j\cdot k_j} = \frac{1}{2\cdot a_m} \end{aligned}\right\} \Rightarrow \sum_{\substack{j\in G\\ j\neq n}} \frac{1}{4\cdot a_j\cdot k_j} - \sum_{\substack{j\in G\\ j\neq m}} \frac{1}{4\cdot a_j\cdot k_j} \cong \frac{1}{2\cdot a_n} - \frac{1}{2\cdot a_m}$$

$$\Rightarrow \quad \frac{1}{4\cdot a_m} - \frac{1}{4\cdot a_n} \cong \frac{1}{2\cdot a_n} - \frac{1}{2\cdot a_m} \quad \Rightarrow \quad -2 \cong 1$$

For the second solution, writing n equations for n generating companies will result in a matrix equation $M\cdot x = N$ as follows:

$$\underbrace{\begin{bmatrix} \frac{b_1+4\cdot a_1\cdot LD}{b_1} & \cdots & 1 \\ 1 & \cdots & 1 \\ \vdots & \vdots & \vdots \\ 1 & \cdots & \frac{b_n+4\cdot a_n\cdot LD}{b_n} \end{bmatrix}}_{M} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{2\cdot LD_1}{b_1} \\ \frac{2\cdot LD_2}{b_2} \\ \vdots \\ \frac{2\cdot LD_n}{b_n} \end{bmatrix}}_{N}$$

If matrix M has an inverse then there would be a solution for x which represents the unique market Nash equilibrium. Otherwise there wouldn't be any Nash point. So Lemma 4 is proved.

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IX. Biographies

Mehdi Kabiri was born on Sep. 21, 1983 in Iran. He received the B.Sc. degree in electrical engineering from K.N.Toosi University, Tehran, Iran, in 2005, and the M.Sc. degree in 2008 from University of Kashan, Kashan.

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