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Performance Evaluation of Indices for Transient Stability

Uros Kerin, *Member, IEEE*, Maras Bojan, and Grega Bizjak, *Member, IEEE*

Abstract—In this paper the established performance indices for screening and ranking of power system contingencies are applied. The indices capture the change in the system state variables and deliver the degree to which it is provoked under different operating states. The indices refer to the generator coherency, transient energy conversion, dot products derived from the classical concept of the transient energy function, and generator angular deviation. The indices are calculated using the time domain simulation and a power system model.

Index terms—screening, ranking, stability indices, transient stability

I. Introduction

POWER SYSTEMS are complex structures subject to disturbances and unexpected fluctuations in operation. Events, such as a three phase fault with a line or generator tripping can cause unforeseen dynamics possibly leading to cascading outages or even loss of synchronous operation. In order to prepare the system to withstand such contingency conditions the disturbances posing a sever threat to the integrity of the system operation need to be determined.

A common approach in screening and ranking of the disturbances is to evaluate the consequent dynamic behavior of a system in respect to the criteria of secure (stable) system operation. The criteria can refer to firm system limits, such as out of step of generators or critical under/over-voltage/frequencies, and derived constraints, such as the ones based on the concept of the transient energy conversion [1].

Also a suitable way of weighting the disturbances according to the severity of their impact is to introduce performance indices [2]. The indices are able to capture a variation in the system state variables in its transition from a pre- to post-contingency condition. Representative results are acquired by referring the indices to the:

- change of rotor angle differences
- change of rotor angle differences with respect to centre of inertia
- change of voltage and currents
- change of generator speed or system frequency
- change of transient energy of generators
- acceleration of generators
- system oscillation and damping

In literature various forms of the indices are available [3]-[4]. However, while application of some is limited to the research level, others can be of great practical value.

In this paper we share our experience in applying the indices from [5] and [6] in stability investigation of a large power system. For the purpose, the indices have been implemented into the dynamic security assessment framework [7] based on the well established power system simulator and calculated using the time domain simulation.

II. Performance Indices

The main aim of system performance indices is to capture the dynamic state of a power system after it has been provoked by a disturbance or contingency. They need to report whether the system fulfils the constraints of secure system operation after outages or severe system faults under different system states.

Using the indices all aspects of the dynamic behavior can be considered; in this work, however, the system transient behavior is of prime concern. The dynamic performance of a system is investigated calculating the indices based on the derived system quantities relating to:

- generator coherency,
- transient energy conversion,
- dot products derived from the classical concept of the transient energy function,
- generator angular deviation over time.

Following is the analytical definition of the indices and their main characteristics.

A. Index based on generator coherency

Generator coherency is defined "as the measure of closeness of all generator rotor angles (related to the center of inertia – COI) after fault-clearing". The concept is illustrated in Fig. 1 and Fig. 2. In Fig. 1, the angles remain close to the COI after fault-clearing. Thus, the operation is coherent and stable. However, if following a fault, large angular deviations

U. Kerin is with Faculty of electrical engineering, University of Ljubljana, 1000 Ljubljana, Slovenia (e-mail: uros.kerin@fe.uni-lj.si).

B. Maras is with Faculty of electrical engineering, University of Ljubljana, 1000 Ljubljana, Slovenia (e-mail: bojan.maras@gmail.com).

G. Bizjak is with Faculty of electrical engineering, University of Ljubljana, 1000 Ljubljana, Slovenia (e-mail: grega.bizjak@fe.uni-lj.si).

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emerge operation coherency could be lost and the system becomes unstable (Fig. 2).

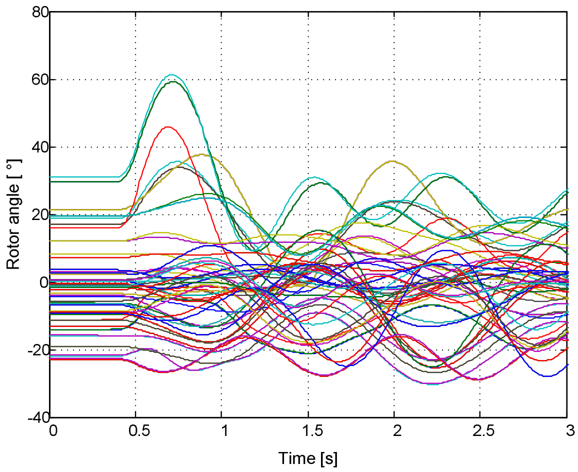


Figure 1 Generator coherent operation

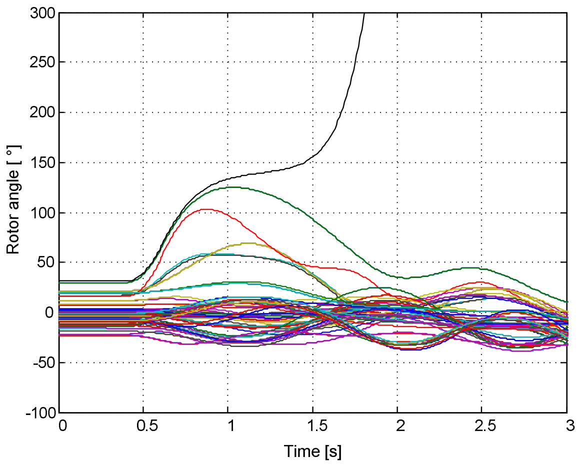


Figure 2 Loss of generator coherency

Performance index derived from these assumptions is defined by (1). It is the maximum difference between the maximum and minimum angle degree of all system generators in a short period after fault clearing:

$$Index1 = \max\left(\max \Theta_i\left(t\right) - \min \Theta_i\left(t\right)\right) \qquad (1)$$
$$i = 1, 2, \ldots N_G$$
$$t_{cl} \le t \le t_{cl} + T$$

where:

- Θ: generator rotor angle relative to COI
- N_G: number of generators
- t_{cl}: time of fault clearing
- T short period after fault clearing

B. *Index based on transient energy conversion*

The index is based on energy conversion between the transient kinetic energy and potential energy of the system machines. The transient kinetic energy is related to the speed of generators, whereas the potential energy includes three parts: position energy of all rotors relative to the COI, magnetic energy, and dissipation energy. In no-fault conditions the energies are balanced and form equilibrium. In case of a fault additional kinetic energy is added to the system and the equilibrium is lost. If the system has enough potential energy to absorb this excessive kinetic energy the system will remain in synchronism and a new stable equilibrium will be reached (Fig. 3). In the opposite case, the point of instability may be reached (Fig 4). The figures provide post-fault energy trajectories of a system after fault clearing.

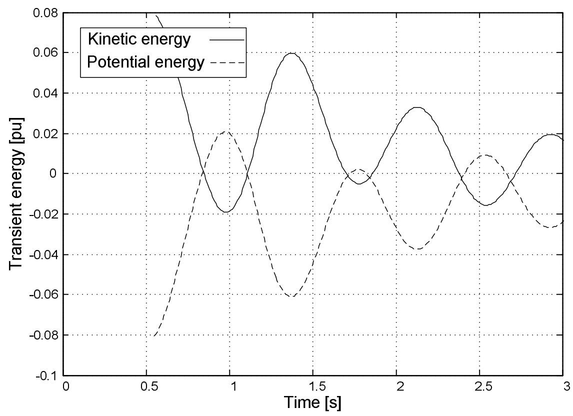


Figure 3 Transient energy conversion

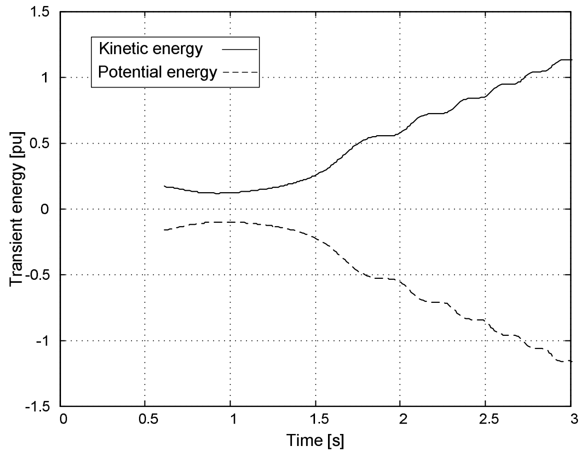


Figure 4 Transient energy conversion at stability loss

Index derived from the concept of energy conversion is defined as the maximum difference between the transient kinetic energy and potential energy of the system in a short period after fault clearing:

$$Index2 = \max\left(\left|V_{ke}\left(t\right) - V_{pe}\left(t\right)\right|\right) \qquad (2)$$
$$t_{cl} \le t \le t_{cl} + T$$

where:

- V_{ke}: transient kinetic energy
- V_{pe}: transient potential energy
- t_{cl}: time of fault clearing
- T: short period after fault clearing

C. Indices based on dot products

Three dot products are established. The first one gives the measure of total accelerating power and the power system (including generator and network) response to this accelerating power. The first dot product is defined by (3)

$$dot1 = \sum_{i=1}^{N_G} f_i \cdot \omega_i \tag{3}$$

$$f_i = P_{mi} - P_{ei} - \frac{H_i}{H_t} P_{COI} \tag{4}$$

$$P_{COI} = \sum_{i=1}^{N_G} \left(P_{mi} - P_{ei} \right) \tag{5}$$

$$i = 1, 2, \ldots N_G$$

where:

- M_i: inertia constant of each generator
- M_t: total inertia constant of all generators
- P_{mi}: mechanical power input of each generator
- P_{ei}: electrical power output for each generator
- ω_i: rotor speed with respect to COI

Using expression (3) as a base and by taking into account the rotor angle vector of i-th machine, expressions (6) and (7) can be derived, defining the second and the third dot product

$$dot2 = \sum_{i=1}^{N_G} f_i \cdot \Theta_i \tag{6}$$

$$dot3 = \sum_{i=1}^{N_G} \omega_i \left(\Theta_i - \Theta_i^{cl} \right) \tag{7}$$

where:

- Θ_i: rotor angle with respect to COI
- Θ_i^{cl}: rotor angle of i-th generator at fault clearing

The typical behavior of the dot products is illustrated in Fig. 5 to Fig. 7. Depicted are stable and unstable cases.

Based on their dynamic response the dot products can deliver system stability measure following a disturbance. The stability indices derived are:

$$Index3 = \max dot1(t) - \min dot1(t) \tag{8}$$

$$Index4 = \max dot2(t) - \min dot2(t) \tag{9}$$

$$Index5 = \max dot3(t) - \min dot3(t) \tag{10}$$

$$t_{cl} \le t \le t_{cl} + T$$

where:

- t_{cl}: time of fault clearing
- T: short period after fault clearing

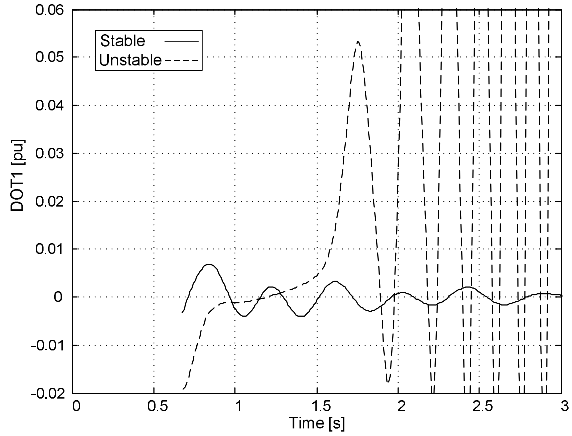


Figure 5 *dot1* post-fault trajectory

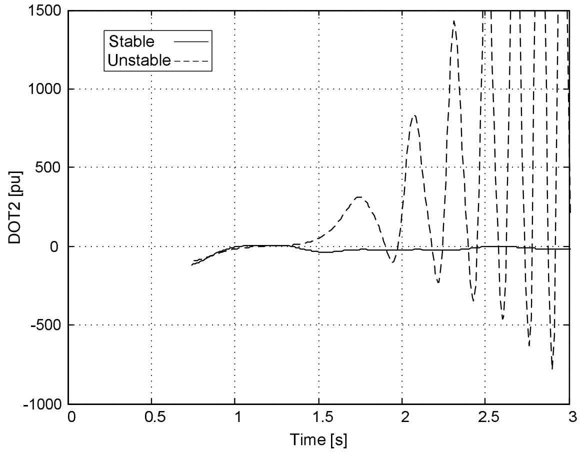


Figure 6 *dot2* post-fault trajectory

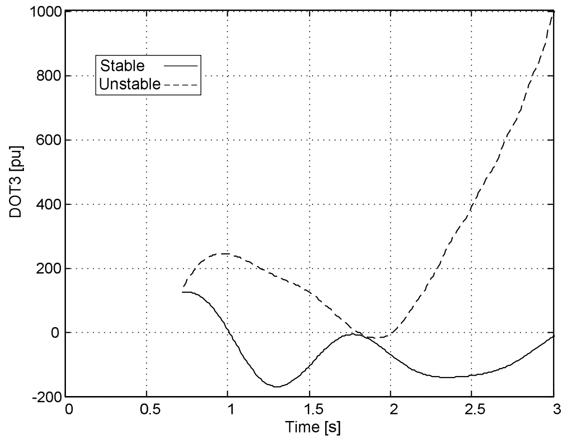


Figure 7 *dot3* post-fault trajectory

D. Integral square generator angle index

The integral square generator angle index (ISGI) aims at judging the severity of stable and unstable transient events considering dynamic behavior of the system generators. The ISGI is defined by (11) and the derived measure for determining the severity is given by (12).

$$ISGI = \int_0^T \sum_{i=1}^{N_G} M_i \cdot \Theta_i^2 \cdot dt \tag{11}$$

$$Index6 = \max\left(ISGI\right) \tag{12}$$

where:

- M_i: are machines inertias
- Θ_i: rotor angle with respect to COI
- T: short period after fault clearing
- N_G: number of generators

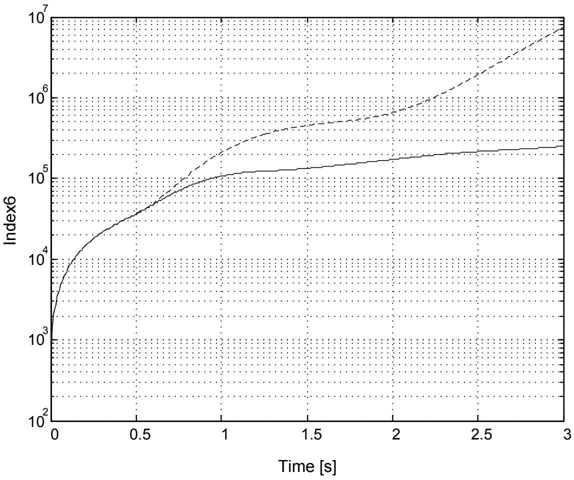


Figure 7 *dot3* post-fault trajectory

The index is a coherency based index (similar as *Index1*). It provides the aggregate measure of angular deviation of generators during transient and equilibrium conditions. The index is usually nonzero when the system is at equilibrium. It increases if changes in generation, load and power transfers that result in larger angle differences take place. The largest index scores follow system transient due to increase in diverging of generators.

E. Composite index

One of the most important characteristics to evaluate the quality of a performance index is its capture ration. The capture ratio is defined as the ratio of the flagged critical contingency cases to the actual contingency cases.

It is clear that each of the defined indices (*Index*1 to *Index*6) has a capture ratio of its own. Some indices can miss critical cases while others can misinterpret the severity of the condition. Referring to the past experience in this field, a composite severity measure is suggested.

A composite index derived is a numerical combination of the individual ones. The most reasonable is to introduce a weighted sum, where each component is weighted in reference to the investigation requirements or system related characteristics. The components can be of absolute or relative magnitude. In either way, it is expected that the composite index would have a better capture ratio than the individual indices. The index is given by (13)

$$\begin{aligned} Index_c = {} & \omega_1 \cdot Index1 + \omega_2 \cdot Index2 + \omega_3 \cdot Index3 + \\ & \omega_4 \cdot Index4 + \omega_5 \cdot Index5 + \omega_6 \cdot Index6 \end{aligned} \tag{13}$$

III. Performance Test

The performance of the indices is evaluated applying the time domain simulation. The indices have been calculated using the simulator PSS™NETOMAC [8] and a power system model.

Main characteristics of the model are: 500-, 230-, 115-kV and some lower voltage levels, 259 transformers, 119 generators, and 515 transmission lines. The main 500- and 230-kV transmission network is shown in Fig. 8. A single contingency case is considered: three phase fault of 200ms in duration. The contingency is applied to all 500-, 230- and 115kV nodes, so in sum 192 study cases are investigated.

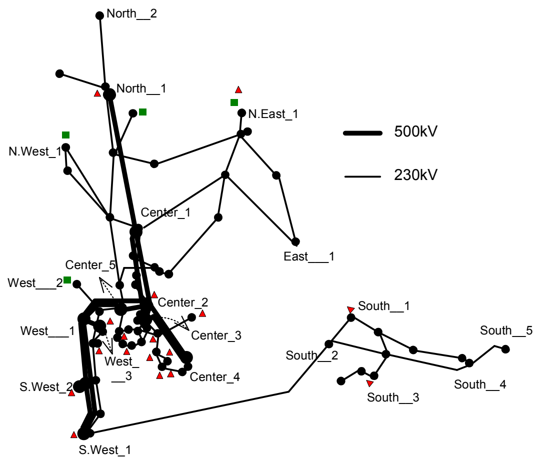


Figure 8: 500 and 230-kV network of the test system

The components of the composite index are defined as a quantitative relation between the case related and the worst case index degree, and are weighted in reference to Table 1. The weights are distributed in a way that the maximum sum of the weighted components equals 1 (see 14). At first, three time frames have been taken into account for the index calculation: 0.5s, 1.0s, and 2.0s after fault clearing. However, since best index performance at the largest of the three is achieved, only the 2.0s time frame is considered further.

Table I
Component Weights

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
0.5	0.1	0.1	0.1	0.1	0.1

$$\begin{aligned} & Index_c\big|_{\max} = \\ & = \omega_1 \cdot 1 + \omega_2 \cdot 1 + \omega_3 \cdot 1 + \omega_4 \cdot 1 + \omega_5 \cdot 1 + \omega_6 \cdot 1 = 1 \end{aligned} \tag{14}$$

TABLE II
RANKING RESULTS WITH THE TEST SYSTEM

case	***Index*1**	case	***Index*2**	case	***Index*3**	case	***Index*4**	case	***Index*5**	case	***Index*6**	case	$\boldsymbol{Index_c}$	$\boldsymbol{\Delta\delta}$
92	8161	171	0.111	172	0.721	172	9529	92	17651	92	83944276	92	0.760	-7981
181	2791	172	0.108	171	0.562	171	6847	184	4524	172	38405192	172	0.495	-2611
177	2435	140	0.099	92	0.163	92	3355	172	4270	171	27633841	171	0.438	-2255
171	2211	150	0.097	181	0.132	181	2418	171	3582	181	22350798	181	0.258	-2031
118	2139	161	0.096	177	0.119	177	2064	181	2181	154	21998260	177	0.222	-1959
184	2096	176	0.095	118	0.107	118	1714	177	1849	177	16074706	184	0.203	-1916
172	2077	180	0.087	184	0.086	154	1594	118	1589	184	15600673	118	0.192	-1897
154	1126	175	0.073	154	0.082	184	1280	154	996	118	11693562	154	0.155	-946
159	730	186	0.072	159	0.057	159	831	159	740	159	9012664	159	0.106	-550
179	202	190	0.062	143	0.034	190	158	186	634	179	705117	140	0.105	-22
72	188	192	0.060	133	0.033	192	152	190	543	186	487469	150	0.103	-8
129	169	189	0.046	142	0.031	186	150	192	520	143	414711	176	0.102	11
186	162	141	0.042	186	0.030	176	128	176	504	168	412497	161	0.101	18
185	160	143	0.039	187	0.029	187	126	143	499	133	375008	180	0.093	20
133	144	185	0.037	190	0.028	189	123	129	496	176	371216	186	0.085	36
168	142	191	0.037	176	0.028	185	123	157	489	190	364489	175	0.080	38
187	133	142	0.036	140	0.028	143	118	133	461	173	361396	190	0.073	47
190	133	187	0.036	150	0.027	140	112	156	452	142	355615	192	0.070	47
173	131	107	0.033	161	0.027	141	111	140	450	140	350529	189	0.054	49
160	126	133	0.033	180	0.025	180	105	102	420	160	347836	141	0.053	54
143	125	159	0.033	158	0.024	175	99	180	418	180	343422	143	0.052	55

To demonstrate the behaviour of the indices, the behaviour of the individual indices and the composite index for the cases 160 to 192 is plotted in the same figure (Fig. 9). The figure shows that the composite index is a sum of the relative components of which the contribution to the sum is reduced in reference to the weights; this is also the reason for the fluctuations in its degree. The composite index as it is tuned is suitable for the purpose of this work; nevertheless custom tuning and adjustments are possible.

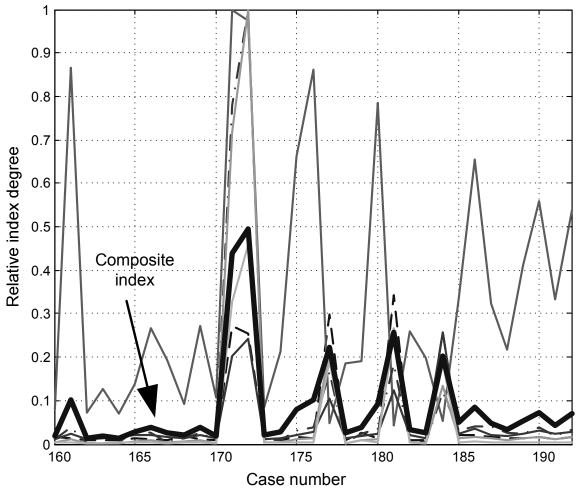


Figure 9 Index behavior for the test system

The calculation results are given in Table II. Each column under ‘case’ represents the case number on the rank list. The column under $\Delta\delta$ delivers angular excursions over the stability limit adopted for the system. The limit refers to the allowable angle difference between system generators and is conservatively selected; the maximum angular deviation allowed is 180 degrees. If the deviation is greater than the limit ($\Delta\delta<0$), the system is said to be unstable. Although the limit may not be the best reference for analyzing the calculation results (because it only refers to a single specific stability parameter, whereas the indices consider and report on many perspectives), it is, however, the most classical screening measure.

The limit of this form implies analyzing the calculation results on basis of comparison of $Index_c$ and *Index*1. Regarding the results the following observations are made. The composite index reports on all unstable cases but two; 72 and 179 which are captured by the *Index*1. The reason is that the case 72 is captured as stable by the rest of the indices (*Index*2 to *Index*6) and is therefore of small weight in calculating the $Index_c$. Similar explanation can be used in clarification of the case 179. The case is captured as unstable only by *Index*1 and *Index*6 and is not of enough significance in $Index_c$.

We tried to reduce the ‘deviations’ in the capture ratio by varying the time at which the indices are calculated. As already mentioned, in addition to the adopted time, also the ones of 0.5s and 1.0s have been taken into account. It has been determined that by applying the times, there is no significant variation in the capture ratio itself, but the sequence of the cases on each index improves if larger time frames are considered.

In order of best performance, thus, a compromise is needed between the accuracy and the calculation speed. The time frame used in this work seems a reasonable solution since the overall capture ratio is approximately 80%. The sequence of the cases on each index may not be exactly correct, but they are on the top side of the ranking list.

The computational burden of the calculation process is summarized in Table III. The complete execution time includes the time domain simulation with time step of 10ms, calculation of the indices (*Index*1 to *Index*6 and $Index_c$), index analysis, and preparation of the analysis results in form of tables and diagrams. The process has been performed on a generic desktop computer.

The average calculation time for investigating a single case is 2.6s. Considering that a typical demand per single processor computer system is 10 load flow cases with about 20 main contingencies checked and reported in 10 minutes, it is concluded that the application of the indices in an online screening and ranking process is a possible.

TABLE III
COMPLETE STUDY EXECUTION TIME

Test system	Cases	Time for complete study
119 generators	192	8.33 min

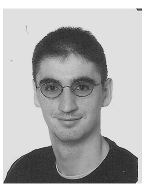
However, at this point an important observation is put forward. The indices alone are not able to distinguish between the stable and unstable cases. There is no firm indication in the index behaviour that would point out the stability limit violation in a clear and definite manner; see marginal index magnitudes in Table II. Since the knowledge of stability violation is crucial for the development of remedial actions it is therefore required that in an actual application the stability measure is referred to.

IV. CONCLUSION

In this paper performance indices for screening and ranking of power system contingencies are investigated. The indices are put to test considering a large power system. The performance of the indices has been aggregated introducing a single all-encompassing index.

Regarding the results the following conclusions can be drawn: The indices are efficient tool in screening and ranking of system events. As given, they can be implemented into conventional step-by-step calculation and calculated using the time domain simulation. Tuning of the indices according to custom needs is possible. Although in the study case the capture ratio was not 100%, by adjusting the weights of the individual indices this can be changed an improved to meet one's requirements. Moreover, with the average time for single case calculation of 2.6s (including the time domain simulation) the indices are suitable for online application.

V. BIOGRAPHIES



Uros Kerin (1979) received his B.Sc. degree from the University of Ljubljana, Slovenia, in 2004. Since then, he has been a member of the laboratory for power networks and devices at the Faculty of Electrical Engineering. In 2004 he was a guest researcher at Siemens AG, PTD SE, Germany. In 2007, he was a guest researcher at Arsenal Research in Vienna, Austria, where he was a member of a renewable-energy group. He is currently working in the areas of power system stability analysis and power system security assessment.

Bojan Maras is a pre-graduate student at the Faculty of electrical engineering, University of Ljubljana. Currently he is pursuing his B.Sc. degree in the field of electrical power systems and devices.

Grega Bizjak received his B.Sc. (1990), M.S. (1993) and Ph.D. (1997) in Electrical Power Engineering from the Faculty of Electrical Engineering at the University of Ljubljana. He started his career as a teaching assistant at the laboratory for power networks and devices at the same faculty. In 1991 and 1992 he was a guest researcher at Siemens AG PTD SE, Germany. He is currently an Assistant Professor and Head of the Laboratory of Lighting Engineering and Photometry at University of Ljubljana. He is active in the field of lighting and photometry as well as in the field of electrical power engineering. His main research interests in power engineering involve power system modeling, modeling of system devices, modeling of circuit breakers and the operation dynamics of distribution and industrial networks. He is the author of more than 90 scientific and conference papers, and a member of the IEEE and CIE.

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