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A Modified Power System Model for AGC Analysis

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Abstract-- **AUTOMATIC generation control (AGC) has been used for several years to meet the objective of maintaining the system frequency at nominal value and the net tie line power interchange from different areas at their scheduled values. One of the main components of AGC is Load frequency control (LFC), one of the major requirements in providing reliable and quality operation in multi-area power systems. In interconnected large power systems, variations in frequency can lead to serious large scale stability problems.**

A new model derived from [3] with substantial modifications is presented in this paper. One principal modification concerns the turbines and consists of the creation of aggregate turbine for each type of turbine, another important modification is the consideration of several types of turbines participating in the secondary control, and the final modification is the consideration of aggregate generation coefficient in forming the rotor angle input of the tie-line model.

***Index Terms*—AGC, LFC, Power System Control, Frequency Control, Tie-Line, Power Exchage**

I. NOMENCLATURE

$\boldsymbol{\Delta P}$ - vector of node active power increments,

$\boldsymbol{\Delta\delta}$ - vector of node voltage angle increments.

$\mathbf{H}$ - Jacobian of the active power as function of voltage angles

$\boldsymbol{\Delta P}_{\mathrm{GA}}$, $\boldsymbol{\Delta P}_{\mathrm{GB}}$ - vector of node active power increments corresponding to generator $\{\mathrm{G_A}\}$ and $\{\mathrm{G_B}\}$

$\boldsymbol{\Delta\delta}_{\mathrm{GA}}$, $\boldsymbol{\Delta\delta}_{\mathrm{GB}}$ - vector of node voltage angle increments corresponding to generator $\{\mathrm{G_A}\}$ and $\{\mathrm{G_B}\}$

$\boldsymbol{1}_{\mathrm{GA}}$, $\boldsymbol{1}_{\mathrm{GB}}$ - unit vectors,

$\Delta\delta_{\mathrm{A}}$, $\Delta\delta_{\mathrm{B}}$ - common generator angle increments of subsystems A and B.

$H_{\mathrm{xy}} = \dfrac{\partial P_{\mathrm{W}}}{\partial \delta_{\mathrm{xy}}} = \dfrac{U_{\mathrm{x}} U_{\mathrm{y}}}{X_{\mathrm{xy}}} \cos\delta_{\mathrm{xy}}$ - tie-line synchronising power of line between nodes x and y.

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II. INTRODUCTION

AUTOMATIC generation control (AGC) has been used for several years to meet the objective of maintaining the system frequency at nominal value and the net tie line power interchange from different areas at their scheduled values. One of the main components of AGC is Load frequency control (LFC), one of the major requirements in providing reliable and quality operation in multi-area power systems. In interconnected large power systems, variations in frequency can lead to serious large scale stability problems. Frequency is one of the stability criteria for large-scale stability of power networks. For stable operation, constant frequency and active power balance must be provided. Frequency is depending on active power. Any change in active power demand/generation at power systems is reflected throughout the system by a change in frequency. In interconnected power networks with two or more areas, the load frequency control scheme has to be with two main control loops. These are primary control and secondary control. Primary control is achieved by the turbine governing system. In this loop, frequency maintenance at the scheduled value cannot be successful. The second control loop is used to control active power at the tie line between neighboring areas. A new model derived from [3] with substantial modifications is presented in this paper. One principal modification concerns the turbines and consists of the creation of aggregate turbine for each type of turbine, another important modification is the consideration of several types of turbines participating in the secondary control, and the final modification is the consideration of aggregate generation coefficient in forming the rotor angle input of the tie-line model. The paper is divided into 3 main sections. Section III is consecrated to a general description of AGC. Section IV describes the aggregate turbine modeling. Section V presents the power system model, in which the modified tie-line model is presented in details. A few conclusions are given in Section VI.

III. THE AGC MODEL

Generally, the load–frequency control is accomplished by two different control actions in interconnected two-area power systems: (a) the primary speed control, accomplished by governors which adjust the turbine valve/gate to bring the frequency back to the nominal or scheduled value. and (b) supplementary or secondary speed control actions. The

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primary speed control performs the initial vulgar readjustment of the frequency. By its actions, the various generators in the control area track a load variation and share it in proportion to their capacities. The speed of the response is only limited by the natural time lags of the turbine and the system itself. The output of each unit at a given system frequency can be varied only by changing its load reference, which in effect moves the speed-droop characteristic up and down. This control is considerably slower and goes into action only when the primary speed control has done its job. Response time may be of the order of one minute. The speed-governing system is used to adjust the frequency. A block diagram of the load-frequency control is presented in Fig. 1.

A model of power system AGC includes a classical integral secondary controller that sets the turbine reference power of each area. Power flows throughout the tie line between areas. Control and balance of the power flows at the tie line are required for supplementary frequency control. Also, damping of oscillations at the tie line is another requirement for successful control of frequency and active power generation. The easiest way of doing this is with the linear combination of the local frequency variation in each area and the tie line power variations as the input of each integral controller, which is called the area control error (ACE).

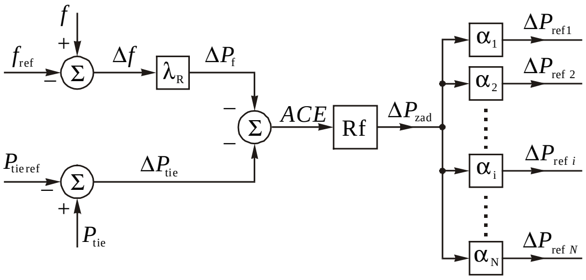

Fig. 1. Load–frequency control

IV. THE TURBINE MODEL

A model of power system is created considering the phenomena which should be taken into account. Most of the times a model is simplified. One of the well-known methods of simplification of mathematical models of power systems is the aggregation of certain power system elements, which has a very slight influence on the accuracy o the model obtained [1][6]. In case of turbines the model aggregation process depends on generator aggregation, and in particular on the type of aggregation used for simplifying the generator mathematical model.

The parameters of an equivalent turbine controller and the equivalent turbine for a given group of turbines can be obtained by fitting the transfer function frequency characteristics of different turbines belonging to a defined group to the transfer function frequency characteristic of the equivalent turbine. More information about frequency characteristic fitting is given by [5][6].

Nonlinearities of the turbine and governor models can be neglected in case of small disturbances analysis. The transfer function of each turbine and governor model are calculated for discrete values within the interval of frequency between 0.01 to 10 Hz. Then the transfer functions of different can be connected as it is shown on Fig. 2. The transfer function of the aggregated turbine is the sum of all transfer functions of coherent turbines of the same type.

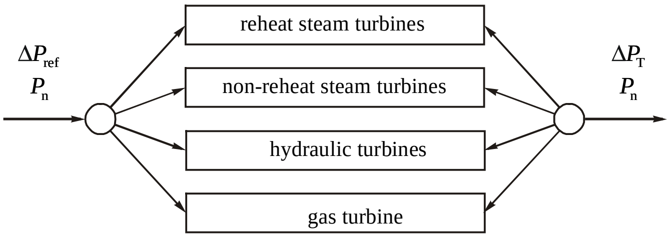

Fig. 2. An example of equivalent model of governor-turbine system

V. THE POWER SYSTEM MODEL

The power system model can be replaced by a 1st order transfer function ($G_{PS}(s)$)[2,3], which represents the equivalent model of coherent synchronous generators described by their rotor swing equations [4]. These assumptions make also possible the derivation of a mathematical model of two similar systems, the procedure of which is as follows:

- Elimination of all variables related to load nodes except terminal nodes of tie-line transmission lines,
- Aggregation of variables related to generating units, which aggregation should be performed separately for each subsystem
- Determination of terminal voltage angle deviations at the ends of each tie-line using the deviations of equivalent emf's of the subsustems.

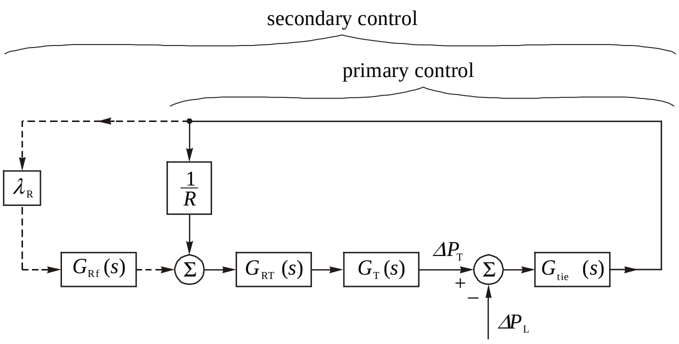

Fig. 3. Block diagram of the LFC o f a Power System [2]

The model of power system interconnection will be derived based on the well-known incremental equation

$$\boldsymbol{\Delta P} = \boldsymbol{H}\ \boldsymbol{\Delta\delta} \tag{1}$$

which describes the power system assuming that the node voltage modules are constant, and the vectors of which the equation are composed are the following:

$\boldsymbol{\Delta P}$ - vector of node active power increments,

$\boldsymbol{\Delta\delta}$ - vector of node voltage angle increments.

$\boldsymbol{H} = \left[\dfrac{\partial P}{\partial \delta}\right]$ - Jacobian of the active power as function of voltage angles

A. The incremental model elimination stage

Eliminating a few nodes will transforms the network model into a new model form composed of equivalent branches. The elimination of load nodes in the incremental model leads to the formation of equivalent synchronising active powers given by the equation (2).

The matrix equation (1) of the power system presented in Fig. 4a can be written in the following form:

$$\begin{array}{c} \{\mathrm{G_A}\} \\ \{\mathrm{G_B}\} \\ \mathrm{a} \\ \mathrm{b} \\ \{\mathrm{L_A}\} \\ \{\mathrm{L_B}\} \end{array} \quad \begin{bmatrix} \boldsymbol{\Delta P}_{\mathrm{GA}} \\ \boldsymbol{\Delta P}_{\mathrm{GB}} \\ 0 \\ 0 \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \\ \cdots & \boldsymbol{H} & \cdots \\ & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta\delta}_{\mathrm{GA}} \\ \boldsymbol{\Delta\delta}_{\mathrm{GB}} \\ \Delta\delta_{\mathrm{a}} \\ \Delta\delta_{\mathrm{b}} \\ \boldsymbol{\Delta\delta}_{\mathrm{LA}} \\ \boldsymbol{\Delta\delta}_{\mathrm{LB}} \end{bmatrix} \tag{2}$$

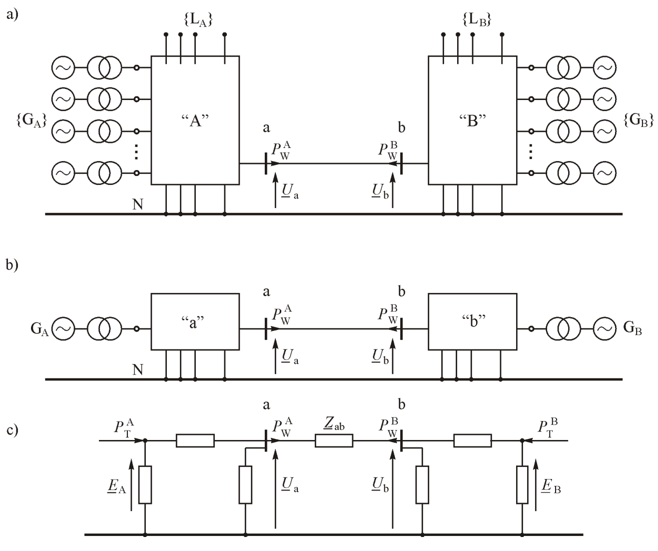

Fig. 4. Different stages of the equivalent power system creation

The most left side of the matrix equation contains symbols of nodes, which the different variables of active power and voltage angle incremental vector refer to. The vectors $\boldsymbol{\Delta P}_{\mathrm{GA}}$, $\boldsymbol{\Delta P}_{\mathrm{GB}}$ correspond to the generator $\{\mathrm{G_A}\}$ and $\{\mathrm{G_B}\}$ active power increments, whereas $\boldsymbol{\Delta\delta}_{\mathrm{GA}}$, $\boldsymbol{\Delta\delta}_{\mathrm{GB}}$ correspond to their rotor angle increments, respectively. They also represent the emf's angle increments of the different generators.

No active power increments correspond to the load nodes a,b of the tie-line terminal nodes. Their increments are equal to zero, which is equivalent to say that $\boldsymbol{\Delta P}_{\mathrm{LA}} = 0$, $\boldsymbol{\Delta P}_{\mathrm{LB}} = 0$ and $\Delta P_{\mathrm{a}} = P_{\mathrm{b}} = 0$. The variables $\boldsymbol{\Delta\delta}_{\mathrm{LA}}$, $\boldsymbol{\Delta\delta}_{\mathrm{LB}}$ are no longer interesting for further analysis and can be eliminated.

Any amount of variables of any matrix equation can be eliminated using the following well-known relationship by means of partial matrix inversion. The following matrix equation

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix} \tag{3}$$

can be transformed into the following form

$$\begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{y}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_{11} - \boldsymbol{a}_{12}\boldsymbol{a}_{22}^{-1}\boldsymbol{a}_{21} & \boldsymbol{a}_{12}\boldsymbol{a}_{22}^{-1} \\ -\boldsymbol{a}_{22}^{-1}\boldsymbol{a}_{21} & \boldsymbol{a}_{22}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{x}_2 \end{bmatrix} \tag{4}$$

For the particular case, in which $\boldsymbol{x}_2 = \boldsymbol{0}$, the matrix equation (4) can be written as follows:

$$\boldsymbol{x}_1 = \left(\boldsymbol{a}_{11} - \boldsymbol{a}_{12}\boldsymbol{a}_{22}^{-1}\boldsymbol{a}_{21}\right)\boldsymbol{y}_1 \tag{5}$$

The matrix $\boldsymbol{a}_{11} - \boldsymbol{a}_{12}\boldsymbol{a}_{22}^{-1}\boldsymbol{a}_{21}$ is called the partially inverted matrix. The matrices of the columns corresponding to the eliminated vector $\boldsymbol{y}_2$ of the expression given by (2) correspond to the column matrices $\boldsymbol{\Delta\delta}_{\mathrm{LA}}$, $\boldsymbol{\Delta\delta}_{\mathrm{LB}}$, respectively. After having used the same transformation as for (5) in (2) the following form of this matrix equation is obtained:

$$\begin{array}{c} \{\mathrm{G_A}\} \\ \{\mathrm{G_B}\} \\ \mathrm{a} \\ \mathrm{b} \end{array} \quad \begin{bmatrix} \boldsymbol{\Delta P}_{\mathrm{GA}} \\ \boldsymbol{\Delta P}_{\mathrm{GB}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{\mathrm{AA}} & \boldsymbol{H}_{\mathrm{AB}} & \boldsymbol{H}_{\mathrm{Aa}} & \boldsymbol{H}_{\mathrm{Ab}} \\ \boldsymbol{H}_{\mathrm{BA}} & \boldsymbol{H}_{\mathrm{BB}} & \boldsymbol{H}_{\mathrm{Ba}} & \boldsymbol{H}_{\mathrm{Bb}} \\ \boldsymbol{H}_{\mathrm{aA}} & \boldsymbol{H}_{\mathrm{aB}} & H_{\mathrm{aa}} & H_{\mathrm{ab}} \\ \boldsymbol{H}_{\mathrm{bA}} & \boldsymbol{H}_{\mathrm{bB}} & H_{\mathrm{ba}} & H_{\mathrm{bb}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta\delta}_{\mathrm{GA}} \\ \boldsymbol{\Delta\delta}_{\mathrm{GB}} \\ \Delta\delta_{\mathrm{a}} \\ \Delta\delta_{\mathrm{b}} \end{bmatrix} \tag{6}$$

The square matrix of (6) is divided into the respective submatrices considering the necessity of performing a further operation, which is the aggregation procedure.

B. The aggregation in the incremental model

Aggregating a group of generating nodes of the admittance model is equivalent to connect those nodes so that an equivalent node is obtained [4]. For the incremental power system this consists of summing the adequate synchronising active powers, resulting from the above-presented analysis.

If the coherence condition is fulfilled the angle changes of each subsystem are the same (co-rotation of all rotors of the same subsystem), which can be expressed as follows:

$$\boldsymbol{\Delta\delta}_{\mathrm{GA}} = \Delta\delta_{\mathrm{A}}\boldsymbol{1}_{\mathrm{GA}} \quad \text{and} \quad \boldsymbol{\Delta\delta}_{\mathrm{GB}} = \Delta\delta_{\mathrm{B}}\boldsymbol{1}_{\mathrm{GB}} \tag{7}$$

where $\boldsymbol{1}_{\mathrm{GA}}$, $\boldsymbol{1}_{\mathrm{GB}}$ are unit vectors, whereas $\Delta\delta_{\mathrm{A}}$, $\Delta\delta_{\mathrm{B}}$ are the common generator angle increments of subsystems A and B. In the admittance model these increments are the emf's angle variations of the equivalent generators. (Fig. 4b, c).

By definition the equivalent generator active power variation is equal to the sum of the active power changes of all generators belonging to the replaced group. This can be expressed as follows:

$$\Delta P_A = \sum_{i=1}^{n_A} \Delta P_i = \mathbf{1}_{GA}^T \, \boldsymbol{\Delta P}_{GA}$$
and
$$\Delta P_B = \sum_{i=1}^{n_B} \Delta P_i = \mathbf{1}_{GB}^T \, \boldsymbol{\Delta P}_{GB} \tag{8}$$

Considering (7) and (8) and after performing a few simple transformations of (6) the resulting matrix equation is expressed as follows:

$$\begin{array}{c} \mathrm{A} \\ \mathrm{B} \\ \mathrm{a} \\ \mathrm{b} \end{array} \begin{bmatrix} \Delta P_A \\ \Delta P_B \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{GA}^T \boldsymbol{H}_{AA} \mathbf{1}_{GA} & \mathbf{1}_{GA}^T \boldsymbol{H}_{AB} \mathbf{1}_{GB} & \mathbf{1}_{GA}^T \boldsymbol{H}_{Aa} & \mathbf{1}_{GA}^T \boldsymbol{H}_{Ab} \\ \mathbf{1}_{GB}^T \boldsymbol{H}_{BA} \mathbf{1}_{GA} & \mathbf{1}_{GB}^T \boldsymbol{H}_{BB} \mathbf{1}_{GB} & \mathbf{1}_{GB}^T \boldsymbol{H}_{Ba} & \mathbf{1}_{GB}^T \boldsymbol{H}_{Bb} \\ \boldsymbol{H}_{aA} \mathbf{1}_{GA} & \boldsymbol{H}_{aB} \mathbf{1}_{GB} & H_{aa} & H_{ab} \\ \boldsymbol{H}_{bA} \mathbf{1}_{GA} & \boldsymbol{H}_{bB} \mathbf{1}_{GB} & H_{ba} & H_{bb} \end{bmatrix} \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_B \\ \Delta\delta_a \\ \Delta\delta_b \end{bmatrix} \tag{9}$$

$$\begin{array}{c} \mathrm{A} \\ \mathrm{B} \\ \mathrm{a} \\ \mathrm{b} \end{array} \begin{bmatrix} \Delta P_A \\ \Delta P_B \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} H_{AA} & H_{AB} & H_{Aa} & H_{Ab} \\ H_{BA} & H_{BB} & H_{Ba} & H_{Bb} \\ H_{aA} & H_{aB} & H_{aa} & H_{ab} \\ H_{bA} & H_{bB} & H_{ba} & H_{bb} \end{bmatrix} \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_B \\ \Delta\delta_a \\ \Delta\delta_b \end{bmatrix} \tag{10}$$

Where all the elements are scalar and are given by the following matrix expressions:

$$\begin{aligned} H_{AA} &= \mathbf{1}_{GA}^T \boldsymbol{H}_{AA} \mathbf{1}_{GA} \\ H_{AB} &= \mathbf{1}_{GA}^T \boldsymbol{H}_{AB} \mathbf{1}_{GB} \\ H_{Aa} &= \mathbf{1}_{GA}^T \boldsymbol{H}_{Aa} \\ H_{Ab} &= \mathbf{1}_{GA}^T \boldsymbol{H}_{Ab} \\ H_{BA} &= \mathbf{1}_{GB}^T \boldsymbol{H}_{BA} \mathbf{1}_{GA} \\ H_{BB} &= \mathbf{1}_{GB}^T \boldsymbol{H}_{BB} \mathbf{1}_{GB} \\ H_{Ba} &= \mathbf{1}_{GB}^T \boldsymbol{H}_{Ba} \\ H_{Bb} &= \mathbf{1}_{GB}^T \boldsymbol{H}_{Bb} \end{aligned} \tag{11}$$

and

$$\begin{aligned} H_{aA} &= \boldsymbol{H}_{aA} \mathbf{1}_{GA} \\ H_{aB} &= \boldsymbol{H}_{aB} \mathbf{1}_{GB} \\ H_{bA} &= \boldsymbol{H}_{bA} \mathbf{1}_{GA} \\ H_{bB} &= \boldsymbol{H}_{bB} \mathbf{1}_{GB} \end{aligned} \tag{12}$$

Some of the matrix equation (6) elements are equal to zero because not all the nodes of the system shown in Fig. 4a have direct connection between one another, therefore:

$$\begin{aligned} \boldsymbol{H}_{AB} &= \boldsymbol{H}_{BA}^T = \boldsymbol{0} \\ \boldsymbol{H}_{Ab} &= \boldsymbol{H}_{bA}^T = \boldsymbol{0} \\ \boldsymbol{H}_{Ba} &= \boldsymbol{H}_{aB}^T = \boldsymbol{0} \end{aligned} \tag{13}$$

Having regard to this fact a part of the matrix equation (10) are equal to zero. Therefore the matrix equation can be rewritten as follows:

$$\begin{array}{c} \mathrm{A} \\ \mathrm{B} \\ \mathrm{a} \\ \mathrm{b} \end{array} \begin{bmatrix} \Delta P_A \\ \Delta P_B \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} H_{AA} & 0 & H_{Aa} & 0 \\ 0 & H_{BB} & 0 & H_{Bb} \\ H_{aA} & 0 & H_{aa} & H_{ab} \\ 0 & H_{bB} & H_{ba} & H_{bb} \end{bmatrix} \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_B \\ \Delta\delta_a \\ \Delta\delta_b \end{bmatrix} \tag{14}$$

The matrix equation (14) describes the equivalent network shown in Fig. 4c The Jacobian of (14) elements correspond to synchronizing active power between nodes of the equivalent network shown in Fig. 4c.

C. The tie-line synchronising power model

Relationships between tie-line terminal voltage $\underline{U}_a, \underline{U}_b$ angle deviations $(\Delta\delta_a, \Delta\delta_b)$ and equivalent emf's $\underline{E}_A, \underline{E}_B$ angle deviations $(\Delta\delta_A, \Delta\delta_B)$ are interesting for further considerations (Fig. 4c). The bottom row of the matrix equation (14) gives the following expression:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} H_{aA} & 0 \\ 0 & H_{bB} \end{bmatrix} \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_B \end{bmatrix} + \begin{bmatrix} H_{aa} & H_{ab} \\ H_{ba} & H_{bb} \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_b \end{bmatrix} \tag{15}$$

Then the following expression is derived from a slight transformation of (15)

$$\begin{bmatrix} \Delta\delta_a \\ \Delta\delta_b \end{bmatrix} = -\begin{bmatrix} H_{aa} & H_{ab} \\ H_{ba} & H_{bb} \end{bmatrix}^{-1} \begin{bmatrix} H_{aA} & 0 \\ 0 & H_{bB} \end{bmatrix} \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_B \end{bmatrix} \tag{16}$$

or

$$\begin{bmatrix} \Delta\delta_a \\ \Delta\delta_b \end{bmatrix} = \begin{bmatrix} \kappa_{aa} & \kappa_{ab} \\ \kappa_{ba} & \kappa_{bb} \end{bmatrix} \begin{bmatrix} \Delta\delta_A \\ \Delta\delta_B \end{bmatrix} \tag{17}$$

where:

$$\begin{aligned} \kappa_{aa} &= +\left(H_{aa}H_{bb} - H_{ba}H_{ab}\right)^{-1} H_{aa}H_{aA} \\ \kappa_{ab} &= -\left(H_{aa}H_{bb} - H_{ba}H_{ab}\right)^{-1} H_{ab}H_{bB} \\ \kappa_{ba} &= -\left(H_{aa}H_{bb} - H_{ba}H_{ab}\right)^{-1} H_{ab}H_{aA} \\ \kappa_{bb} &= +\left(H_{aa}H_{bb} - H_{ba}H_{ab}\right)^{-1} H_{bb}H_{bB} \end{aligned} \tag{18}$$

The tie-line active power of the system shown in Fig. 4c is in function of the angle difference $\delta_{ab} = \delta_a - \delta_b$. If the power losses at both sides of the transmission line are neglected the active power at both line ends are the same and is given by the following expression:

$$P_{\mathrm{W}} = \frac{U_{\mathrm{a}} U_{\mathrm{b}}}{X_{\mathrm{ab}}} \sin \delta_{\mathrm{ab}} \tag{19}$$

The tie-line active power variation can then be expressed as follows:

$$\Delta P_{\mathrm{W}} \cong \frac{\partial P_{\mathrm{W}}}{\partial \delta_{\mathrm{ab}}} \Delta\delta_{\mathrm{ab}} = H_{\mathrm{ab}} \Delta\delta_{\mathrm{ab}} = H_{\mathrm{ab}} \left(\Delta\delta_{\mathrm{a}} - \Delta\delta_{\mathrm{b}} \right) \tag{20}$$

where the partial derivative given by the following expression:

$$H_{\mathrm{ab}} = \frac{\partial P_{\mathrm{W}}}{\partial \delta_{\mathrm{ab}}} = \frac{U_{\mathrm{a}} U_{\mathrm{b}}}{X_{\mathrm{ab}}} \cos \delta_{\mathrm{ab}} \tag{21}$$

is the tie-line synchronising power. It is worthwile remembering that the expression (21) appears also in the matrix expression (2).

The matrix expression (17) can be expressed as follows:

$$\begin{aligned} \Delta\delta_{\mathrm{a}} &= \kappa_{\mathrm{aa}} \Delta\delta_{\mathrm{A}} + \kappa_{\mathrm{ab}} \Delta\delta_{\mathrm{B}} \\ \Delta\delta_{\mathrm{b}} &= \kappa_{\mathrm{ba}} \Delta\delta_{\mathrm{A}} + \kappa_{\mathrm{bb}} \Delta\delta_{\mathrm{B}} \end{aligned} \tag{22}$$

then:

$$\Delta\delta_{\mathrm{ab}} = \Delta\delta_{\mathrm{a}} - \Delta\delta_{\mathrm{b}} = (\kappa_{\mathrm{aa}} - \kappa_{\mathrm{ba}}) \Delta\delta_{\mathrm{A}} + (\kappa_{\mathrm{ab}} - \kappa_{bb}) \Delta\delta_{\mathrm{B}} \tag{23}$$

Substituting the angle difference $\Delta\delta_{\mathrm{a}} - \Delta\delta_{\mathrm{b}}$ of (20) by the equivalent expression (23) gives:

$$\Delta P_{\mathrm{W}} = H_{\mathrm{ab}} \left[(\kappa_{\mathrm{aa}} - \kappa_{\mathrm{ba}}) \Delta\delta_{\mathrm{A}} + (\kappa_{\mathrm{ab}} - \kappa_{\mathrm{bb}}) \Delta\delta_{\mathrm{B}} \right] \tag{24}$$

That means that the tie-line power deviation can be expressed by the emf's angle variations of both power systems.

Considering the primary and secondary control, an approximate analysis of load frequency control after a sudden load unbalance occurring in one or two systems can be carried out. The unbalance is indicated as $\Delta P_0^{\mathrm{A}}(\mathrm{s})$ or $\Delta P_0^{\mathrm{B}}(\mathrm{s})$ (Fig. 5). They are introduced as an additional input to the sum blocks $G_{\mathrm{PS}}^{\mathrm{A}}(s)$ and $G_{\mathrm{PS}}^{\mathrm{B}}(s)$, representing the power systems of areas A and B(Fig. 5). Block elements marked as "TURBINES A" and "TURBINES B" (Fig. 5). are composed of different types of turbines as shown in Fig. 2.

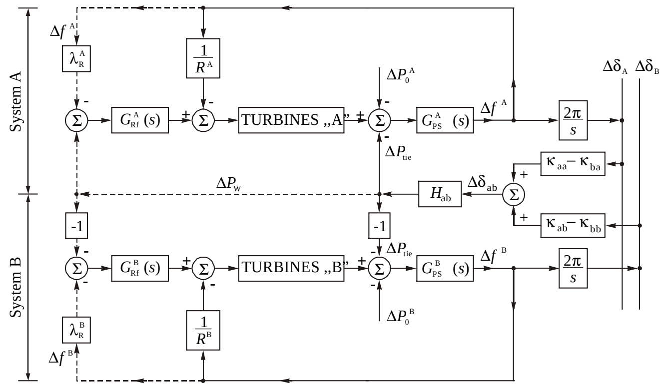

Fig. 5. The load-frequency control model for a two-area power system

VI. Conclusion

Substantial modifications of the Kundur model [4] allows to assess the real contribution of each power system in a new power model presented in this paper. One principal modification introduced in this paper concerns the turbine model - an aggregate turbine for each type of turbine, another important modification is the consideration of several types of turbines in the AGC model, and the final modification is the consideration of aggregate generation coefficient in forming the rotor angle input of the tie-line model. The mathematical model has shown more details about the procedure to be followed for considering the participation of aggregated models of turbines and generators on the formation of the feedback signal as an input for the tie-line model.

Test results of a 3-system LFC model obtained from the above-presented derivation is presented in Fig. 6. The simulated contingency is a sudden unbalance between load and generation in the system A (200MW). It could be seen that after a few long-period oscillations, frequency and tie-line power return to their original level due to the intervention of the generators of System A. it can be seen that the deviation of power is completely recovered by the generators of this area (Generation power A). The main problem in this regulation is the amplitude of oscillations during the transient state. This could be solved by using special stabilizing devices, the analysis of which is presented in [7] in details.

VII. Acknowledgment

The author gratefully acknowledges the contribution of prof. J. Machowski for his help on verifying the righteousness of all assumptions related to the aggregation procedure of some elements of the power system model.

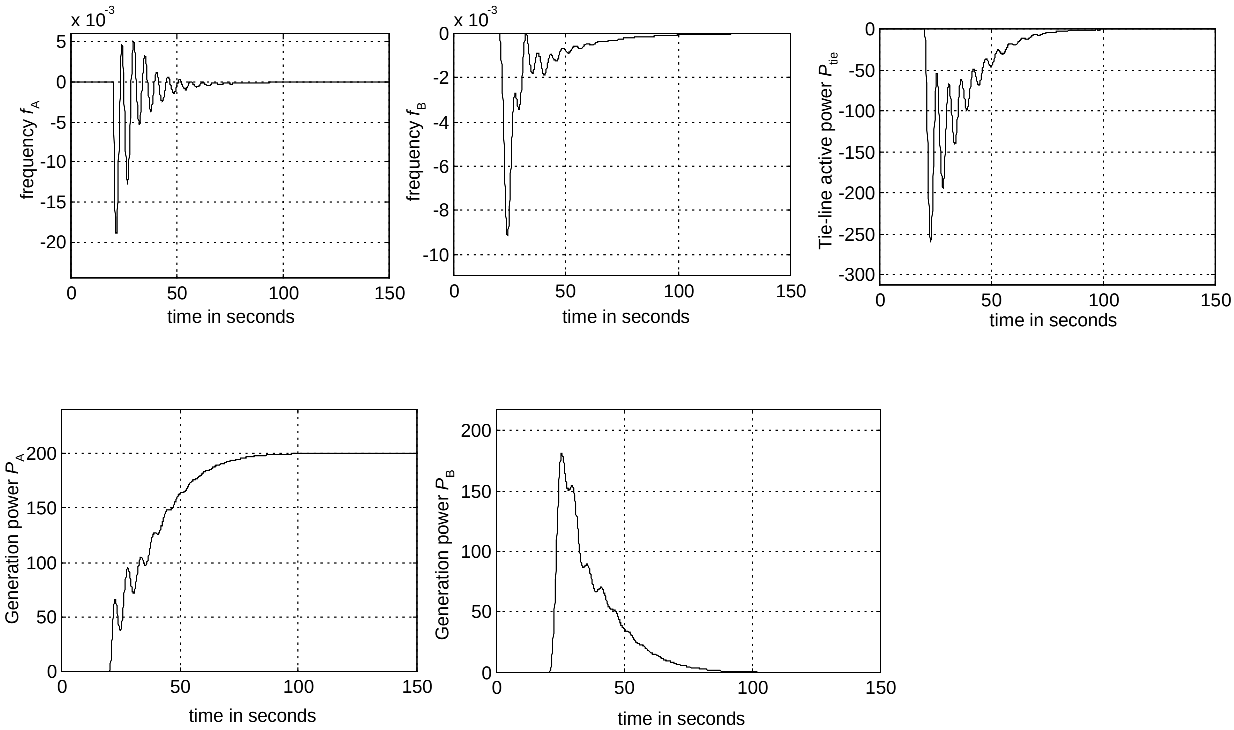

Fig. 6. Tests results of the modified AGC power system model

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IX. Biographies

Desire Dauphin Rasolomampionona (M'2005) was born in 1963 in Madagascar. He received his MSc (1988) PhD (1994) and Habilitation (2008) in Electrical Engineering from Warsaw University of Technology. He joined the Warsaw University of Technology Faculty of Elctrical Engineering in 1994 at the Power System Protection Division, Institute of Electric Power Engineering. Presently He is the Head of Power System Protection Division. His research interests include protection and control of power stem and computer networking