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Auctions Application for Electricity Markets Modeling

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***Abstract* — The paper gives an overview of the present situation in power industry in Russia, caused by electricity market introduction. Staff training, the pricing simulation methods and algorithms development on the market are of crucial importance at this stage. Developing new relations between participants of the electric power market makes is necessary to consider and analyze various pricing mechanisms.**

The model based on auctions usage as a pricing mechanism on the competitive electric power market is proposed. The model implies auction application under the conditions of the fuzzy information. The participants are expected to select the optimal strategy of their behavior. Each participant may choose a bidding strategy based on the information available and bids expected from other participants. The main goal is to maximize their own welfare function. The algorithm based on game theory is used.

***Keywords—auction, electric power market, optimal strategy*.**

I. INTRODACTION

Economical and structural reorganization ongoing today in Russia has affected power industry. The structure and interrelation concepts of the Russian UPS have been modified along with computing facilities and software updating, power generation, transmission and distribution technologies improvement as well as operation modes of controlling power systems and power system interconnections (IPS). At present, in the Russian power industry the transition from government approved tariffs to competition pricing takes place. Power is a commodity of a specific nature due to continuous production and consumption balance maintenance. It is essential to note that electricity market operation principles are similar to gas market mechanisms as the following similar technology stages exist: production, transmission and consumption. Therefore, the auction mechanism can be used in order to develop mathematical instruments for competitive power pricing.

For the proper electricity market operation in Russia (along with other engineering problems being solved) it is insufficient to recruit qualified operating personnel capable to effectively use market principles and select an optimal market behavior strategy. In future, during the transition to a completely competitive market, the personnel requirements are to increase. Mathematical models and power system simulators are designed for personnel training. They ensure a better understanding of the competitive market pricing and selecting an optimal market strategy for different purposes.

The Automated Electrical Systems Department of the Ural State technical University – UPI actively cooperates with “System Operator of Central Dispatch Unit of Unified Power System” – Unified Dispatch Office of Urals in working out training programs and techniques for the staff of new structures, such as “SO CDU of UPS” and territorial Regional Dispatching Units. One of these joint projects is working out electric power modeling teaching programs.

The developed market simulation software is based on the pool model and designed for studies in a form of role-play games [1]. The purpose of the software instruments used is to give players an insight into the electricity market auction mechanisms nature and to provide on-the-job training.

A wholesale electricity market model introduced in Russia is based on the world experience and the use of a pool model. The model application has been necessitated to adapt electric energy trade to the competition conditions and was developed further at electricity market introduction stages. According to the model market participants interact with each other through the spot market, where the demanded volumes of generation are determined in terms of conventional economic distribution by means of common welfare function maximization.

All the market participants’ bids and offers are placed at a unified finance utility (Administrator of Trade System in Russia); power system mode operation and scheduling are performed by Independent System Operator (JSC “SO – CDU UPS”). Bids are of a standard form and contain suggested energy prices and volumes which the market participants want to purchase or sell on the electricity market. If it is possible to assume that suggested prices fairly accurately reflect real generation costs, the pool market model permits to achieve economic efficiency by means of selecting the most inexpensive generation units. Hence, it is possible to assume that within the framework of the market model, centralized pricing is held along with the opportunity to optimize the operation mode.

A pool-based realization power system model can be regarded as auction [2, 3]. The distinguishing features of auctions are shown in Table 1. In anyone type of auctions the objective function is of the same form:

$$\max\left(\sum_{j=1}^{m} c_{d_j} P_d - \sum_{i=1}^{n} c_{g_i} P_{g_i}\right), \qquad (1)$$

where d denotes node consumption, g is node generation; n and m are the number of generation and load nodes accordingly. The use of auction of this type is referred to as a bilateral auction; bids are placed both by sellers and purchasers. Besides bilateral auctions, there are unilateral auctions: only purchasing or only selling bids are placed. The objective function of unilateral markets of both types contains separate components. In unilateral auctions the objective function is represented either by sellers’ bids:

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$$\max\left(-\sum_{i=1}^{n} c_{g_i} P_{g_i}\right) = \min\left(\sum_{i=1}^{n} c_{g_i} P_{g_i}\right), \qquad (2)$$

or purchasers' bids:

$$\max\left(\sum_{j=1}^{m} c_{d_j} P_{d_j}\right), \qquad (3)$$

price bids.

The auction includes four constituent parts: market participants, commodities, accepted pricing mechanism and participant strategies [4]. Market players participating in the auction (participants) must have an idea of the auction mechanism functioning principles. Selecting a behavior strategy, the participants are guided by the previous experience and prospective (forecasted) bid placing of other participants. The main purpose of each auction participant is to choose an optimal behavior strategy ensuring maximization of the personal objective function (personal welfare function).

TABLE 1
AUCTION DISTINGUISHING FEATURES

№	Distinguishing feature	Characteristic types	
1	By the number of the players	Unilateral: • Seller auction • Purchaser auction	Bilateral
2	By auction performance	Oral, open bid	Sealed bid
3	By price value for participants	Discriminatory auction – Yankee auction (different prices for participants - you get, what you bid)	Nondiscriminatory auction (price is uniform for all the participants)
4	By market pricing	Marginal (first-price sealed bid)	Marginal (second-price sealed bid)

Auction mechanism functioning can be studied and forecasted by two ways: theoretically and by an experimental approach. The former implies mathematical modeling which makes it possible to study different factors affecting pricing. The latter is performed on basis of training for power market simulation. The suggested auction realization model is designed for role-play games involving market participants.

II. AUCTIONS FOR ELECTRICITY MARKET

A mathematical model being developed is designed for studying market participants' pricing principles. The algorithmic approach to modeling can arbitrarily be divided into two procedures:

- The first one demonstrates the auction mechanism application for electricity market pricing;
- The second one reflects the influence of the strategy preferred by the participant upon placing the price bid and financial outcome.
- A more detailed examination of these procedures is given below.
- Linear expressions were used for better understanding by the market participants the auction mechanism functioning. The model allows the following assumptions to be made:
- Matrices of coefficients of current distribution (sensitivity coefficients) are used to connect power flows in tie-lines with the nodal capacities;
- All the nodal capacities are the positive values;
- Price bids are assumed to be single-stage ones. A linear programming approach is used to find an optimal solution;
- A model assumes a node pricing approach; however the loss cost component is not taken into account.

Auction simulating software suggests the following stages:

1. Defining the most inexpensive demanded generation for the node loads rated values. Computation requires an expanded matrix of current distribution coefficients $|\alpha|$ divided into two components $|\alpha_g|$, $|\alpha_d|$ - for generation and load nodes correspondingly. Each matrix is of $L\times(n+m)$ dimension, where L is the number of tie-lines, n, m are the number of generation and load nodes correspondingly.

2. Defining marginal cost of generation and real power constraints of tie-lines. Formulating a dual problem, defining real power constraints and constraint costs.

3. Defining the welfare function value for i-th participant under the fixed strategy and a welfare function deviation from the maximum value.

A mathematic auction model for the particular hour k of the trade day (for electricity market in Russia $k=\overline{1,\ 24}$), is given as follows. The objective function:

$$F^k = \min\left[\sum_{i=1}^{n} C_{g_i}^k P_{g_i}^k\right], \qquad (4)$$

where $C_{g_i}^k$ is the price bid of the i-th generation unit.

The equation of power balance corresponds to the constraint in the form of equality:

$$\sum_{i=1}^{n} P_{g_i}^k - \sum_{j=1}^{m} P_{d_j}^k = 0\ . \qquad (5)$$

Constraints in the form of inequalities are as follows:

$$\begin{cases} |\alpha_g|\cdot \mathrm{P}_g^k - |\alpha_d|\cdot \mathrm{P}_d^k \le \mathrm{P}_l^{\max}; \\ |\alpha_g|\cdot \mathrm{P}_g^k - |\alpha_d|\cdot \mathrm{P}_d^k \ge \mathrm{P}_l^{\min}; \\ \mathrm{P}_g^k \ge \mathrm{P}_g^{\min};\ \mathrm{P}_g^k \le \mathrm{P}_g^{\max}; \\ \mathrm{P}_g^k \ge 0;\ \mathrm{P}_d^k \ge 0;\ k=\overline{1,24} \end{cases}, \qquad (6)$$

where $\mathrm{P}_l^{\min}$, $\mathrm{P}_l^{\max}$ are the lower and upper tie-line l power flow constraints.

After selecting the most inexpensive generation unit and all the constraints taken into account, the following dual problem of linear programming is formulated:

$$D = \min\left[\sum_{h=1}^{n+m} K_h^k \cdot Y_h^k + K_{(n+m+1)} \cdot (\sum_{j=1}^{m} P_{d_j}^k)\right], \tag{7}$$
$$\left|\alpha^R\right|^T \cdot Y^k \geq -C^k,$$

where $\left|\alpha^R\right|$ is the matrix of current distribution coefficients, containing an additional row necessary to allow for a power balance equation.

Solving the above-mentioned dual problem enables the costs of real (achieved) constraints to be defined together with node prices. Thus, by the end of the second stage the volumes of generation demanded as well as node prices are determined. A linear programming method is used to solve the problem.

A separate procedure involved in the simulator includes dominating variables identification, participants' strategy elaboration as well as determination of the strategy influence upon financial outcome. Defining variables the modification of which doesn't depend upon the participant's decision is of crucial importance for selecting a strategy. E.g. a participant has to take into consideration price fluctuations due to other participants' behavior. the complexity of real power systems can make it difficult to represent the objective function and constraints in an analytic form therefore a simplified representations of real systems is often used thus making it possible to identify dominating variables and to select the appropriate level of abstraction in objects modeling.

An analytical auction outcome representation uses the game theory which permits to elaborate simulation model of participants' behavior strategy. Strategy A_i for *i*-th participant can be determined quantitatively by means of the welfare function U_i .

It is not determined unambiguously by the strategy itself and depends of a quantity upon a number of strategies accepted by other market participants $U_i = f(x_1^k, x_2^k, ..., x_n^k)$, where k is the number of the rated time intervals, n is the number of market participants. Using the welfare function $U(X^k)$, every result depending upon x_i^k case and every alternative are assigned one value of welfare $U(x*_i^k)$.

With a single-stage bid, a market participant anticipates the strategies $X*_{-i}^k$ accepted by other participants. For himself he chooses the $x*_i^k$ parameter maximization strategy.

$$U_i(x*_i^k, X*_{-i}^k) = \max_{\alpha \in A_i}\left(U_i(\alpha, X*_{-i}^k)\right) \tag{8}$$

Defining the function minimum/maximum, its derivative is equal to zero therefore condition (8) can be formulated as follows:

$$\frac{\partial U_i(x*_i^k, X*_{-i}^k)}{\partial x_i} = 0, \tag{9}$$

if all the participants except for the i-th accept the forecasted strategies, it is can be expressed as:

$$U_i(x_i^k, X*_{-i}^k) = U_i(x_i^k) \tag{10}$$

The unilateral sellers auction was accepted for simulating; the welfare function for the i-th participant $U_i(x_i^k)$ was stated by three different ways; settlements were fixed for every hour:

- The participants express the intention to maximize profit (with a definite price bid): $U(P_i^k) = C_0^k \cdot P_i^k - C(P_i^k)$, where C_0^k is an equilibrium marginal price determined by auction; P_i^k is power demanded from the i-th participant; $C(P_i^k) = a_i(P_i^k)^2 + b_i P_i^k + c_i$ – are hourly costs of the i-th participant [1] (first strategy);
- The participant intended to reach the maximum income (second strategy): $U(P_i^k) = \dfrac{C_0^k \cdot P_i^k - C(P_i^k)}{C(P_i^k)}$;
- The participant considered it as the main purpose to carry the maximum possible load $P_{i_{\max}}^k = \text{const}$, $k = \overline{1,24}$ with the profit of not less than a constant share of r, %, in this case the welfare function assumes the following form: $U(P_i^k) = C_0^k \cdot P_{i_{\max}}^k - C(P_{i_{\max}}^k) = r \cdot C(P_{i_{\max}}^k)$ (third strategy).

In each case the participant had to elaborate an optimal strategy basing on the auction outcome analysis, using different rules of decision-making.

At the last stage, the welfare functions values for every hour were presented as well as maximum values of these functions.

III. Results

Let us take an example of a test power system scheme consisting of five nodes.

Let a quadratic function $C_i(P_{g_i}) = a_i P_{g_i}^2 + b_i P_{g_i} + c_i$ be known for generation unit (GU) number 5. It determines for production (consumption) costs of a definite power value P_{g_i} Let the numerical expression be known as $C_i(P_{g_i}) = 0{,}1 \cdot P_{g_i}^2 + 5P_{g_i} + 1960$, rubles/h, where $P_{g_i}^{\min} = 100$ MW, $P_{g_i}^{\max} = 200$ MW, $i = 5$. For generation unit 5 let us determine marginal $MC_i = \dfrac{\partial C_i(P_{g_i})}{\partial P_{g_i}} = 0{,}2 \cdot P_{g_i} + 5$, rubles/MW·h and average $AC_i = \dfrac{C_i(P_{g_i})}{P_{g_i}}$, rubles/MW·h costs with the power being changed from minimum to maximum values. Additionally the real power constraints are stated for GU 1 and 2.

According to the above described algorithm price bids of GU 1 and 2 are constant, and GU 5 varies its price bids from the value corresponding to the equality of marginal and av-

[1] While cost function modeling, various functional dependences can be used; polynomial functions of the fourth to the second order or linear dependences are mostly used.

erage costs ($MC = AC = 33$), 50 rubles/MW·h up to the value of maximum price bid of GA 1 – 50 rubles/MW·h.

It is a common practice to regulate pricing on the basis of mean (average) costs; however, under competitive conditions the price for producers is to be based on marginal costs or the last kilowatt-hour electricity production cost. Let us in detail how the power prices are formed on electricity market and how GU 5 is loaded when different price bids are placed.

TABLE 2

TEST COMPUTATION RESULTS

Price bid of GU 5 (C_5), rubles/MW·h	33	45	48[2]	49	50
Market price, rubles/MW·h	48	48	48	49	50
Load P_{g5}, MW	200	200	200/ 180	180	0
1-st strategy					
$U_5(P_{g5})$, rubles/h	2640	2640	2640/ 2540	2720	0
$U_5^{max}(P_{g5})$, rubles/h	2640	2640	2640	2840	3040
2-nd strategy					
$U_5(P_{g5})$	0,38	0,38	0,38/ 0,42	0.45	0
$U_5^{max}(P_{g5})$	0,46	0,46	0,46	0,49	0,52

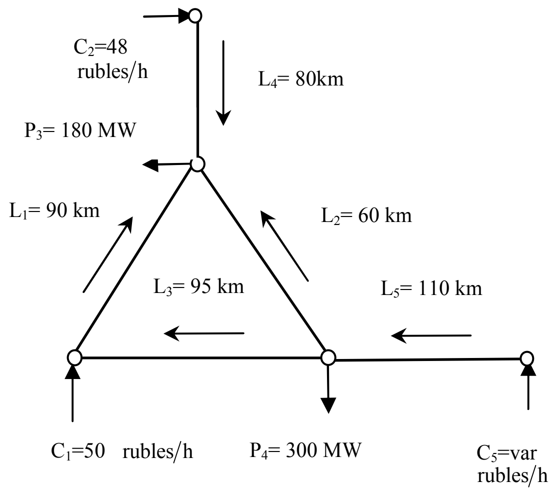

Fig. 1. EPS test scheme

The above-given results show that depending upon the price bid placed, the participant can obtain different profit, increasing or decreasing his income. The example described is transparent enough; however even with the system constraints taken into consideration, the price bid value will greatly influence the GU load and nodal price.

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[2] Under price bid of GU 5 equal to 48 rubles/MW·h the different generation allocation is possible depending of slack bus selection.

V. CONCLUSTION

- The article presents a training power system simulator involving two procedures:
- Auction mechanism used as a pricing method for electricity market;
- Optimal participants' behavior strategy selection with the welfare function used.

With an auction simulation, nodal loads schedules can be represented as regular and casual components. These auctions are arbitrarily performed for several days that allows for accumulating some statistic data for each participant. On the basis of this data and using certain rules of decision-making it is possible to analyze how effective each participant was placing bids.

During the training process the participants have no access to other participants' bid information but are completely equipped with the statistic data about their own trade results. Thus, mathematical model ensures a better understanding of competitive market pricing and helps select the optimal market strategy. The use of software instruments enables special training programs reflecting specific features of electricity market participants' behavior to be implemented.

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