

Probabilistic Load Flow Calculation Based on an Enhanced Convolution Technique

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Abstract—The increasing uncertainties grid operators have to face in their every-day work lead to the necessity for fast and accurate information about the probability of certain operational states of the operated network. Algorithms traditionally used in grid operation and planning are only able to evaluate discrete operational states of the particular power grid. A consideration and integration of probabilistic data can only be done by the analysis of selected discrete states followed by an interpolation. The algorithm presented in this paper falls into the category of Probabilistic Load Flow Calculation with a new approach for the modeling of the underlying network, as well as measures to reduce the inaccuracy introduced by linearization. Aspects like accuracy and calculation time in comparison to existing approaches are covered in detail.

Index Terms— linear map, power system simulation, probabilistic load flow.

I. NOMENCLATURE

x	real valued scalar
\bar{x}	complex valued scalar
X, \underline{x}	matrix or vector
X^\dagger	pseudoinverse

II. INTRODUCTION

THE increasing utilization of renewable energy sources, as well as the unbundling of formerly integrated utilities pose new challenges to grid operation. At the same time as power flow patterns get less predictable, the necessity for an operation of the grid closer to its limits leads to the need of more appropriate approaches for dealing with uncertainties in grid usage. The classical deterministic load flow calculation techniques are not able to cope with uncertainties and can only be used within tight limits.

As a reaction to this shortcoming the field of Probabilistic Load Flow (PLF) evolved. The main aim of PLF is to calculate the probability of operational grid states from the probability of a certain grid usage profile. In [8] an overview on the presently available approaches is given.

In general two main types of PLF algorithms can be distinguished. The first type makes use of classical load flow calculation for a selected set of grid usage profiles. The calculated grid state is associated with the overall probability

of the used grid usage profile. Due to the computational complexity Monte-Carlo approaches are used to find an approximate result for load flow probabilities in a reasonable amount of time.

The second type bases on an analysis of the underlying grid in order to find a model allowing for the usage of convolution techniques [1][10]. The main drawback is the inherent imprecision due to model simplification. Most of these methods use a linearization of the Load Flow equation at a certain expansion point. Hereby the inaccuracy increases for widespread Probabilistic Density Function (PDF) of the nodal power. This is compensated by a significant reduction of computation time, as well as the consideration of all possible network usage profiles and not only a selection like with Monte-Carlo based approaches. To use the advantages of both methods a combination of Monte-Carlo-Simulation (MCS) and linearization is possible [2]. Another method of PLF calculation uses the Gram-Charlier series [3]. In this approach the PDF are analysed using the advanced probabilistic method of cumulants. After calculating the cumulants for the given PDF their propagation through the network is analyzed in order to calculate the cumulants of line flow PDF. The actual PDF of line flows is then calculated from the values of its cumulants.

The solution of the PLF is very complex whereby the most methods made assumptions of a constant network configuration and of the independence of the input parameters. But through the need for the active power balance in grid operation a correlation between the nodal powers exists that is usually neglected. A consideration of these correlations by methods based on convolution techniques is difficult [5].

To this context the authors want to contribute a novel approach for the network model simplification, as well as the application of a modified convolution technique. The modification allows for a significant reduction of imprecision imposed by the linearization of the grid model. Assumptions of the independence of the nodal powers and of a constant network topology are made, but possibilities to consider correlation and topology changes will be depicted.

III. PRESENTED ALGORITHM

Starting point of every PLF is the Probabilistic Density Function of nodal power stated for all nodes of a given network. In our presented algorithm not only the probability of active power, but also the probability of reactive power injection/consumption is taken into account by two

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dimensional PDF. In a first step total independence between the nodal power variables is assumed.

Used network model

The presented algorithm is based on a linear complex valued map between nodal currents and line currents which is derived from the following two maps: inverted map between nodal voltages and nodal currents and the map between nodal voltages and line currents.

Equation (1) shows the first map the nodal admittance matrix.

$$\underline{Y}_n = \begin{pmatrix} \underline{Y}_{1,0} + \sum_{i=1}^n \underline{Y}_{1,i} & -\underline{Y}_{1,2} & \cdots & -\underline{Y}_{1,n} \\ -\underline{Y}_{2,1} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ -\underline{Y}_{n,1} & \cdots & \cdots & \underline{Y}_{n,0} + \sum_{i=1}^n \underline{Y}_{n,i} \end{pmatrix} \quad (1)$$

$\underline{Y}_{i,k}$: Admittance between node i and k

$\underline{Y}_{i,0}$: Admittance between node i and earth

n : Number of nodes

Neglecting line capacitance and shunt conductance this matrix is singular. But to every singular matrix it is possible to find a pseudoinverse [6]. One method to compute pseudoinverses is the singular value decomposition (SVD). Based on the result of the SVD of a matrix \underline{A} , stated in (2), the pseudoinverse of \underline{A} can be stated according to (3), where \underline{U}^* is the conjugate transpose of \underline{U} .

$$\underline{A} = \underline{U} \cdot \underline{\Sigma} \cdot \underline{V}^* \quad (2)$$

$$\underline{A}^\dagger = \underline{V} \cdot \underline{\Sigma}^\dagger \cdot \underline{U}^* \quad (3)$$

The matrix $\underline{\Sigma}$ in (2) is a rectangular matrix that contains on its main diagonal the singular values of \underline{A} in descending order, while in $\underline{\Sigma}^\dagger$ of eq. (3) their reciprocal value. Eq. (4) illustrates the structure of $\underline{\Sigma}^\dagger$.

$$\underline{\Sigma}^\dagger = \begin{pmatrix} \frac{1}{\sigma_1} & \cdots & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ \vdots & & \frac{1}{\sigma_r} & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \quad (4)$$

σ_i : i-th singular value of matrix \underline{A}

r : rank of the matrix \underline{A}

Applying this to the before stated singular nodal admittance matrix results in a matrix mapping a nodal current profile \underline{I}_n to one voltage profile \underline{V}' that causes this specific nodal current profile (5).

$$\underline{V}' = \underline{Y}_n^\dagger \cdot \underline{I}_n \quad (5)$$

The calculated vector \underline{V}' is only one of the voltage profiles that cause the nodal current profile \underline{I}_n . In case of neglecting line capacitance and shunt conductance the nodal currents only depend on the voltage differences between the nodes. Considering the line capacitance and shunt conductance the nodal admittance matrix is regular and the pseudoinverse pass into the normal inverse. Due to the sparse of this matrix the calculation is numerically superior.

As usually there is no value given for the current of the reference node we express the nodal current as the negative signed sum of all other nodal currents, which is true for the stated singular nodal admittance matrix.

$$\bar{I}_{n,1} = - \sum_{j=2}^n \bar{I}_{n,j} \quad (6)$$

In case of consider line capacitance and shunt conductance the nodal admittance matrix is regular and the effect of these elements must be considered by the calculation of the current of the reference node. (7). The estimate of the complex nodal voltage is presented in the next section.

$$\bar{I}_{n,1} = \sum_{j=2}^n \bar{V}_j \cdot \bar{Y}_{j,0} - \sum_{j=2}^n \bar{I}_{n,j} \quad (7)$$

The mapping between nodal voltages and line currents is also a linear map based on a singular matrix (8).

$$\underline{I}_l = \underline{Y}_l \cdot \underline{V} \quad (8)$$

The line admittance matrix can be stated like sketched in (9), with the positive signed serial admittance of the respective line in the column of its starting node and the negative signed admittance in the column of its terminal node.

$$\underline{Y}_l = \begin{pmatrix} \bar{Y}_{12} & -\bar{Y}_{12} & & \\ & \bar{Y}_{23} & -\bar{Y}_{23} & \\ & & \bar{Y}_{34} & -\bar{Y}_{34} \\ & & & \bar{Y}_{24} & -\bar{Y}_{24} \end{pmatrix} \quad (9)$$

The stated line admittance matrix \underline{Y}_l is always singular, as the line current profile does only depend on the voltage differences between the nodes of the network and not their absolute values. Combining (5) and (8) results in a direct mapping of nodal current profiles to the corresponding line current pattern, denoted \underline{Z}_{nl} in (10).

$$\underline{I}_l = \underline{Y}_l \cdot \underline{Y}_n^\dagger \cdot \underline{I}_n \quad (10)$$

\underline{Z}_{nl}

With the nodal currents given for all nodes, except the reference node, it is possible to directly calculate the according line current pattern.

$$\begin{pmatrix} \bar{I}_{l,1} \\ \vdots \\ \bar{I}_{l,m} \end{pmatrix} = \underline{Z}_{nl} \cdot \begin{pmatrix} -\sum_{j=2}^n \bar{I}_{n,j} \\ \bar{I}_{n,2} \\ \vdots \\ \bar{I}_{n,n} \end{pmatrix} \quad (11)$$

Integration of convolution techniques

Usually the transmission capacity of a given network is limited by the thermal limits of its lines. Therefore PDF of the line currents are among the most interesting information on the network state.

Usually the input quantities of the PLF are PDF of the nodal active/reactive power. To calculate the nodal current information about the absolute value and complex argument of the nodal voltages is necessary. The absolute value can be approximated as being the nominal voltage. The deviations of the nominal voltage are not more than $\pm 10\%$.

The complex arguments of the nodal voltages heavily depended of the active power injection/consumption so that a DC Load Flow approach is used to estimate the complex voltage argument. Derived from the basic DC Load Flow model stated in (12) it is possible to determine the approximate voltage angle profile with (13).

$$\underline{P} = \underline{D} \cdot \underline{\delta} \quad (12)$$

$$\underline{\delta}' = \underline{D}^\dagger \cdot \underline{P} \quad (13)$$

Adjusting the voltage angle profile according to the reference nodes voltage angle being equal to zero, it is possible to calculate the approximate complex nodal current corresponding to the nodal complex power (14).

$$\bar{I}_{n,i} = \frac{\bar{S}_i^*}{V_n \cdot e^{-j \cdot \delta_i}} \quad (14)$$

Applying (14) to the PDF of nodal complex power equals to a rotation of this two-dimensional PDF. In this way the approximate PDF of nodal current, denoted $PDF(\bar{I}_i)$ in the following, can be calculated. Hereby $\underline{\delta}'$ is calculated using (13) with the vector of the expected values for the active power of all nodes.

Recalling that with (10) the line currents can be expressed as a weighted sum of the nodal currents, they can be stated as (15).

$$\bar{I}_{l,j} = \sum_{i=1}^n (\bar{z}_{nl,j,i} \cdot \bar{I}_{n,i}) \quad (15)$$

When considering PDF rather than concrete values for the nodal currents, this leads to the convolution of weighted PDF

as stated in (16).

$$PDF(\bar{I}_{l,j}) = PDF\left(\frac{\bar{I}_{n,1}}{\bar{z}_{nl,j,1}}\right) * \dots * PDF\left(\frac{\bar{I}_{n,n}}{\bar{z}_{nl,j,n}}\right) \quad (16)$$

As the weighting factors in (16) all have an absolute value between zero and one, they can be interpreted as describing shrinking and rotating of the PDF prior to the actual convolution.

In a last step of the calculation, the linear PDF of absolute line currents have to be calculated from the two-dimensional PDF of line currents. This equals to the circular integral stated in (17).

$$PDF(|\bar{I}_i|) = \int_0^{2\pi} \frac{PDF(|\bar{I}_i| \cdot e^{j \cdot \delta})}{2\pi \cdot |\bar{I}_i|} \cdot d\delta \quad (17)$$

Summary and discussion

The approach presented in this chapter allows for the calculation of the approximate line current absolute value PDF from the nodal complex power PDF. In contrast to other convolution based approaches, its underlying network model is not developed at a certain expansion point and thus is valid also for wide spread input PDF. The imprecision introduced by the linearization are eased by the employment of a DC Load Flow to countervail the deviations originating in the neglect of the complex arguments of the nodal voltages.

IV. COMPARISON TO OTHER METHODS

In order to prove the legitimacy of our approach its accuracy is evaluated against two other algorithms. The reference algorithm combines a Newton-Raphson based load flow calculation with a Monte-Carlo approach, denoted MCS in the following. The second one uses a first order Taylor series linearization of the Load Flow equations with the expansion point being the mean value of the nodal power [10], denoted LLF in the following. The evaluation includes an analysis of the accuracy and the computation time. The calculations are carried out on the 20 lines test network depicted in Fig. 1.

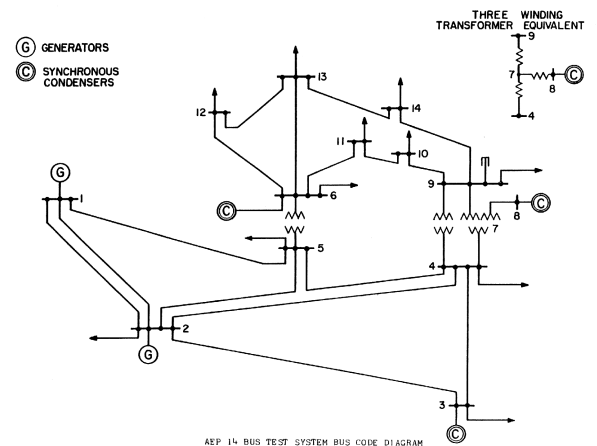


Fig. 1: Test network with 14 nodes and 20 lines [11]

A. Precision comparison

For the comparison of the achieved precision all three approaches are applied to different Gaussian distribution PDF for the nodal power. The Gaussian distributions have standard deviations between 0,05 pu and 0,5 pu, while their expected value is for all different deviations the same.

Especially for the approach using a linearization of the power flow equation at a certain expansion point it is expected, that inaccuracy increases with increasingly wide spread PDF.

The comparison is made using the 14 node, 20 line network depicted in Fig. 1. Fig. 2 and Fig. 3 illustrate exemplary results for the three approaches for narrow input PDF, while Fig. 4 and Fig. 5 show the results for widespread PDF. The calculation is performed for the line 2-5 (Fig. 2) and 9-11 (Fig. 3).

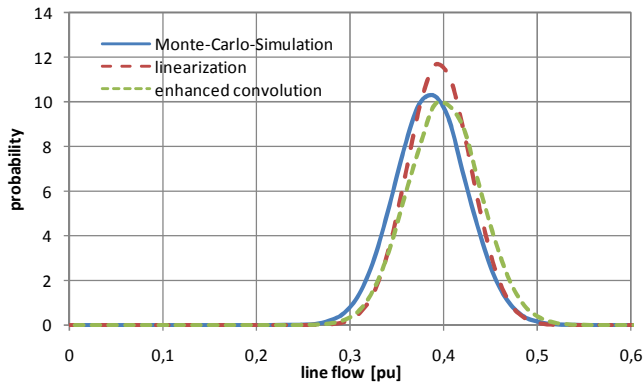


Fig. 2: Precision comparison for narrow PDF, line 2-5

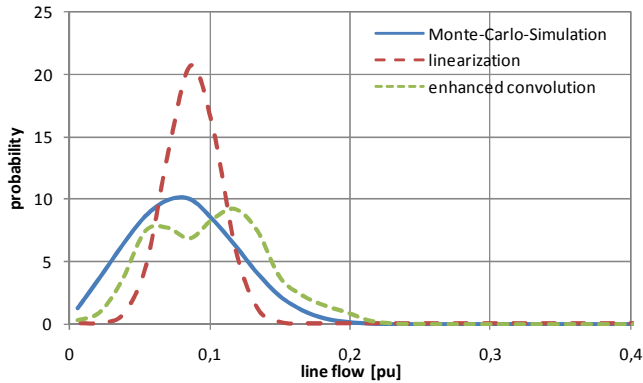


Fig. 3: Precision comparison with narrow PDF, line 9-10

In comparison to the reference algorithm the MCS the LLF shows better results by narrow PDF than for widespread PDF. This is due to the limited extend of the PDF around the expected value that is used as the expansion point of LLF. By line 9-10 the extension of the PDF is underestimated and the deviation to the reference algorithm is larger than by line 2-5. The result of the presented approach is slightly shifted to an overestimation of the line currents. This is mostly due to the neglect of nodal voltage increase caused by a power injection, what in reality leads to smaller line current. Nevertheless the deviation between the MCS and the presented approach is fairly small.

For PDF with a higher extend the advantage of the

presented algorithm over the LLF becomes clear (Fig. 4, Fig. 5). It has to be mentioned that, although it is used as the reference algorithm, the MCS also contains deviations from the real result, due to its stochastic nature. But it becomes clear that the correlation between the MCS result and the result generated with the presented approach is much higher than between MCS and LLF. Concrete numbers for the correlations are given in the Appendix.

One of the main reasons for the observed high deviation of the LLF result is the increasing distance to the expansion point in case of wide PDF. In contrast to the LLF, the presented algorithm uses a map between nodal and line currents that isn't developed in an expansion point what leads to smaller deviation in the case of wide PDF.

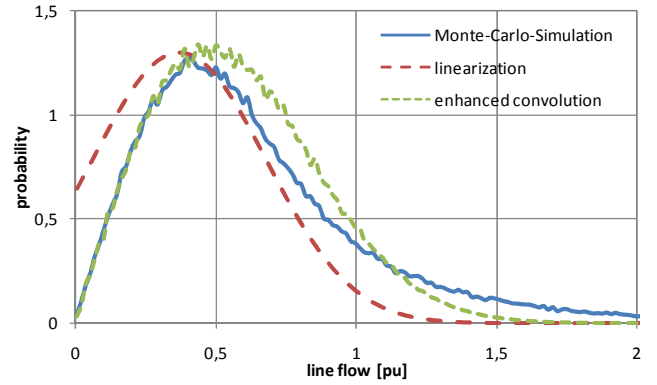


Fig. 4: Precision comparison for widespread PDF, line 2-5

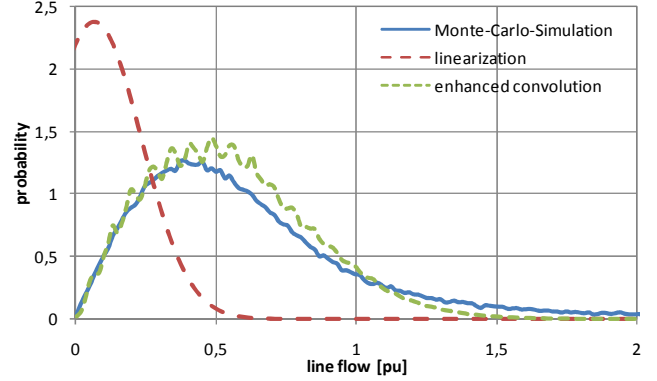


Fig. 5: Precision comparison for widespread PDF, line 9-10

In some cases the LLF suffers an additional source of imprecision due the underlying Jacobian matrix of the load flow equation is close to being singular. This introduces an increased risk for severe deviations. The line connecting node 7 and 8 (see Fig. 6) is one example for the effect which a nearly singular Jacobian has in LLF.

B. Computation time

Another important aspect of PLF is computation time. An applicable algorithm has to be able to provide results in a sensible amount of time. Although the used two-dimensional convolution is time consuming and requires a large amount of storage at an adequate level of precision. The MCS and LLF require less storage because the absolute values of line

currents are calculated and stored without the intermediate result for the complex valued line current PDF.

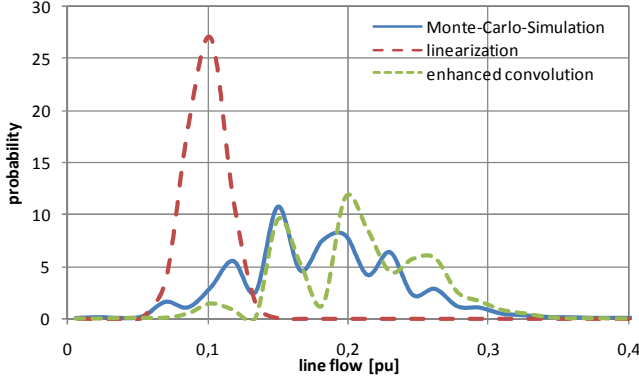


Fig. 6: Precision comparison for narrow PDF, line 7-8

Due to its stochastic nature the MCS has to be performed with a very high amount of experiments. The effort, necessary for a certain level of precision, scales exponentially over the number of nodes. This makes it unpractical for the application to large networks.

Table I depicts the execution time of all three mentioned PLF-algorithms for the test network displayed in Fig. 1 that was necessary to calculate the line current PDF shown in the respective figures. The MCS was performed with 10.000 experiments.

TABLE I
COMPUTATION TIME IN SECONDS

Method	narrowed PDF Fig. 2, Fig. 3	widespread PDF Fig. 4, Fig. 5
Monte-Carlo-Simulation	10.211	11.184
LF-Linearization	94	83
Enhanced Convolution	8.217	4.676

The computation time already shows a big advantage over the reference algorithm for small systems.

For larger systems the computation time of the presented algorithm increases, because the requirement for storage and the number of necessary convolution is augmented. But this effort scales with app. square of the number of nodes, while the MCS scales app. exponentially over the number of nodes.

Another advantage of the presented algorithm is the possibility to limit the calculation to certain lines of the network and calculate their line current PDF independently. This is not possible with the MCS, where for every experiment the load flow equations have to be solved for the whole network.

C. Summery and Discussion

The MCS was selected as the reference method because it uses the exact solution of the non-linear load flow equations. The main drawback of the MCS is its need for a high number of experiments which is not precisely determinable in advance. It is not possible to determine the number of experiments necessary to achieve a certain level of accuracy.

In contrast the two algorithms basing on a convolutionary

approach include all possible combinations of power injections and consumptions and only lead to deviation caused by the respective model simplifications.

The presented algorithms achieve a fairly good correlation with the results of the reference algorithm. But especially for large networks the necessary computation time is significantly smaller than the reference methods and there is the possibility to calculate the line current PDF for selected lines independently from all other lines of the network.

V. POSSIBLE EXTENSION OF THE NEW METHOD

Integration of DC load flow into convolution

Possible future extensions to the presented algorithm are e.g. the integration of the DC load flow directly into the convolution. The changeover of the PDF of nodal powers to the PDF of nodal currents requires the acknowledgment of the voltage angles which are mainly depended of the nodal active powers. This dependence can be considered in an enhancement of the convolution.

Integration of a V-Q-model

Based on the imaginary part of the network impedance matrix a V-Q-model can be integrated. Estimating the change of voltage magnitudes due to the expected value of the nodes reactive power balances results in an individual expected voltage magnitude for each node. This can be used to improve the precision of (14).

$$\bar{I}_{n,i} = \frac{\bar{S}_i^*}{U_i \cdot e^{-j \cdot \delta_i}} \quad (18)$$

Modeling of frequency control

The basic assumption that all nodal power balances are independent is not true under the presence of a frequency control [8] [5]. Future improvements of the presented algorithm may include the introduction of conditional probability based frequency control model.

VI. CONCLUSION

In this paper a probabilistic load flow method based on linear maps and DC-improved convolution techniques is proposed. In contrast to existing linear map based approaches the interdependent influence of active power flows on voltage angles is taken into account while convoluting the input PDF. By this the inaccuracy introduced by the linearization is reduced significantly for widespread input PDF.

Basis for the algorithm is a complex valued network model derived from the networks structure and parameters.

Results show the legitimacy of our approach and that this method has clear advantages over existing algorithms when it comes to widely extended PDF. The presented method requires more computation effort in comparison to other convolution based method, but has clear advantages in terms of precision.

Possible fields of application are transmission system

expansion planning where uncertainties about the future development of generation capacities and load are inherent.

Future developments will focus on integration of reactive power and voltage magnitude relationship into the convolution.

VII. APPENDIX

TABLE II
CORRELATION COEFFICIENTS

Line		narrow PDF $\sigma = 0,05$		$\sigma = 0,25$		widespread PDF $\sigma = 0,5$	
starting node	terminal node	Linearization	Enhanced convolution	Linearization	Enhanced convolution	Linearization	Enhanced convolution
1	2	1,00	0,93	0,99	0,99	0,92	0,96
1	5	1,00	0,94	0,99	0,99	0,93	0,98
2	3	0,97	0,94	0,99	0,99	0,97	0,99
2	4	1,00	0,95	0,99	0,99	0,94	0,99
2	5	0,98	0,97	0,99	0,99	0,93	0,99
3	4	0,09	0,99	0,89	1,00	0,83	0,99
4	5	0,59	0,99	0,92	0,99	0,95	1,00
4	7	0,97	0,94	0,89	0,99	0,77	0,98
4	9	0,99	0,90	0,94	0,99	0,83	0,99
5	6	0,98	0,96	0,94	0,99	0,82	0,98
6	11	0,35	0,92	0,38	1,00	0,39	0,99
6	12	0,20	0,93	0,32	0,99	0,27	0,99
6	13	0,05	0,91	0,44	0,99	0,40	0,99
7	8	0,23	0,85	0,51	0,96	0,29	0,97
7	9	0,59	0,97	0,75	1,00	0,71	0,99
9	10	0,86	0,94	0,54	1,00	0,36	0,99
9	14	0,78	0,79	0,57	0,99	0,37	0,99
10	11	0,33	0,99	0,57	1,00	0,36	0,99
12	13	0,55	0,92	0,34	0,99	0,21	0,99
13	14	0,61	0,84	0,58	1,00	0,36	0,99

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IX. BIOGRAPHIES



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