Paper accepted for presentation at 2009 IEEE Bucharest Power Tech Conference, June 28th - July 2nd, Bucharest, Romania

Steady-State Calculation of Electrical Power System by the Newton's Method in Optimization.

Andrew V. Pazderin and Sergey V. Yuferev

Abstract--**In this paper the application of the Newton's method of optimization for power flow calculation is considered. Convergence conditions of the suggested method using an example of a three-machine system are investigated. It is shown, that the method allows to calculation non-existent operating points and automatically remove them onto the feasibility boundary of the power flow solution.**

Index Terms—**Newton method, Hessian matrices, convergence of numerical methods, steady state stability.**

I. Introduction

The solution of the power flow problem is the basis on which others problems of managing the operation and development of electrical power systems (EPS) are solved. The complexity of the problem of power flow calculation is attributed to nonlinearity of steady-state equations system and its high dimensionality, which involves iterative methods. The basic problem of the power flow calculation is that of the solution feasibility and iterative process convergence [1]. Involving infinite buses provides the solution of the power system steady-state stability problem [2]. Misconvergence normally implies that the state is unstable and cannot be realized in actual practice. There are situations when various methods of power flow calculation and different software packages yield different results concerning existence of the power flow solution, given the same initial data. The continuing problem is the development of methods and algorithms of power flow calculation, providing reliable convergence to the solution. The problem relevance is also attributed to the necessity of ensuring a guaranteed solution as one of the requirements for the system with initial data being outside the scope of marginal solution. Engineers may have to apply a lot of efforts to understand the iterative process divergence. Experience and qualification of engineers who are responsible for power flow calculation, do count in operating and designing EPS. Even for small EPS, there is a wide range of solutions to pull the operating point into the existence and feasibility domains. The problem is normally solved on the basis of preliminary calculations within the course which marginal characteristic of power flow (limits of transferred capacity of sections, limit values of magnitude nodal voltages) are defined by the data of static aperiodic stability.

A. V. Pazderin is with Department of Automated Electrical Systems, Ural State Technical University – UPI, Ekaterinburg, 620002 Russia (e-mail: pav@daes.ustu.ru).

S. V. Yuferev is with Department of Automated Electrical Systems, Ural State Technical University – UPI, Ekaterinburg, 620002 Russia (e-mail: Yuferev@uralweb.ru).

978-1-4244-2235-7/09/$25.00

II. Formulation of the Problem

A. Steady-state equations

The system of steady-state equations in a general can be expressed as follows:

$$\overline{\mathbf{W}} - \mathbf{W}(\mathbf{X},\mathbf{Y}) = 0 \qquad (1)$$

Where $\overline{\mathbf{W}}$ is the vector of parameters, given for power flow calculation. In power flow calculation real and reactive powers are set in each bus except for the slack bus. In generation buses the modulus of voltage can be fixed. $\mathbf{W}(\mathbf{X},\mathbf{Y})$ is the nonlinear vector function. Variables $\mathbf{Y}$ define the quasi-constant parameters connected with an equivalent circuit of an electrical network. $\mathbf{X}$ is required state vector, it defines operating point of EPS. The dimension of state vector coincides with the number of nonlinear equations of the system (1). There are various known forms of notation of the steady-state equations. Normally, they are nodal-voltage equations in the form of power balance or in the form of current balance. Complex quantities in these equations can be presented in polar or rectangular coordinates, which lead to a sufficiently large variety of forms of the steady-state equations notation. There are variable methods of solution of a nonlinear system of steady-state equations. They are united by the incremental vector of independent variables $\Delta\mathbf{X}$ being searched and the condition of convergence being assessed on the each iteration. For the last decades, a huge number of scientific researches concerning methods of power flow calculation have been performed. According to [1], all methods of calculation can be divided into three groups.

The methods of zero order are based on progressive approximation to a solution on separate nodes of the circuit. Among these methods, the Gauss-Seidel method is the most commonly used. It was applied in programs of power flow calculation on computers, when memory capacity restriction was of critical importance.

The first order methods allow searching out approximation to a solution of the system of steady-state equations for all buses of design model in one cycle. Multiple solving of a simplified system of equations is required for the accounting nonlinearity of the steady-state equations. Now, Newton's method and its modifications are widely used for power flow calculation.

The second order methods allow solving quadric system of equations in one iteration.

B. The method suggested

Another way of solving the problem of power flow calculation is related to defining a zero minimum of objective function of squares sum of discrepancies of steady-state equations:

$$F = \left[\overline{\mathbf{W}} - \mathbf{W}(\mathbf{X},\mathbf{Y})\right]^{\mathbf{T}} \left[\overline{\mathbf{W}} - \mathbf{W}(\mathbf{X},\mathbf{Y})\right] \qquad (2)$$

Such formulation of the problem is close to that of the power flow calculation on the basis of telemetry, that is to the problem of state estimation of EPS [3]. In the problem of state estimation the objective function includes not only squares of discrepancies of nodal equations, but also squares of discrepancies of equations for power flows in branches, for currents and for modulus of nodal voltages. The number of the square discrepancies can be more than state vector length $\mathbf{X}$. The solution of state estimation problem is not necessarily connected with zero value of the objective function being minimized. The number of square discrepancies is equal to number of solution variables in the typical formulation of the power flow calculation problem. The effective solution of system (1) reduces (2) to zero. In the case when initial data are incompatible, in other words there is no solution (1) with zero power mismatches, the use of the objective function (2) implies receiving the approximate solution (1) with minimum difference between calculated $\mathbf{W}(\mathbf{X},\mathbf{Y})$ and given $\overline{\mathbf{W}}$ nodal capacities. Such solution allows to estimate the number of set vector of initial data overstepping borders of the solution existence domain. Nonzero power mismatches can be considered as operational actions for pulling the operating point back onto the feasibility boundary.

The function minimum (2) is reached in the point where derivatives on all required variables are equal to zero:

$$\mathbf{dF}/\mathbf{dX} = -2\,\mathbf{dW}/\mathbf{dX}\left[\overline{\mathbf{W}} - \mathbf{W}(\mathbf{X},\mathbf{Y})\right]^{\mathbf{T}} \qquad (3)$$

It is necessary to solve a nonlinear set of equations (3) to find the solution for the problem. Calculating the power flow, which is made by the system of the linear equations with a Hessian matrix at each iteration, is referred to as the Newton's method of optimization [4]:

$$\mathbf{G} \cdot \Delta\mathbf{X} = -\mathbf{dF}/\mathbf{dX} \qquad (4)$$

The Hessian matrix contains two items:

$$\mathbf{G} = (\mathbf{dW}/\mathbf{dX})^{\mathbf{T}} (\mathbf{dW}/\mathbf{dX}) - \mathbf{d}^2\mathbf{W}/\mathbf{dX}^2 \left[\overline{\mathbf{W}} - \mathbf{W}(\mathbf{X},\mathbf{Y})\right] \qquad (5)$$

If the second item is neglected and the matrix of the first derivatives (Jacobian matrix) is designated as $\mathbf{J} = \mathbf{dW}/\mathbf{dX}$, then the system of the linear equations (4) can be expressed as follows:

$$\left[\mathbf{J}^{\mathbf{T}} \cdot \mathbf{J}\right] \cdot \Delta\mathbf{X} = \mathbf{J}^{\mathbf{T}} \left[\overline{\mathbf{W}} - \mathbf{W}(\mathbf{X},\mathbf{Y})\right] \qquad (6)$$

Such simplification has found universal application for the solution of the state estimation problem. In [3] it is noticed, that such simplification in no way worsens the convergence, but even improves it. As such, the given simplification is still more valid, the nearer is the system of the equations (1) to the linear system. In the problem of the power flow calculation based on the Newton's method of optimization, this simplification is equivalent of Gauss transformation for a well-known expression of the power flow calculation by the Newton-Raphson method:

$$\mathbf{J} \cdot \Delta\mathbf{X} = \overline{\mathbf{W}} - \mathbf{W}(\mathbf{X},\mathbf{Y}) \qquad (7)$$

For the typical problem of power flow calculation, that is according to nodal capacities when Jacobian matrix is square, the use of (6) is not approved, as the solution of system of the linear equations (6) and system of the linear equations (7) will give identical result. The volume of calculations for the solution (6) is greater in comparison with (7), as the amount of nonzero elements of matrix $[\mathbf{J}^{\mathbf{T}} \cdot \mathbf{J}]$ is greater in comparison with Jacobian $\mathbf{J}$ matrix. Convergence of the iterative process based on (6) can be appreciably worse than that based on the use of Newton-Raphson method (7) because the power of conditionality of system of the linear equations (6) in comparison with the power of conditionality of system of the linear equations (7) is worse:

$$\mathrm{cond}\left(\mathbf{J}^{\mathbf{T}} \cdot \mathbf{J}\right) = (\mathrm{cond}\mathbf{J})^2 \qquad (8)$$

III. Investigations on the Test Scheme

According to the aforesaid, it is worthy carrying out the power flow calculation based on the Newton's method of optimization, basing on a full Hessian matrix. Convergence of the Newton's method of optimization with a full Hessian matrix has been investigated. Calculations were made based on program MathCAD for a network comprising three buses the parameters of which are presented in Figure 1. Dependant variables were angles of vectors of bus voltage 1 and 2 $\mathbf{X} = \{\delta_1, \delta_2\}$, independent variables were capacities in nodes 1 and 2, and absolute values of voltages of nodes 1, 2 and 3 were fixed.

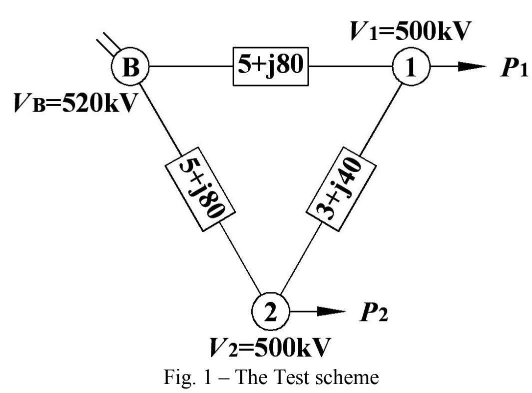

Fig. 1 – The Test scheme

In the Figure 2, the border of existence domain for a solution of the steady-state is presented in angular coordinates

$\delta_1 - \delta_2$. This border conforms to positive value of the Jacobian determinant:

$$\det(\mathbf{J}) > 0 \qquad (9)$$

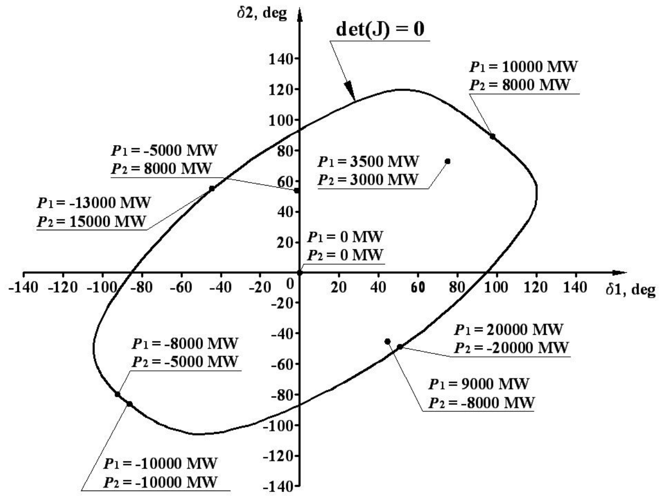

Fig. 2 – Domain of Existence for a Solution

As a result of the power flow calculation based on the Newton method of optimization, the angle values have been received, received, these corresponding to the given capacities in Fig.2 (generation is a positive value and loading is a negative value).

For the operating points which are inside the existence domain, the objective function (2) has been reduced to zero. For the operating points which are on the border of the existence domain, objective function has not been reduced to zero and the calculated values of capacities differed from the given capacities.

In Fig.3, the borders of the existence domain and convergence domain for the Newton's method of optimization are presented in coordinates of capacities $P_1 - P_2$. Operating points occurring on the border of the existence domain (9) have been set by the capacities which were outside of the existence domain. As a result of power flow calculation by minimization (2) based on the Newton's method of optimization, the iterative process converges to the nearest limiting operating point. It is due to the fact that surfaces of the equal level of objective function (2) in coordinates of nodal capacities are proper circles having the centre on the point defined by given values of nodal capacities $\overline{\mathbf{W}}$. The graphic interpretation of surfaces of the equal level of objective function for operating point state with 13000 MW loading bus 1 and 15000 MW generating bus 2 is presented in Fig.3.

Hessian matrix is remarkable in its being not singular on border of existence domain. The determinant of a Hessian matrix (5) is a positive around zero and a negative value of the determinant of Jacobian. This fact allows the power flow to be calculated, even for the operating points, which are outside of existence domain. The iterative process based on system of the linear equations (4) solution has converged to the limiting operating point for 3-5 iteration. Naturally, the iterative process based on Newton-Raphson method is divergent for such unsolvable operating points.

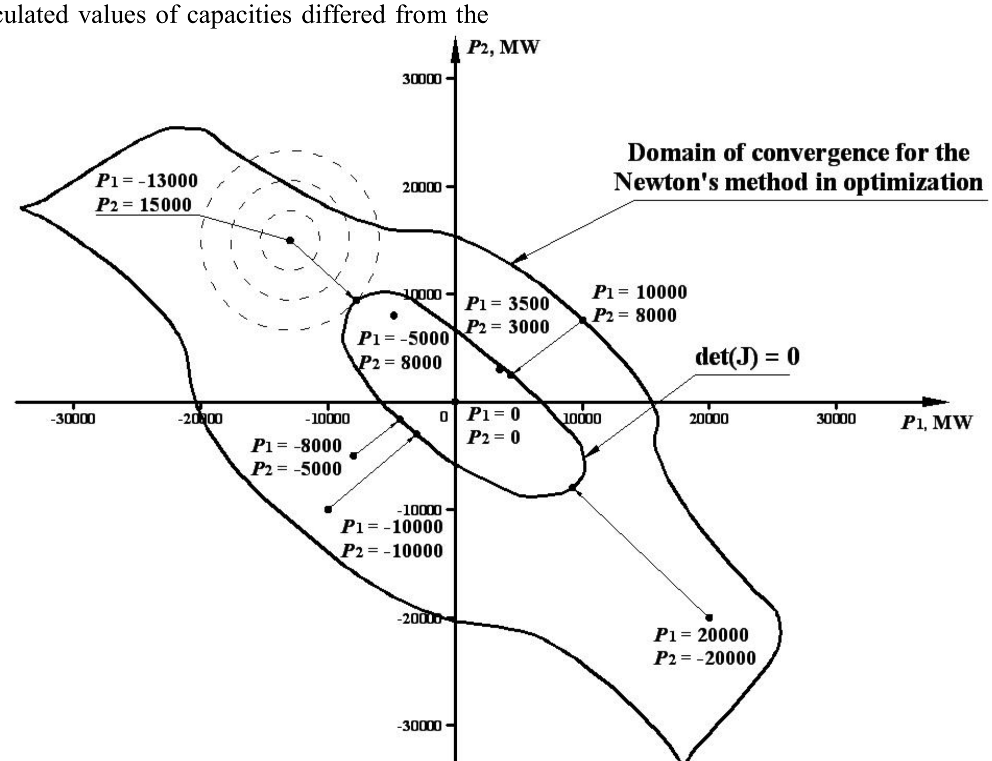

Fig.3 - Borders of domain of existence and domain of convergence for the Newton's method of optimization

The convergence domain of the method under consideration has been investigated. What is meant is that not all unsolvable operating points will be pulled onto the border of existence domain. A certain threshold having been exceeded the iterative process has begun to converge to the imaginary solution with the angles exceeding $360°$.

The border of convergence for the Newton's method of optimization has been defined by the method of successive weightings. In Figure 3 the solid curve presents «Border of domain of convergence». In the domain limited by the given curve and the curve $\det(\mathbf{J})=0$, the iterative process converged to the latter. It is necessary to note that the border of the convergence domain tends to infinity if to use the steady-state equations in rectangular coordinates form in other words, $\mathbf{X}=\left\{\mathbf{U}_1',\mathbf{U}_1'',\mathbf{U}_2',\mathbf{U}_2''\right\}$. The iterative process converged steadily to a operating point with $\det(\mathbf{J})=0$ for operating point which exceeded the bounds of the existence domain at distances tens and hundreds times exceeding critical values of nodal capacities.

During the experiments the convergence domain to the solution for the Newton's method of optimization was investigated. It was shown, that this domain for the Newton's method of optimization was narrower, than for Newton-Raphson method that proved to be true [3]. Newton's method of optimization is known to converge steadily to the solution in case:

$$\det(\mathbf{G})>0 \tag{10}$$

The lines of zero level for the determinant of Hessian matrix were plotted in angular coordinates. These lines limit the domain of initial approximation, from which the iterative process based on the Newton's method of optimization will converge to the correct solution. Domains of change of sign for determinants of Jacobian and Hessian matrixes in angular coordinates for different nodal capacities are presented in Figure4.

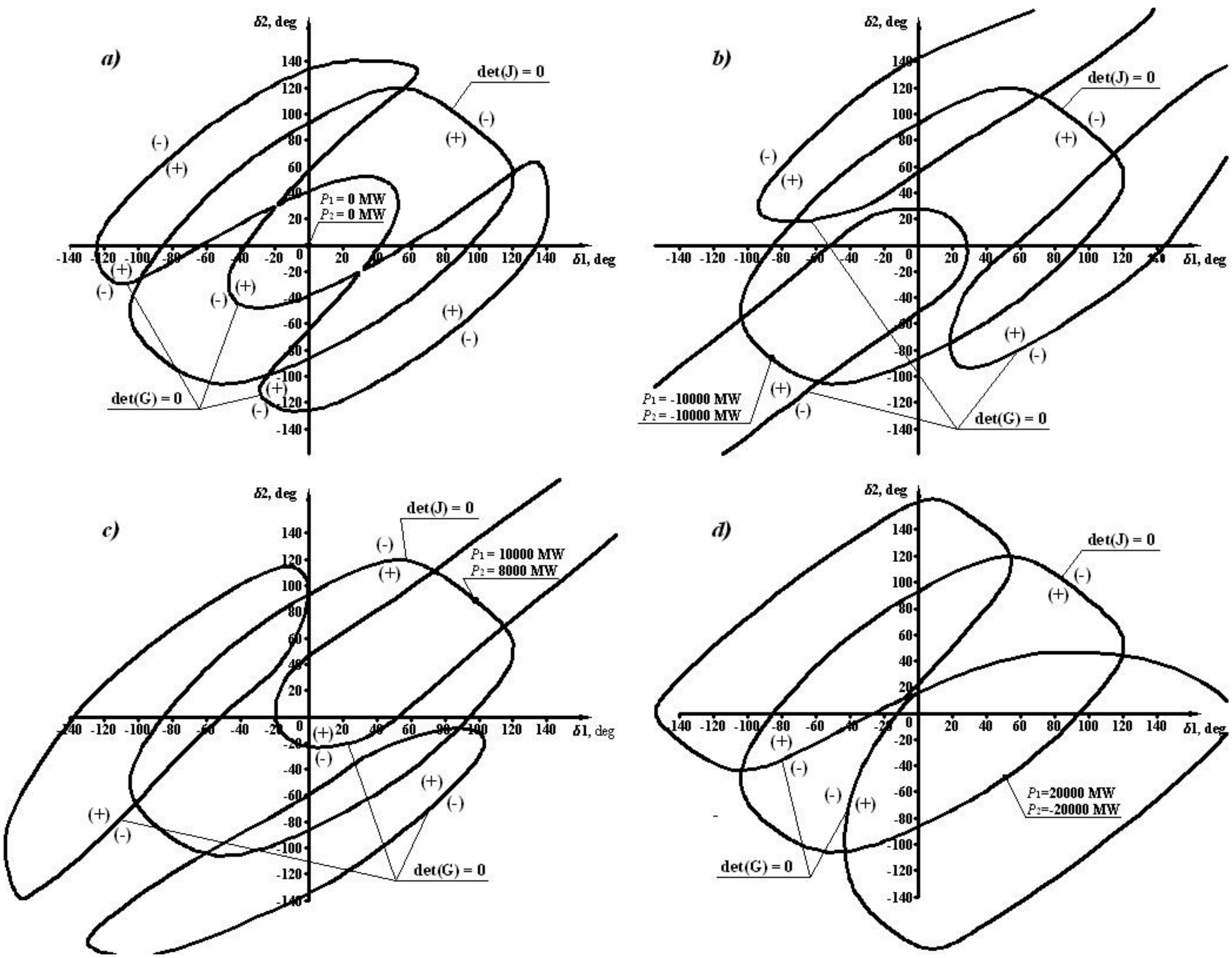

Figure 4 - Domains of Change of Sign for Determinants of Jacobian and Hessian Matrixes

As it may be seen from Fig. 4a domain $\det(\mathbf{G})>0$ considerably differs from domain $\det(\mathbf{J})\geq 0$. Calculations have confirmed that area $\det(\mathbf{G}) > 0$ is the area from which the iterative process based on the Newton's method of optimization converges to the solution. The power flow calculation converges to the imaginary solution if it begins from approximation which was outside the domain of convergence. So, for the power flow, the calculation for zero capacities from initial approximation $\delta_1 = 80°$ and $\delta_2 = 80°$ converges to the point with angles $\delta_1 = 93.6°$ and $\delta_2 = 93.6°$ ($P_1 = 3438\ \text{MW}$ and $P_2 = 3438\ \text{MW}$, accordingly) which is on border of the existence domain (9). Convergence of iterative minimization process (2) to point with zero value of Jacobian has been marked in [2] and it is connected with unsuccessful initial approximation which does not belong to the convergence domain. The computing experiments show that it is appropriate to use zero approximation which always belongs to domain of convergence to the solution and for which the determinant of Hessian matrix is positive.

It is necessary to note that elements of Hessian matrix depend not only on the vector of decision variables $\mathbf{X} = \{\delta_1, \delta_2\}$ but also on the vector of given nodal capacities $\overline{\mathbf{W}}$. In this connection, configurations of the convergence domain to the solution $\det(\mathbf{G}) > 0$ for different initial data are various. Configurations of these domains for three nonoperable states are presented in Figures 4a-d. The given figures show, that the decision condition is in the center of the convergence domain of to the solution with $\det(\mathbf{G}) > 0$. This domain is less than the domain with $\det(\mathbf{J}) \geq 0$.

The path of correcting nodal capacities in coordinates of capacities $P_1 - P_2$ is perpendicular to border of existence domain. Therefore, it provides the opportunities of using the given method for search of optimal operational actions within the limits of algorithms of emergency control system for the operating point input the feasibility domain.

It is necessary to note, that to receive a limiting operating point in a case when initial nodal capacities are set outside the border of the existence domain, there is no necessity to make any accounting by the method of making successive heavier (lighter) conditions as the iterative process converges automatically to the nearest limiting operating point.

IV. Reduction of Computational Costs

If to compare the Newton's method of optimization for power flow calculation to Newton- Raphson using a Jacobian matrix the method computational costs on each iteration will be several times greater as a property of being filled up of a Hessian matrix nonzero elements in 2,5-3 times greater than at Jacobian one. In each row of Jacobian matrix corresponding to any bus, contains nonzero elements corresponding to all incident buses of scheme. Each row of Hessian matrix contains nonzero elements in a matrix corresponding not only to the neighboring buses, but also their neighbors that is buses of the first and second belt. However it is possible to compensate this disadvantage through the combination Newton-Raphson method with Newton's method of optimization for power flow calculation. It means that the part of buses can be calculated by conventional Newton method, and the remained buses will be computed by Newton's method of optimization. In the first group of passive buses consist of buses in which it is not possible change nodal capacity or it is not expedient. Hence emergency control actions are possible only in small group of the buses connected to channels of telecontrol. Most of buses including purely transit buses are passive. Active buses are generating buses in which operating actions are provided. Such approach allows to withstand nodal capacity contingencies for all passive buses of the scheme which have been calculated by Newton-Raphson method. In active buses which have been calculated by Newton's method in optimization, deviations from set values of nodal capacity are possible. These deviations can be considered as operating actions. The power flow calculation algorithm based on combination Newton – Raphson method and Newton's method of optimization can be presented in follows:

1. Linear equation system based on Jacobian matrix is generated for buses of scheme.
2. All passive nodes are eliminated by Gauss method. Factored equations are kept.
3. The matrix of nodal admittance is generate from not factored the part of Jacobian matrix corresponding to active buses. This admittance matrix contains network parameters of the active equivalent.
4. Linear equation system with Hessian matrix is generated for the active equivalent by Newton's method of optimization.
5. Linear equation system with Hessian matrix is calculated and changes of independent variables are defined for active buses.
6. Factored equations of passive buses are calculated, and changes of independent variables are defined for passive buses.
7. The vector of in independent variables is updated using the changes of independent variables for all buses.
8. New nodal capacities in all buses of the network are defined; constraints are checked; if it needs the list of active buses will be corrected.
9. Convergence of the iterative process is checked. If changes of variables are significant it is need to turn to item 1.

Taking into account that number of active buses in the network is not large computational costs of such algorithm slightly exceed computational costs of the Newton- Raphson method.

V. Conclusions

1. The power flow calculation of an electric network by minimizing the square sum of discrepancies of nodal capacities based on the Newton's method of optimization materially increases the productivity of deriving of a solution for heavy as to the conditions of stability states and the unstable states outside the existence domain of the solution.

2. During the power flow calculation, the determinant of Hessian matrix is positive around zero and negative value of a

determinant of Jacobian.

3. The iterative process automatically converges to the nearest marginal operating point during the power flow calculation, when initial operating point has been outside of the existence domain.

4. The convergence domain of the solution of the Newton's method of optimization is limited by a positive value of the determinant of Hessian matrix. The iterative process even for solvable operating point can converge to an incorrect solution if initial approximation has been outside of convergence domain.

5. It is necessary to use zero approximation for which the determinant of Hessian matrix is positive.

6. The power flow calculation based on the Newton's method of optimization is effective for problems of pulling the operating point back onto feasibility boundary since the power mismatches can be considered as optimal operational actions.

VI. References

[1] D. A. Arzamastsev, P. I. Bartolomej, A. M. Holjan, Automatic control system and optimization of modes of electric power systems, Moscow: Vysshaya Shkola, 1983.

[2] V. I. Tarasov, Nonlinear minimization methods for calculating steady states of electric power systems, – Novosibirsk: Nauka, 2001, p. 214.

[3] A. Z. Gamm, Statistical methods of power system state estimation, Moscow: Nauka, 1976, p. 220.

[4] V. M. Gornshtejn, B. P. Miroshnichenko, A.V. Ponomarev, Methods of optimization of power system modes, Moscow: Energiya, 1981, p. 336.

VII. Biographies

Andrey V. Pazderin was born in Ekaterinburg, Russia, on June 12, 1960. He graduated from the Urals Polytechnic Institute, Ekaterinburg, in 1982. He obtained PhD in 1987. In 2005 the Doctoral degree was received by him.

From his graduation A.V. Pazderin is employed by Ural State Technical University as a Professor and head of the Power Engineering department (from 2006).

His special fields of interest included power system state estimation, energy meters verification, power loss calculation.

Sergey V. Yuferev was born in Russia on May 14, 1985. He received the B.Eng. degree (Hons) and M.Sc degree from Ural State Technical University, Ekaterinburg, Russia, in 2006 and 2008, respectively.

His research interests lie in power flow calculation, power system analysis and steady-state stability.