Mathematical model of Reliability Centered Maintenance (RCM). Power transmission and distribution networks applications

D. Sarchiz, D. Bică, O. Georgescu

Abstract-- **A basic component of the power quality generally and energy supply in particular is the management of maintenance actions of electric transmission and distribution networks. Starting from this fact, the paper develops a mathematical model of external interventions upon a system henceforth called Renewal Processes. These are performed in order to reestablish system performances i. e. its availability.**

***IndexTerms*--Mathematical model,Mentenability, Reliability, Renewal**

I. PRINCIPLES OF MODELLING

LIBERALIZATION of the energy market and separation of production, transport and distribution activities brought to light the importance of increased reliability of transport and distribution generally with a special focus in reduction of maintenance costs for networks. The purpose of maintenance action is to keep and reestablish parameters of availability of equipment and installations at the designed parameters.

The Reliability Centered Maintenance (RCM) is based on planning future action (T^+) on the availability of the studied system at the given time (T^0). This state are also estimated based on the past events succession i.e. the available data relating system behavior over the period (T^-).

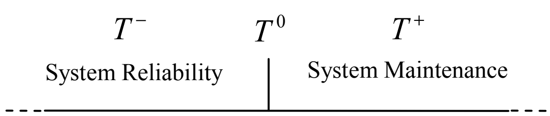

Fig. 1. Estimation and planning time of RCM.

II. EXPRESSION OF SYSTEM RELIABILITY

To estimate the reliability of the studied system (20 kV voltage overhead electric line), from the database of Incident/Intervention Charts belonging to Electrica Distribution South Transylvania Company, Tg.Mures Subsidiary, we select to kind of random variables: a) time moments t_i (expressed in days) corresponding to incident *i* when the functioning of line was interrupted, considering only corrective instances to restore line function; b) duration of corrective instances T_{ri} (expressed in minutes) to restore the line after corresponding incident *i*.

TABLE I
THE INCIDENTS, MOMENTS AND CORRESPONDING INTERRUPTION TIME

Incident number (*i*)	1	2	3	…	52	53	54
t_i (days)	62	68	80	…	1608	1638	1712
T_{ri} (min)	213	118	352	…	403	178	841

Power networks generally and electric lines particularly contain mechanical and electrical subassemblies and their function is directly influenced by environmental fluctuation, so that we can safely state that failures of these elements are due to wear and slow aging. Having these considerations in view, to model such survival processes Weibull distribution is used which characterizes the wear systems such as overhead electric lines.

Let's consider as mathematical model for estimation and evolution of reliability over the period $[0,t]$ the Weibull distribution as follows:

$$R(t,\beta,\lambda_w) = \exp(-\lambda_w t^\beta) \qquad (1)$$

where $\lambda_w > 0$ represents a scale parameter corresponding to measure units of time variable *t* (days/hours/minutes), $\beta > 0$ is the shape parameter for Weibull distribution ($\beta = 1$ corresponds to the exponential distribution function) and *t* is time variable.

Relation (1) expresses the probability that the event occurs in the time interval $(0,t)$ i.e. the faultless function probability up to moment *t*.

$$t_i = [62 \;\; 68 \;\; 80 \;\; ...1608 \;\; 1638 \;\; 1712] \qquad (2)$$

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Estimation of parameters λ_w and β of Weibull distribution using data from Table I was made using 2 methods, checking the correlation with experimental data each time.

A. Estimation using Baron model

In [2] the author T. Baron presents a mathematical method to establish the parameters λ_w and β of biparametric Weibull distribution under the form (1) on basis of statistical data obtained by analysis of operating state of a system. Based on t_i, random variables and using Baron method, the parameters are calculated:

$$\lambda_w = 1.2668827 \cdot 10^{-4}$$

$$\beta = 1.2939825283\,3252 \qquad (3)$$

With these values, the expression of Reliability Function of the 20 kV overhead electric line is:

$$R_N(t_i) = \exp\left(-0.0000126688\ t_i^{\ 1.29398252}\right) \qquad (4)$$

B. Estimation using MATLAB model

Another estimation procedure of reliability function parameters based on Weibull repartition is given by MATLAB software through *weibfit* command from *Statistics Toolbox*. This command allows for estimation of a and b parameters in the standard form of the Weibull model for the random variable string *x*, given as:

$$R_W = f(x) = abx^{(b-1)}\ \exp\left(-ax^b\right) \qquad (5)$$

with commands *phat = weibfit(x)* where variable $x = t_i$ resulting phat vector with two dimension corresponding to *a* and *b* coefficients:

phat(1)=a; *phat(2)=b*

or:

a= 1.41103661765 $\cdot 10^{-6}$; *b*=1.95737893053 (6)

After theoretical identification that best models function/wear probability of the studied phenomenon, the correspondence between the measurement database and the chosen analytical model *A* or *B* was imposed. As check method for models (4) and (5) and experimental data (2) *Test* ζ^2 by command *Hi2=chi2inv* in MATLAB *Statistic Toolbox* was applied. The test confirms the hypothesis that the two attributes correspond to experimental data, confirming that behavior of overhead electric line wear is a Weibull distribution.

It can be conclude that (4) models with accuracy the Reliability Function of overhead electric line so it can be used in the study of Reliability Centered Maintenance (RCM) of this element. The graphical representation of experimental Reliability Function R_N and standard Reliability Function R_W is shown in Fig. 2.

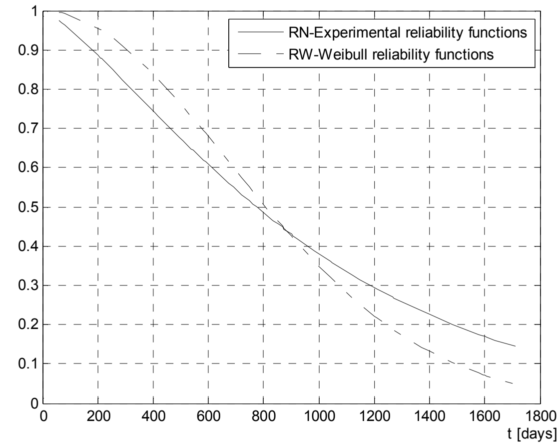

Fig. 2. Experimental and standard Reliability Functions

III. SYSTEM MAINTENABILITY MODEL AS TIME FUNCTION

It is necessary to present a maintenability model together with the reliability model already presented, in order to restore system performance, to constitute an overall model of reliability time evolution i.e. system availability.

To define this model we use the concept of Renewal [4] as any external intervention on the system in view of restoring its performance as a result of wear accumulated in time. With preventive maintenance under renewals, the Reliability Function (4) is [3]:

$$R_r(t) = \exp\left[-\lambda(r+1)^{(1-\beta)} t^{\ \beta}\right] \qquad (7)$$

where *r* is the preventive renewals number on the time interval *t* and α, β are the parameters of the Reliability Function from (1).

Evolution of a Renewal System is thus represented with the succession of a renewal number $r_1, r_2, ..., r_i$ and the intervals between them $\Delta T_1, \Delta T_2, ..., \Delta T_i$ with $i = 1,2,...n$ (Fig. 3).

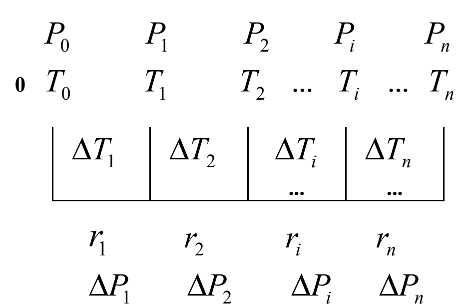

Fig.3. Parameters of renewal time ΔT_i.

If in $t = T_0$ the system is operating, function probability for another $T_1 = T_0 + \Delta T_1$ period presupposes simultaneous determination of two independents events: the function in $t = T_0$ and uninterrupted function for ΔT_1 days:

$$P_1(T_1) = P_1(T_0, T_0 + \Delta T_1) = P_0(T_0)P_1(\Delta T_1) \quad (8)$$
$$P_2(T_2) = P_1(T_1)P_2(\Delta T_2) = P_0(T_0)P_1(\Delta T_1)P_2(\Delta T_2)$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$P_n(T_n) = \ldots = P_0(T_0)P_1(\Delta T_1)P_2(\Delta T_2)\ldots P_n(\Delta T_n)$$

or:

$$P_n(T_n) = P_0(T_0)\prod_{i=1}^{n} P_i(\Delta T_i) \quad (9)$$

where:

$$P_i(\Delta T_i) = \exp\left[-\lambda_i (r_i + 1)^{(1-\beta)} \Delta T_i^{\beta}\right] \quad (10)$$

for $i = 1,2,\ldots n$.

Replacing (10) in (9) the function probability of system for the time interval $T_n = T_{n-1} + \Delta T_n$ depends by number of renewals r_i in each time interval:

$$P_n(T_n) = P_0(T_0)\prod_{i=1}^{n} \exp\left[-\lambda_{i-1} (r_i + 1)^{(1-\beta)} \Delta T_i^{\beta}\right] \quad (11)$$

for $i = 1,2,\ldots n$.

For $\lambda_i = \lambda_0 \prod_{j=1}^{i} (r_j + 1)^{(1-\beta)}$ the following recurrence relation of probability function P_n is obtained, for a system with ΔT_i time period and for each number of renewals r_i:

$$P_n = P_0 \exp\left\{-\lambda_0 \sum_{i=1}^{n}\left[\prod_{j=1}^{i}(r_j + 1)^{(1-\beta)} \Delta T_j^{\beta}\right]\right\} \quad (12)$$

Let us study the influence of the number of renewals *r* on time variation of Reliability Function (12) in a given time period. We carry out this study for different values of the numbers of renewals *r* for an interval ΔT or $t_i = [62 \div 1712]$ days and for the next parameters: $\lambda_0 = 0.00012668827$; $\beta = 1.29398252833255$; *n*=54; $P_0 = 0.991863877536537$. The graphics are presented in Fig. 4 and highlight the influence of the number of renewals on the increase of Reliability Function $P(t)$ at the end of the interval ΔT =1712 days (Table II).

The second study is the evaluation of the number of renewals *r* on variation of Reliability Function for three equal time intervals ΔT_i =1000 with *i=1,2,3*. The evaluation is made for different values of the numbers of renewals, corresponding to each ΔT_i interval (Table III) considering the next parameters: $\lambda_0 = 0.00012668827$; $\beta = 1.29398252833$ and $P_0 = 0.991$ (Fig. 5).

TABLE II
NUMBER OF RENEWALS AND RELIABILITY FUNCTION

Number of Renewals *r*	Reliability Function P_f
r_0 =0	P_{f0} =0.15
r_1 =10	P_{f1} =0.39
r_2 =50	P_{f2} =0.55
r_3 =100	P_{f3} =0.65

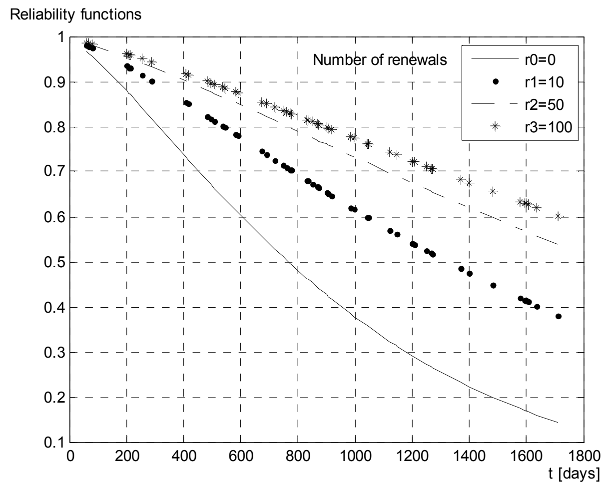

Fig. 4. Influence of the number of renewals on the increase of Reliability Function.

TABLE III
TIME INTERVALS AND CORRESPONDING NUMBER OF RENEWALS

Time interval ΔT (days)	Number of renewals *r*
ΔT_1 =1000	r_1 = 0, 5, 20, 50
ΔT_2 =1000	r_2 = 0, 3, 15, 30
ΔT_3 =1000	r_3 = 0, 1, 10, 20

As a direction to continue the Reliability Centered Maintenance (RCM) studies, it must to establish the optimum value for the number of renewal r_i in a interval ΔT_i that leads to minimizing maintenance costs in condition of maintaining the system reliability within limits imposed by safety operating.

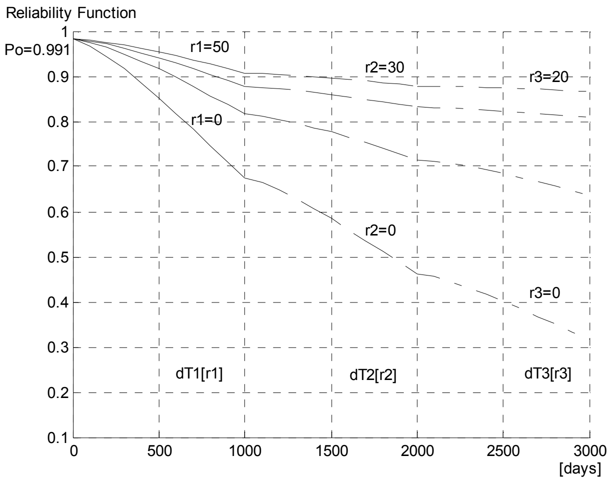

Fig. 5. Influence of the number of renewals r_i on the Reliability Function for time intervals ΔT_i.

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V. Biographies

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