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Power System Reliability Modeling with Aging Using Thinning Algorithm

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***Abstract*--** **Renewal process has been frequently employed as a mathematical model of components in power system reliability analysis. However, some of the components may enter an aging stage and this paper illustrates the effects of aging in reliability modeling. To implement the history of working and failure of system, Stochastic Point Process modeling based on Sequential Monte Carlo simulation is proposed. In the proposed method, the concept of thinning is used for Non-homogeneous Poisson Process characterization of lifetime of aging components. This simulation technique is applied to the Single Area IEEE RTS and the results are compared.**

***Index Terms*—Aging, Intensity Rate Function, Loss of Load Expectation, Monte Carlo Simulation, Mean Time to Failure/ Repair, Non-Homogeneous Poisson Process, Power Law Process, Power System Reliability, Renewal Process, Stochastic Point Process, Thinning.**

I. INTRODUCTION

Reliability analysis and assessment are important in the planning and operation of power systems[1-2]. Power systems typically enjoy high levels of reliability and customer demand is continuously satisfied unless there is equipment failure. Operation and maintenance costs [4] due to low reliability can be reduced by adequate planning, monitoring system behavior and taking proper control actions. An acceptable level of reliability needs be achieved at the minimum possible cost. This requires multi-objective optimization considering cost and reliability [3].

In general, the inter-failure time of a component shows three patterns of failure rate: decreasing indicating reliability growth, constant indicating random failures and increasing rate indicating wear out. In the renewal process commonly employed for reliability modeling, after repair the failure rate is set to the as new condition. So there may be aging of the component during the inter-failure time but there is no aging in the inter-failure times sequentially. In this paper, we focus on the aging in the sequential intervals leading to long term degradation.

When considering reliability analysis, there are two approaches: analytical and simulation method [5]. Analytical or numerical methods include state enumeration, network reduction, and minimal cut method. State space method is quite general and can be also applied to a system with dependent failure modes. However, for large power systems, direct application of this method is inefficient, considering storage and computational requirements. Network reduction method is based on reliability block diagram which is used for analyzing reliability of systems with only parallel and series combinations. By sequentially reducing the network, we can find reliability indices, like probability of failure, availability, frequency of failure, or mean up and down time. Minimal cut method can be used for complicated networks. However, it is generally not applied to systems involving dependent failure modes. All these methods assume a basic renewal model of components. So for systems not modeled by renewal processes, simulation approach is more flexible. This method virtually simulates all modes of each component during operation time, based on probabilistic laws.

This paper does not consider transmission systems, since it is mainly interested in illustrating the effect of aging components on quantitative system reliability. So models only for generators are constructed as components. Section III describes problem formulation, and sections IV and V present reliability modeling and simulation methodology. Sequential Monte Carlo Simulation [8-10] is introduced for occurrence of success and failure behaviors of the system. This technique is illustrated by the application to the Single Area IEEE Reliability Test Systems (RTS) [6-7], which includes generator capacities, transmission network, failure/ repair rate, load profile, and so on. This is illustrated by Section VI. MATLAB software is programmed for this implementation. Conclusion is provided by Section VII. In closing, Section VIII and IX represent references and biographies respectively.

II. BACKGROUND

Power system is considered repairable [11], that is, if it fails, it is put back into normal operation after repair of the failed component. This paper considers only two states of a component: success and failure, and it is assumed that initial state of all operating components is working. For renewal process [5], [12], same probability distribution is independently repeated after each failure. Fig. 1 shows the failure rate variations for different distributions. Each vertical dotted line denotes the moment of repair. So its duration is one cycle. As the term ‘renewal’ implies, failure rate after repair becomes renewed, whether it increases or not during its working period. So renewal process has a zero trend over sequential cycles. If the inter-failure time in renewal process is exponentially distributed, it is called Homogeneous Poisson

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Process (HPP) [13]. On the other hand, for an aging component, it has a positive or negative trend over sequential intervals. Uptime tends to get smaller as the component ages.

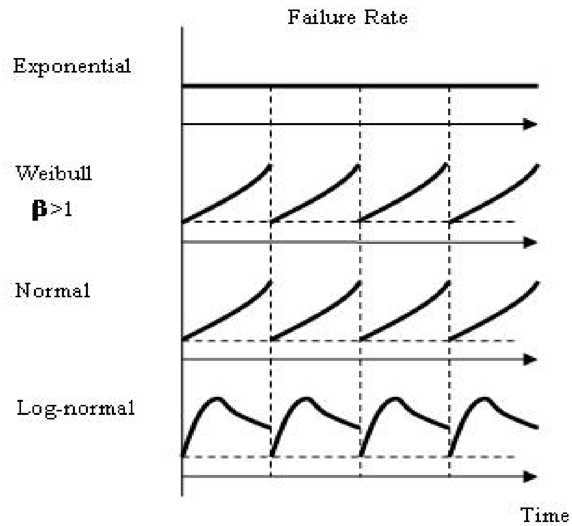

Fig.1. Failure rate comparison for different probability distributions

A significant index of reliability evaluation in power systems is Loss of Load Expectation (LOLE) [5]. Loss of load event is determined by using difference between generation capacity and load, if the transmission constraints are ignored. Then, LOLE is obtained through the sum of load loss duration for operation time. As the equipment ages, some equipment may have a positive trend. So, it is essential to observe and compare LOLE in both cases of non-aging and aging for system reliability evaluation.

III. PROBLEM FORMULATION

A. Stochastic Point Process

Stochastic Point Process $\{N(t), t \geq 0\}$, $N(0) = 0\}$ [12] is proposed for reliability modeling of power systems. N (t) is the number of events during (0, t]. Time x_k indicates the k-th time between successive arrivals, and t_k is the arrival time. This is described by equations (1), (2), and (3). Then expectation of N (t) is given in (4).

$$t_0 = 0 \tag{1}$$

$$t_k = \sum_{n=1}^{k} x_n \tag{2}$$

$$x_k = t_k - t_{k-1}, k \geq 1 \tag{3}$$

$$\Lambda(t) = E[N(t)] \tag{4}$$

$$\Lambda(t) = \int_0^t \lambda du = \lambda t \tag{5}$$

The integral of intensity rate $\lambda(t)$ is equal to $\Lambda(t)$ which is called renewal function. If random variable t is the time to failure, intensity is the failure rate. If t is time to repair, it is the repair rate. In a HPP with constant rate λ, renewal function is given by (5). On the other hand, failure rate is a function of time in a Non Homogeneous Poisson Process (NHPP) [13-14]. If it increases over time, the process shows an aging trend. If it decreases over time, it shows reliability growth.

IV. SAMPLING TRANSITION TIME

A. Transition time for non-aging model [5], [15]

For non-aging model, renewal process is applied. In (6), time x is interval-time, Z is a uniform random variable with an interval on (0, 1], and function F is a probability distribution function. Then the value of x can be taken in (7)

$$Z = \Pr(x \leq Z) = F(x) \tag{6}$$

$$x = F^{-1}(Z) \tag{7}$$

Renewal process includes several kinds of probability distributions. Here we briefly introduce commonly used four probability distributions.

(1) Exponential

For a stochastic point process, if N (t) is given by Poisson distribution, the interval-time is exponentially distributed. Intensity rate of a component is constant. Using (7) and (8), time x is given by simple function.

$$F(x) = 1 - e^{-\rho x} \tag{8}$$

$$x = \frac{-\ln(Z)}{\rho} \tag{9}$$

$$E(x) = \frac{1}{\rho} \tag{10}$$

Where ρ represents constant intensity, and is the reciprocal of mean up time.

(2) Weibull

Weibull distribution is characterized by probability distribution described in (11). Similar to the previous case, interval-time is taken from (7). The expected value is given by (13). When β is equal to one, it is exactly the same as exponential.

$$F(x) = 1 - e^{-(x\rho^{\frac{1}{\beta}})^{\beta}} \tag{11}$$

$$x = \left(\frac{-\ln(Z)}{\rho}\right)^{\frac{1}{\beta}} \tag{12}$$

$$E(x) = \frac{\Gamma(1+\frac{1}{\beta})}{\lambda^{\frac{1}{\beta}}} = \frac{\frac{1}{\beta}\Gamma(\frac{1}{\beta})}{\lambda^{\frac{1}{\beta}}} \tag{13}$$

Where $\Gamma(\bullet)$ is a gamma function.

(3) Normal

Normal distribution follows.

$$F(x) = \frac{1}{2}[1 + \mathrm{erf}(\frac{x - m}{\sigma\sqrt{2}})] \tag{14}$$

$$x = m + \sigma\sqrt{2}\mathrm{erf}^{-1}(2Z - 1) \tag{15}$$

Where m is mean of x, σ is variance of x, and erf indicates error function. Time x is given by (15).

(4) Log-Normal

In general, Log-normal is used more for repair time than the failure time. Time x and its mean value are given in (17) and (18).

$$F(x) = \frac{1}{2}[1 + \mathrm{erf}(\frac{\ln(x) - m}{\sigma\sqrt{2}})] \tag{16}$$

$$x = e^{m+\sigma\sqrt{2}\mathrm{erf}^{-1}(2Z-1)} \tag{17}$$

$$E(x) = e^{m+\frac{\sigma^2}{2}} \tag{18}$$

B. Transition time for aging model [13-14]

It should be clear that the aging is concerned with time to failure and the repair times may have nothing to do with aging. So the time to repair can be modeled as a non aging renewal process.

NHPP is introduced as a model for the aging failures. Specially, Power Law Process (PLP) [15] is used for this model and is described by (19-21). As shape parameter β varies, three types of trend are generated. If β is one, it is a zero trend. If β is greater than one, the process has an aging trend. If β is less than one, it has a negative trend, i.e., reliability growth. PLP is actually based on Weibull distribution because of equivalent failure rate function in (19). However, as can be seen from the comparison of Fig. 1 and Fig. 2, they are different.

$$\lambda(t) = \lambda\beta t^{\beta-1} \tag{19}$$

$$\Lambda(t) = E[N(t)] = \int_0^t \lambda(u)du = \lambda t^{\beta} \tag{20}$$

$$\Pr(N(\Delta t) = k) = \frac{\lambda^k (\Delta t)^{\beta k} e^{-\lambda(\Delta t)^{\beta}}}{k!} \tag{21}$$

Just as in Fig 1, vertical dotted lines represent the repair action. Failure rate is, however, not renewing, instead is the same as immediately before failure, which is called as good as old. This is minimal repair, while the repair action of Weibull renewal distribution is perfect - after repair, it is as good as new. In practice, however, a component of a system may be having an imperfect repair, which is between perfect repair and minimal repair.

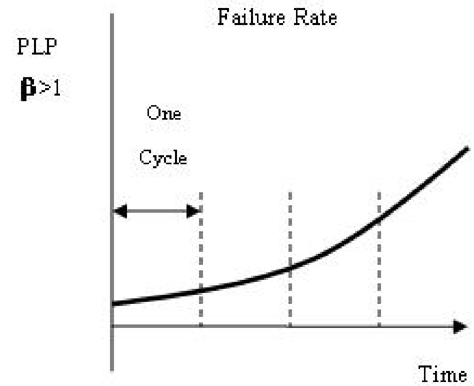

Fig.2. Failure rate variation with time of PLP model

There are several techniques [18], [22] to sample the transition time of a NHPP. This paper uses one of the techniques called Thinning Algorithm [17].

(a) Thinning Algorithm of a NHPP:

Step 1. Set time t to k-th arrival time, $t_k, k \ge 1$

Step 2. Generate two uniform random variables Z_1, Z_2.

Step 3. Set time t to $t - \frac{\ln(Z_1)}{\lambda^H}$.

Where λ^H is maximum value of $\lambda(t)$ during time T.

Step 4. If $\frac{\lambda(t)}{\lambda^H} \ge Z_2$, set (k+1)th arrival time, t_{k+1} to time t.

and if $t_{k+1} > T$, the process terminates

Or, Go back Step 2 and repeat the sequences.

If $\lambda(t)$ increases over time, $1 - \lambda(t)/\lambda^H$ gets smaller and, removing the arrival time, which is called thinning-out occurs less. However, when $\lambda(t)$ decreases over time, thinning-out process occurs more often. Then interval-time becomes bigger over time.

Here is a consideration. Basically, aging will start after the first cycle of the process. So Mean Time to First Failure (MTTFF) of PLP should be the same as $1/\lambda e$. And Mean up time during only the first cycle of PLP is the same as that of Weibull distribution, shown by Fig.1 and Fig. 2. Using this fact, following equations are derived. Equation (22) is the same as (13), since Weibull is Independently, Identical Distributed (IID). Similarly, λ is updated for different β in aging model to satisfy this property.

$$\mathrm{MTTFF} = \int_0^{\infty} \Pr(N(\Delta t) = 0) = \frac{\frac{1}{\beta}\Gamma(\frac{1}{\beta})}{\lambda^{\frac{1}{\beta}}} = \frac{1}{\lambda_e} \tag{22}$$

So that,

$$\lambda = {\lambda_e}^{\beta}(\frac{1}{\beta})^{\beta}\Gamma(\frac{1}{\beta})^{\beta} \tag{23}$$

V. SIMULATION METHODOLOGY

Simulation methodology for Stochastic Point Process is sequential Monte Carlo approach. Based on Thinning, interval-time for every generator of RTS is calculated. Then, arrival time is taken using (3). The smallest interval-time is selected. And the state of the corresponding generator is updated. If the state of the generator is success (failure), it is converted into failure (success) after selected minimum time. The remaining generators have residual time. Then operating time is updated.

The load values for every demand point are changed on an interval of one hour. To get a reasonably converged LOLE, coefficient of variation is calculated and [18-19] applied. This method is shown below.
The simulation is repeated by the condition of convergence criterion.

- I_i Reliability index-In the case of this paper, load loss value[h] during simulation time T
- *N* Number of reliability samples
- *SD* Standard deviation of the estimate of the index

Then, the estimate of the expected value of the index is calculated using (24) and standard deviation of the estimate in (25) is also determined. Simulation is continued until Coefficient of Variation (COV) is less than a preset value. This paper uses $\varepsilon = 5\%$.

$$\bar{I} = \frac{1}{N}\sum_{i=1}^{N} I_i \qquad (24)$$

$$SD = \sqrt{\frac{V}{N}} \qquad (25)$$

Where

$$V = \frac{1}{N}\sum_{i=1}^{N}\left(I_i - \bar{I}\right)^2 \qquad (26)$$

$$COV = \frac{SD}{\bar{I}} \qquad (27)$$

VI. CASE STUDIES

Fig. 3 shows the Single Area IEEE RTS, which consists of 24 buses, 32 generators, 38 transmission lines and transformers. This paper uses MATLAB to implement proposed technique. From load profile [6-7] expressed by the percent of annual, weekly, and daily peak load data, hourly peak load value is modeled during one year. Simulation time T [years] can be extended, by sequentially repeating load data.

Before considering the issue of aging, let us examine the non-aging model. If Mean Time to Failure (MTTF) or Mean Time to Repair (MTTR) of different distributions used in renewal process is the same, it should be unchanged even after each failure or repair, since it is as good as new. To get the identical mean up times, expected value of Exponential, Weibull, Normal, Log-normal distributions is set to the same value, for example $1/\lambda_e = 950$, where λ_e indicates intensity of generator G27 from Table V when the interval-time is exponential.

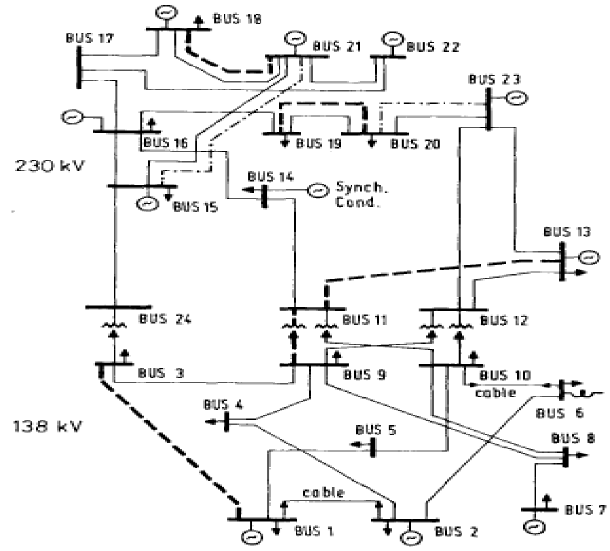

Fig.3. Single Area IEEE RTS

Setting value for each distribution is illustrated by (28-33). In Exponential, mean value is simply set to reciprocal of intensity. In Weibull, β is input data. λ should be changed for different β to get the same MTTF. In a case of Normal or Log-normal, variance of the variable is input data. It is assumed that variance is one. If we take high variance, simulation will require a more time by convergence criterion.

Exponential	$\frac{1}{\lambda_e} = 950$ (28)
Weibull	$\frac{\frac{1}{\beta}\Gamma(\frac{1}{\beta})}{\lambda^{\frac{1}{\beta}}} = \frac{1}{\lambda_e}$ (29) $\lambda = \lambda_e^{\beta}(\frac{1}{\beta})^{\beta}\Gamma(\frac{1}{\beta})^{\beta}$ (30)
Normal	$m = \frac{1}{\lambda_e}$ (31)
Log-normal	$\frac{1}{\lambda_e} = e^{m+\frac{1}{2}}$ (32) $m = \ln(\frac{1}{\lambda_e}) - \frac{1}{2}$ (33)

Table I-IV shows the mean values of up to 10 successive up times of G27 for different probability distributions. As you see from the Tables, mean up time of each distribution is maintained as the age of G27 grows. Also, mean up times for different distributions have approximately the same value. Small differences between them are caused by randomness.

TABLE I

G27 MEAN UP TIMES USING EXPONENTIAL

1st	2nd	3rd	4th	5th
951.211	950.266	951,311	949.561	950.918
6th	7th	8th	9th	10th
951.751	948.991	952.534	950.505	950.232

TABLE II
G27 MEAN UP TIMES USING WEIBULL ($\beta = 2$)

1st	2nd	3rd	4th	5th
952.241	950.322	953.030	950.212	951.876
6th	7th	8th	9th	10th
950.312	952.287	951.112	950.199	951.819

TABLE III
G27 MEAN UP TIMES USING NORMAL

1st	2nd	3rd	4th	5th
949.900	950.800	951.300	949.800	950.900
6th	7th	8th	9th	10th
950.700	949.900	951.200	950.600	951.100

TABLE IV
G27 MEAN UP TIMES USING LOG-NORMAL

1st	2nd	3rd	4th	5th
953.387	950.221	950.435	951.436	953.466
6th	7th	8th	9th	10th
949.452	953.322	950.599	951.646	952.426

Table VI shows LOLE for different renewal distributions. Simulation time is one year. The reliability indices are almost the same, since MTTF or MTTR is set to equivalent value using (28-33).

TABLE V
GENERATOR RELIABILITY DATA

Generators	Capacity [MW]	Failure Rate [h]	Repair Rate [h]
G1-G5	12	1/2940	1/60
G6-G9	20	1/450	1/50
G10-G15	50	1/1980	1/20
G16-G19	76	1/1960	1/40
C20-G22	100	1/1200	1/50
G23-G26	155	1/960	1/40
G27-G29	197	1/950	1/50
G30	350	1/1150	1/100
G31-G32	400	1/1100	1/150

TABLE VI
LOLE COMPARISON FOR THREE DIFFERENT DISTRIBUTIONS

Distributions	LOLE [h] (operation time=one year)
Exponential	9.171
Weibull	9.147
Normal	9.211
Log-Normal	9.253

Now, to examine the impact of aging components on system reliability, following two cases shown in Table VII are proposed. It is assumed that the remaining generators are exponentially distributed. In general, for system planning, long operation time is required. So simulation results are estimated for five years.

TABLE VII
DESCRITION FOR APPLICATIONS

Case	Applications
1	G16-19, G30 are aging
2	G23-26, G30 are aging

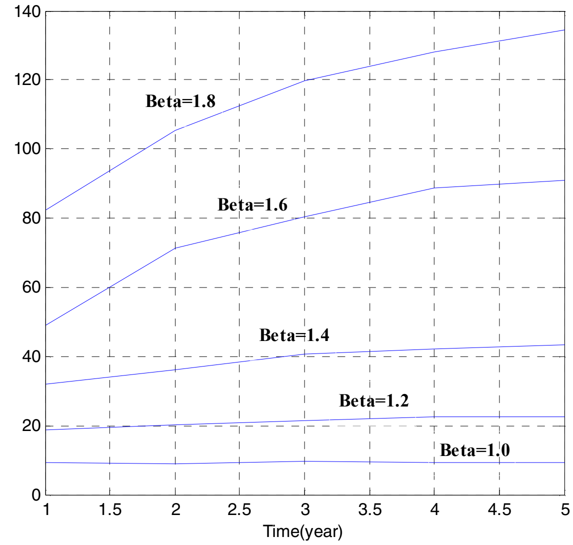

Fig.4. LOLE change for different β in case 1

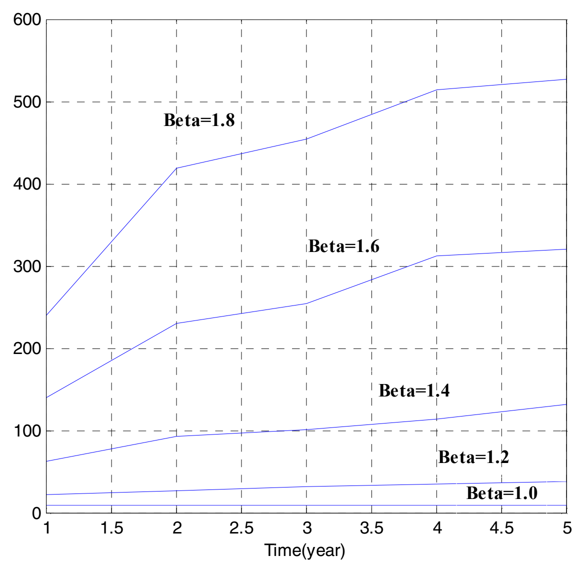

Fig.5. LOLE change for different β in case 2

Fig. 4 and Fig. 5 show the LOLE variations with different β during five years in the two cases, respectively. This index is evaluated only during each one year interval and is not accumulated. In the case of $\beta = 1$, LOLE is hardly changed over time for both cases and the value is also the same as the results from the Table VI. This is because that failure rate of PLP is constant in case of $\beta = 1$.

In case 1, the total capacity of aging generators is equal to 815 MW, constituting 26.6% from the total generator capacity 3055 MW. In case 2, the aging capacity is 605 MW, constituting 19.9% of the total capacity. So, aging capacity of case 1 is bigger than that of case 2. And failure rates of case 1 are higher than those of case 2. From these facts, it should be obvious that LOLE of case 1 increases faster, as β increases, or as the age of the system grows.

VII. CONCLUSIOS

In most power systems, some of equipment may enter aging or wear out stage. So it is important to monitor system reliability with quantitative indices. This helps system engineers to predict future system behavior and to construct proper resource and maintenance planning.

For non-aging model, Exponential, Weibull, Normal, Log-normal distributions are used to sample transition time. These distributions are identically repeated every cycle. So it is observed that mean uptimes and LOLE have almost the same values for different distributions because of this renewal property. For aging model, PLP is introduced. And to get inter-arrival time, Thinning methodology is applied to Sequential Monte Carlo.

The Single Area IEEE RTS is used for illustration of the proposed Stochastic Point Process and simulation method. When some components of the system show an aging trend, LOLE increases. To handle aging characteristics, parameter β of PLP model is properly regulated. As β increases, or the age of the system grows, it is observed that LOLE increases. Simulation time is set to five years, considering long term planning in reliability analysis.

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IX. BIOGRAPHIES

Hagkwen Kim (S'08) was born in Dae-gu in South Korea, on September 17, 1881. He graduated from the Kangneung National University, Korea, and is studying in the Power Systems Engineering division of electrical and computer engineering department at Texas A&M University, College Station, TX, USA. He is a student member of IEE. His specific research interest is reliability analysis and evaluation in power system network.

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Dr. Singh is a Fellow of IEEE and the recipient of the 1998 Outstanding Power Engineering Educator Award given by the IEEE Power Engineering Society. For his research contributions, he was awarded a D.Sc. degree by the University of Saskatchewan, Saskatoon, SK, Canada, in 1997. In 2008, he was recognized with the Merit Award by the PMAPS International Society.