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An alternative method for estimating mean life of power system equipment with limited end-of-life failure data

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***Abstract*-- Traditional statistical reliability analysis relies on failure data for a population of devices. If a complete data set is available, statistical reliability analysis can provide predictions, such as mean-time-to-failure for a particular device, percentages of devices that will fail at a particular time or before a particular age, a statistical distribution of failure ages, and other statistical measures of device failures. However, a typical population includes devices that have not yet failed. In reliability analysis, such populations are often denoted as "right censored populations".**

In this paper, we propose a new method to evaluate mean life of power system equipment with limited end-of-life failure data. The method is based on the generalized exponential distribution. This method can be used as an alternative to methods based on normal and Weibull distribution models.

***Index Terms*— Equipment aging, power systems, distribution systems, generalized exponential distribution, Weibull distribution, reliability, life data analysis.**

I. Introduction

Increasingly, industry is turning to statistical reliability analysis for equipment and products. In electricity industry, such statistical reliability analysis is useful in planning and budgeting for maintenance, predicting costs associated with electricity service, and making decisions about maintenance of particular electric network equipment. In addition, some utilities have turned to statistical reliability analysis for assessing the service life of electric network equipment.

Traditional statistical reliability analysis relies on failure data for a population of devices. If a complete data set is available (i.e., failure ages are known for each device within the population), statistical reliability analysis can provide predictions, such as mean-time-to-failure for a particular device, percentages of devices that will fail at a particular time or before a particular age, a statistical distribution of failure ages, and other statistical measures of device failures. However, a typical population includes devices that have not yet failed, termed "suspensions". In reliability analysis, such populations are often denoted as "right censored populations" [1].

The two-parameter Weibull (scale and shape parameters) is one of the most popular distributions for analyzing any lifetime data [2]. In recent years, the Weibull distribution is becoming very popular to analyze lifetime data mainly because in presence of censoring it is much easier to handle, at least numerically, compared to a gamma distribution.

In reference [3], it is presented two methods to estimate the mean life and its standard deviation of a power system equipment group with limited end-of-life or aging failure data. One is for the normal distribution model and another for the Weibull distribution model. An equipment group containing 100 reactors with only four dead (retired) units was used as an application example to illustrate the methods.

This paper presents a new distribution, named generalized exponential distribution, as an alternative to Weibull and normal distributions to evaluate the mean life and standard deviation of power system equipment with limited end-of-life failure data. The generalized exponential distribution (GED) was introduced by Gupta and Kundu in 1999 [4], and it has been proposed in the literature as an option to analyze lifetime data in place of gamma, Weibull and log-normal distributions. To demonstrate the application of the GED, the same reactors group reported in [3] was used.

II. The Method

In this paper, we propose a new method to evaluate mean life of power system equipment. The method is based on the generalized exponential distribution (GED) [4]. The GED has the form:

$$F(t;\alpha,\lambda) = (1-e^{-\lambda t})^{\alpha} \qquad t;\alpha,\lambda > 0 \tag{1}$$

The density function of the GED, the survival function, and the hazard function are the following:

$$f(t;\alpha,\lambda) = \alpha\lambda(1-e^{-\lambda t})^{\alpha-1} e^{-\lambda t} \tag{2}$$

$$S(t;\alpha,\lambda) = 1-(1-e^{-\lambda t})^{\alpha} \tag{3}$$

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$$h(t;\alpha,\lambda)=\frac{\alpha\lambda(1-e^{-\lambda t})^{\alpha-1}e^{-\lambda t}}{1-(1-e^{-\lambda t})^{\alpha}} \quad (4)$$

Here α is the shape parameter and λ is the scale parameter, α, $\lambda > 0$. The GED can have constant, increasing and decreasing failure rates depending on the shape parameter; that means that the GED can be used to model all the three portions of the basin curve for failure rate of equipment.

We have the following relationship for each pair of F_k and t_k and the error ε_k :

$$\ln F_k = \alpha \ln\left(1-e^{-\lambda t_k}\right)+\varepsilon_k \quad (5)$$

The cumulative failure probability is calculated as follows [3]:

$$F_k = F_{k-1}+\frac{d_k}{n_k} \quad (6)$$

where t_k is the age of the dead component k in the group, n_k denotes the number of components in the group over the age of $(t_k - 1)$, and d_k is the number of components in the group that have died at the age of t_k. The age of a component is calculated as follows: for a dead component, its age is the difference between the retired year and the in-service year; for a surviving component, its age is the difference between the current year and the in-service year.

The parameters α and λ are obtained by minimizing the sum of squares of errors $L(\alpha,\lambda)$:

$$L(\alpha,\lambda)=\sum_{k=1}^{N}\left[\ln F_k - \alpha\ln\left(1-e^{-\lambda t_k}\right)\right]^2 \quad (7)$$

where N is the total number of dead components. The mean life and standard deviation for a group of power system equipment are given by:

$$Mean\ Life = \frac{1}{\lambda}\left[\psi(1+\alpha)-\psi\right] \quad (5)$$

$$Sd = \sqrt{\frac{1}{\lambda^2}\left[\psi'(\alpha)-\psi'(1+\alpha)\right]} \quad (6)$$

where $\psi(.)$ and its derivatives are the digamma and polygamma functions respectively; and (5) and (6) are formulated based on the mean and standard deviation reported in [4].

III. APPLICATION EXAMPLE

A group of 100 single-phase 500 kV reactors at BC Hydro reported in [3] is used, with only four end-of-life failures in the past 31 years. The reference year used in the study is Year 2000. The mean life and standard deviation were recalculated using the normal and Weibull distribution, and they were compared against the results of the reference [3]. The recalculated results were also compared against the results obtained by using the proposed method based on the GED model.

Table I shows the raw data for this case. Table II shows the numbers of exposed and dead (retired) reactors for each age year. Table III shows data used in the parameters estimation for the GED. In this table, it is assumed that the cumulative failure probability before the first en-of-life failure is a negligibe value (0.001), which has a very small effect on the result. Similarly, it is assumed that the cumulative failure probability after the last end-of-life failure is the same as that corresponding to the age year of the last end-of-life failure component, indicating that the cumulative failure probability after the last end-of-life failure has no increase until the current year.

The estimates of the mean life, standard deviation and distribution parameters for the reactor group using the normal, Weibull distributions, GED and sample mean method are shown in Table IV. The corresponding distribution density functions plots are shown in Fig. 1.

TABLE I
RAW DATA OF REACTORS

No.	Setting in service (Year)	Retired Year	No.	Setting in service (Year)	Retired Year
1	1979	0	51	1976	0
2	1979	0	52	1976	0
3	1979	0	53	1976	0
4	1981	0	54	1970	0
5	1981	0	55	1970	0
6	1981	0	56	1970	0
7	1985	0	57	1981	0
8	1985	0	58	1981	0
9	1985	0	59	1981	0
10	1979	0	60	1983	0
11	1979	0	61	1983	0
12	1979	0	62	1983	0
13	1969	0	63	1984	0
14	1969	0	64	1984	0
15	1969	0	65	1984	0
16	1969	0	66	1983	0
17	1969	0	67	1983	0
18	1969	0	68	1983	0
19	1970	1996	69	1984	0
20	1996	0	70	1984	0
21	1970	0	71	1984	0
22	1970	0	72	1983	0
23	1970	1989	73	1983	0
24	1989	0	74	1983	0
25	1970	0	75	1984	0
26	1970	0	76	1984	0

27	1970	0	77	1984	0
28	1970	0	78	1978	0
29	1970	0	79	1978	0
30	1970	0	80	1978	0
31	1970	0	81	1969	1996
32	1970	0	82	1996	0
33	1976	0	83	1969	0
34	1976	0	84	1969	0
35	1976	0	85	1969	0
36	1976	0	86	1969	0
37	1976	0	87	1969	0
38	1976	0	88	1969	0
39	1976	0	89	1969	0
40	1976	0	90	1969	0
41	1976	0	91	1969	0
42	1976	0	92	1969	0
43	1976	0	93	1969	0
44	1976	0	94	1969	0
45	1976	0	95	1969	1997
46	1976	0	96	1997	0
47	1976	0	97	1969	0
48	1976	0	98	1969	0
49	1976	0	99	1969	0
50	1976	0	100	1969	0

TABLE II
NUMBERS OF EXPOSED AND RETIRED REACTORS(CASE FOR FOUR DEAD REACTORS)

Year No.	Exposed number	Retired number	Year No.	Exposed number	Retired number
0	100	0	16	93	0
1	100	0	17	84	0
2	100	0	18	75	0
3	100	0	19	75	1
4	99	0	20	68	0
5	97	0	21	68	0
6	97	0	22	62	0
7	97	0	23	59	0
8	97	0	24	59	0
9	97	0	25	38	0
10	97	0	26	38	1
11	97	0	27	37	1
12	96	0	28	36	1
13	96	0	29	35	0
14	96	0	30	35	0
15	96	0	31	22	0

TABLE III
DATA USED IN THE PARAMETERS ESTIMATION FOR THE GED

Age	F_k
18	0.00100
19	0.01433
26	0.04065
27	0.06768
28	0.09545
31	0.09545

TABLE IV
MEAN LIFE AND ITS STANDARD DEVIATION (CASE FOR FOUR DEAD REACTORS)

Parameter	Normal	Weibull	GE	Sample mean
Mean life (years)	36.130	48.826	47.486	25.0
Sd(years)	7.587	15.735	16.661	
Shape parameter		3.961	0.075	
Scale parameter		53.898	20.066	

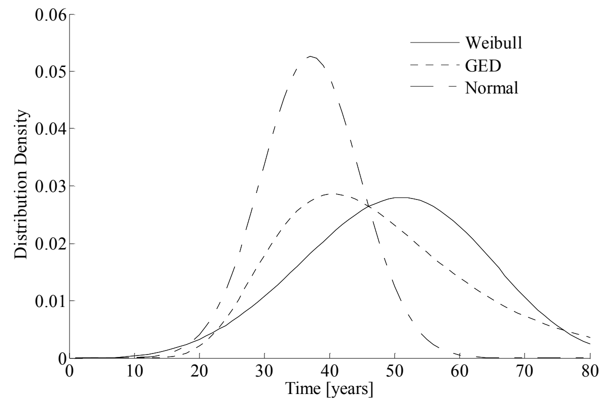

Fig. 1. The distribution density functions for the values in Table IV.

The proposed method was also applied to the case of twenty retired units reported in [3]. Table V shows the numbers of exposed and dead (twenty retired) reactors. In this case, it is artificially assumed to have 16 more reactors retired at ages from 27 to 31. The mean life and standard deviation estimates for this case are presented in Table VI and Fig. 2.

TABLE V
NUMBERS OF EXPOSED AND RETIRED REACTORS (CASE FOR 20 DEAD REACTORS)

Year No.	Exposed number	Retired number	Year No.	Exposed number	Retired number
0	100	0	16	93	0
1	100	0	17	84	0
2	100	0	18	75	0
3	100	0	19	75	1
4	99	0	20	68	0
5	97	0	21	68	0
6	97	0	22	62	0
7	97	0	23	59	0
8	97	0	24	59	0
9	97	0	25	38	0
10	97	0	26	38	1
11	97	0	27	37	2
12	96	0	28	35	3
13	96	0	29	32	3
14	96	0	30	29	4
15	96	0	31	15	6

TABLE VI
MEAN LIFE AND ITS STANDARD DEVIATION (CASE FOR 20 DEAD REACTORS)

Parameter	Normal	Weibull	EG	Sample mean
Mean life (years)	30.307	29.874	36.328	29.0
Sd (years)	4.515	4.1830	9.938	
Shape parameter		28.254	0.1283	
Scale parameter		30.460	59.047	

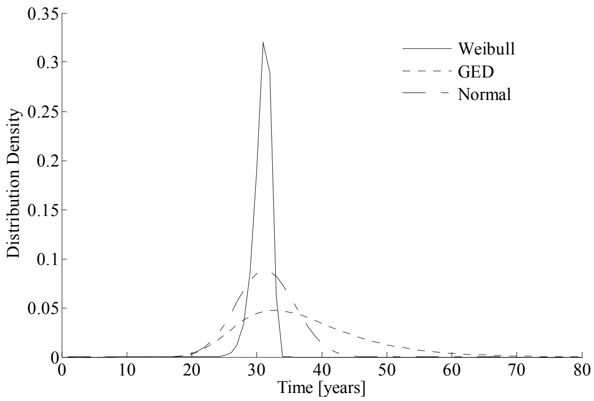

Fig. 2. The distribution density functions for the values in Table VII.

IV. CONCLUSIONS

In this paper, a generalized exponential distribution (GED) is applied to estimate the mean life of power system equipment with limited end-of-life failure data. The results obtained by the GED were compared with the corresponding estimates using the normal and Weibull distributions which are usually applied to the aging failure model of power system components [3]. Two equipment groups containing 100 reactors with four and twenty retired units respectively were used as an application example.

The results of estimation of the mean life using the Weibull and GED for the case of four retired components are very close; whereas the mean life calculated with the use of the normal distribution is approximately ten years less. The possible reason of this is that the normal distribution is not quite appropriate for estimating the mean life of equipments because it supposes the existence of nonzero probabilities of the negative life-times. In addition, in this case, the number of end-of-life failure data is very small. The Weibull and the GED are deprived these shortcomings.

On the contrary, the conventional sample mean technique cannot create a good estimate in this case because it uses the information of ages of only died components.

In the twenty-retired-units case, while increasing the number of retired components, the estimates of the mean life and standard deviation using the normal, Weibull and GED models show a tendency to approach to the corresponding estimates using the sample mean technique.

The results of reference [3] have been recalculated with more accuracy. As a result, the mean life and the standard deviation in the case of Weibull distribution for four retired components have been obtained very different to the results of reference [3]. The minimum of the corresponding sum-of-squares-of-errors function obtained in that reference is investigated with very high precision. It is shown that the parameters calculated in the reference [3] are faraway from the real minimum of the function. Therefore, a criterion of precision for calculating distribution parameters is required.

Currently, the GED is being used to analyse other power system equipment. The results will be reported in the future.

In summary, it is clear that the GED model provides reasonable results. It can be used as an alternative to the models based on the normal and Weibull distributions.

V. REFERENCES

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