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POWER NETWORKS' ROBUSTNESS ORIENTED EXTENSION

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Abstract

The safety of power networks is formulated in terms of probability, considering the failure processes as inherent stochastic events of large systems. This approach is important to design reliable power systems based on some optimal values for the system's availability and the involved costs. The vulnerability expressed visa the disturbances initiated by natural disasters, adverse weather, technical failures, human errors, sabotage, terrorism, and acts of war. Hence, the system's vulnerability is described as a sensitivity to threats and hazards, and it's measured by negative societal consequences, affecting large human communities. In this approach, the robustness is the network's capacity to support random destroying experiences. Today, one general accepted approach in the vulnerability quantification is made in terms of scale-free network, networks having the capability to conserve their robustness. Our paper formulates some proposals on the enlargement of the electrical distribution networks resulting from the old and the new, enlarged network properties. As a first step in our study we'll try to formulate appropriate methods for optimal adjustments, intending to transform a not-robust networks to the nearest free-scale, robust network.

Key words: networks' vulnerability, scale free networks, critical threshold fraction

I. INTRODUCTION

The safety of some large systems is well settled in many civil areas by national standards regarding the design of power systems, the goal being to connect the reliability with the costs to obtain this reliability. The most known way to quantify the vulnerability is to use the probably- based approach of systems with renewing processes, introducing in the system fail- oriented analyze new states corresponding to the nature of disasters.

The critical services are used to distribute energy, information, water, goods and people, disturbances in the flow of services can be initiated by natural disasters, human errors, sabotage, terrorism etc. All that infrastructural systems have a common trace – the network form. Hence a natural way to analyze the vulnerability of these large systems is to use one network-based approach.

In this letter, the static robustness of small distribution network are investigated .

II. NETWORKS' CONNECTIVITY QUANTIFIERS [1]

Generally, networks considered as graphs can be characterized by a number of parameters:

- The nod's degree *k*, which tells how many links the node has to other nodes. An undirected network with *n* nodes and L links is characterized by an average degree

$$<k> \approx 2L/N \qquad (1)$$

where :

- L is the total number of the network's edges;
- N denotes the nodes' number, <>denotes the average.

- The degree distribution *P(k)* gives the probability that a selected node has exactly k links. *P(k)* is obtained by counting the number of nodes *N(k)* with *k = 1, 2…* links and dividing by the total number of nodes n. Hence, the correct expression of the average degree will be:

$$<k> = \sum_k k \cdot P(k) \qquad (2)$$

- The clustering coefficient:

$$C_I = \frac{2n_I}{k_I(k_I - 1)} \qquad (3)$$

where n_I is the number of links connecting the k_I neighbors of node to each other, $k_I(k_I - 1)/2$ is the total number of triangles that could pass through node I, should all of I's neighbors be connected to each other.

A. THE CALCULUS OF THE CRITICAL FRACTION

To quantify the networks' robustness, random series of removed nodes (random attacks) are generated: after every node deletion, a maximal cluster dimension S, depending to the fraction of deleted nodes is kept. Graphically, there is a point corresponding to the critical fraction, from where the decreasing of the remaining clusters dimension changes its slope.

One computable measure of the critical threshold *fc* for random fragmentation was proposed by Cohen et al. (2000). Resuming their argumentation, a criterion for *fc* is realized when [1]:

$$\frac{<k^2>}{<k>} = 2 \qquad (4)$$

Continuing, for a node with initial degree k_0 and initial distribution $P(k_0)$, after the random removal of fN nodes,

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the probability that the degree of that node becomes k is a binomial – form probability:

$$P' = C_{k0}^{k}(1-f)^{k} f^{k0-k} \qquad (5)$$

Hence, the degree distribution, applying (5) for all the terms of the initial distribution becomes:

$$P(k) = \sum_{k0=k}^{\infty} P(k_0)P' \qquad (6)$$

Finally, with the new $P(k)$ the average degree $<k>$ and its second moment $<k^2>$ are computed, resulting:

$$\begin{aligned} &< k > = < k_0 > (1-f_c) \\ &< k^2 > = < k_0{}^2 > (1-f_c)^2 + < k_0 > f_c(1-f_c) \end{aligned} \qquad (7)$$

Replacing (7) in (4), a general formula permeates to calculate the critical fraction f_c:

$$f_c = 1 - \frac{1}{\frac{< k^2 >}{< k >} - 1} \qquad (8)$$

For scale-free networks with the degree distribution:

$$P(k) = Ak^{-b}, \quad k = m, m+1, ..., K \qquad (9)$$

the value of $\frac{< k^2 >}{< k >}$ is approximated with:

$$\frac{< k^2 >}{< k >} = \begin{cases} m^{b-2}K^{3-b} & if\ 2 < b < 3 \\ K & if\ 1 < b < 2 \end{cases} \qquad (10)$$

For $2 < b < 3$, a practical approximation for the critical threshold f_c can be used [2]:

$$f_c = 1 - \frac{1}{2b-1} \qquad (11)$$

B. ADJUSTING NON-SCALE FREE NETWORKS TO THE NEARLY SCALE-FREE NETWORK

Let done one a NT_0 network with the empiric degree distribution *P(k)*. We assume that the network adjustment will be realized by adding only new edges to the existing connections. In a logarithmic coordinates system, $\log(P(k))$ and $\log(N(k))$ define lines:

$$\log(P(k)) = A - b\cdot\log(k), \log(N(k)) = A_1 - b\cdot\log(k) \quad (12)$$

A nearest line will funded by linear regression method, representing a theoretical (9)- form degree distribution function, characterized by the values of *A* and *b*. The problem consists in getting the $P^*(k)$, so that for all integer k's to verify the existence of one graph N^*, with the same nodes' number *n* as the initial, non-scale-free network, with the rational demand to assure a certain minimal critical fraction f_c. That is a typical optimization problem. The second step will to identify the elementary transformations from NT_0 *to* NT^*. We decide to choose for adjustments, only the nodes with $N_k^0 < N_k^*$, considering that in the contrary case , the additional connections' number can be eliminated .

B.1 Networks enlargement by new edges addition

Let be the initial network ant its graph G_0 histogram:

$$G_0\begin{bmatrix} k \\ N_k \end{bmatrix} = \begin{bmatrix} 1 & ... & i & ... & j & ... & n \\ N_1 & ... & N_i & ... & N_j & ... & N_n \end{bmatrix} \qquad (13)$$

An elementary enlargement is made adding a single new edge, denoted by the operation:

$$G_{r+1} = e_{ij}(G_r), 1 \le i \le j \le n \qquad (14)$$

where:

- G_r, G_{r+1} are two successive graphs;
- e_{ij} - connection of one nodes with *i* edges with the other one, with *j* edges.

The new graphe's histogram will be:

$$G_{r+1} = \begin{cases} \begin{bmatrix} ... & i & i+1 & ... & j & j+1 & ... \\ ... & N_1 & N_i & ... & N_j & N_{j+1} & ... \end{bmatrix} & if\ j > i+1 \\ \begin{bmatrix} ... & i & i+1 & i+1 & ... & j & ... \\ ... & N_1 - 1 & N_i & N_{i+1}+1 & ... & N_j & ... \end{bmatrix} & if\ j = i+1 \\ \begin{bmatrix} ... & i & i+1 & ... \\ ... & N_i - 2 & N_{i+1}+2 & ... \end{bmatrix} & j = i \end{cases} \qquad (15)$$

Assertion: Let be G_o, G_F the initial, respectively the enlarged (final) graphs. Hence, the final graph can by obtained by m elementary enlargements, where:

$$\begin{aligned} \max(m) &= \sum\left[N_k^F - N_k^0\right]\Big|_{N_k^F - N_k^0 > 0} = \\ &= \sum\left[N_k^F - N_k^0\right]\Big|_{N_k^F - N_k^0 < 0} \end{aligned} \qquad (16)$$

Proof: If for all k, $N_k^F - N_k^0 \ge 0$, the enlargement is done. For $\sum(N_k^F - N_k^0) = 1$, the assertion is evident. Let ordering the elementary extension operators as an increasing array:

$$e_{11} < e_{12} < e_{22} < \ldots < e_{i-1,i} < \ldots \quad (17)$$

Assume that the elementary extensions will be made in this manner. So, all these elementary extensions will be $e_{i,i}$ or $e_{i,i+1}$ form, and Γ_{r+1} can be writing simpler as:

$$G_{r+1} = \begin{cases} \begin{bmatrix} \ldots & i & i+1 & i+1 & \ldots & j & \ldots \\ \ldots & N_1 - 1 & N_i & N_{i+1}+1 & \ldots & N_j & \ldots \end{bmatrix} & if\ j = i+1 \\ \begin{bmatrix} \ldots & i & i+1 & \ldots \\ \ldots & N_i - 2 & N_{i+1}+2 & \ldots \end{bmatrix} & j = i \end{cases} \quad (18)$$

For simplicity, let choose G_o and G_F like:

$$G_0 = \begin{bmatrix} 1 & 2 & 3 \\ a+b & N_2 - a & N_3 - b \end{bmatrix}, \quad G_F = \begin{bmatrix} 1 & 2 & 3 \\ 0 & N_2 & N_3 \end{bmatrix}, \quad N_2 > N_3 \quad (19)$$

Fix $a = 2K_1, b = 2K_2, \quad K_1 > K_2$

Examining G_o, G_F from (19),

$$\sum \left[N_k^F - N_k^0 \right] \Big|_{N_k^F - N_k^0 > 0} = 2K_1 + 2K_2$$

We'll perform successively:

$$G_1 = e_{11}(G_0) = \begin{bmatrix} 1 & 2 & 3 \\ 2(K_1+K_2)-2 & N_2 - 2K_1 + 2 & N_3 - 2K_2 \end{bmatrix},$$

....

$$G_{K1} = e_{11}(G_{K1-1}) = \begin{bmatrix} 1 & 2 & 3 \\ 2(K_1+K_2)-2K_1 & N_2 - 2K_1 + 2K_1 & N_3 - 2K_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2(K_1+K_2)-2K_1 & N_2 & N_3 - 2K_2 \end{bmatrix}$$

$$G_{K1+1} = e_{12}(G_{K1}) = \begin{bmatrix} 1 & 2 & 3 \\ 2(K_1+K_2)-2K_1-1 & N_2 & N_3 - 2K_2 + 1 \end{bmatrix}$$

.........,

$$\Gamma_{K1+2K2} = e_{12}(G_{K1+K2-1}) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & N_2 & N_3 \end{bmatrix} \quad (20)$$

Thus, the assumption (16) is realized:

In the similar way are discussed the other cases.

C. APPLICATION

Let be the initial graph depicted in fig. 1 and its histogram, respectively the degree repartition:

$$G_0 = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 20 & 3 & 1 & 1 \end{bmatrix}$$

The node numbers of the nearest scale free G'$_F$ graph, having the slope $tg(S) > 2$ and keeping as constant the nodes' number, $\sum N(k) = 25$ found by a linear regression method are: N'$_2$=14.0238 N'$_3$= N25.8551 N'$_4$=3.1506 N'$_5$=1.9482. Rounding these values, we obtain the final graph's degree distribution as:

$$G_F = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 14 & 6 & 3 & 2 \end{bmatrix}$$

$$\begin{pmatrix} k \\ P(k) \end{pmatrix} = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0.56 & 0.24 & 0.12 & 0.08 \end{bmatrix}$$

Note the value, in logarithmic coordinates, of the covariance coefficient: *corrrcoef* (log(G$_F$), log(G'$_F$))= 0.9992 (fig.2)

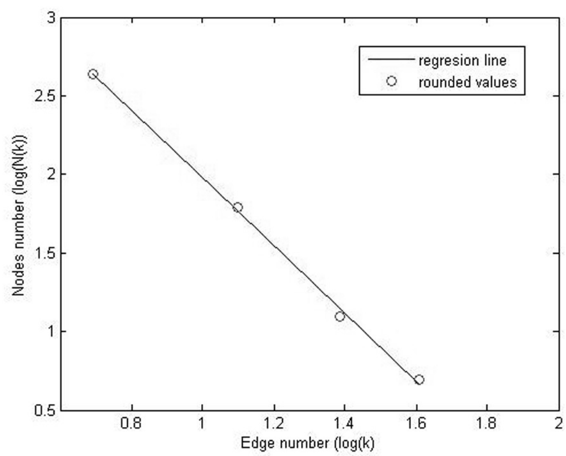

Fig.1 The degree distribution of G'$_F$, and G$_F$,

The set E of elementary extensions from G_o to G_F, following the algorithm presented in section A is:

$$E = \{e_{11}, e_{11}, e_{12}, e_{12}, e_{23}\}$$

The initial and the final graphs are depicted in fig. 3: the added connections are evidenced.

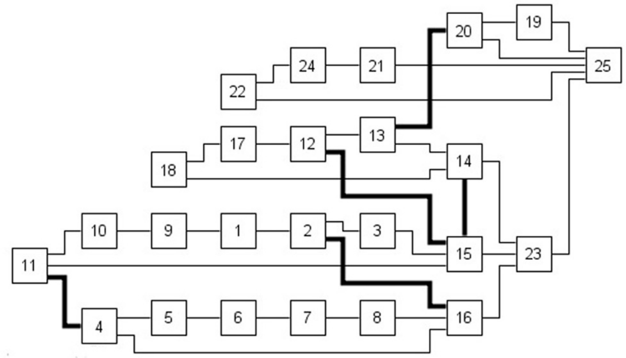

Fig.2 The initial and the new networks

Performing the elementary extensions of E in an indefinite order, a G_1, G_1,…, G_9 graph array is generated. We'll compute for all $G_i, i = 1:5$:

- The distances to $G_F = G_5$, computed by the Chi-square concordance criteria, expressing that in $Chi_{p,m}$ units, where: $p = 0.95$ is the risk value and m=3 is the degree of freedom: $Chi_{0.95,3} = 7.8147$;
- $(d_{i5})_{R.U.} = \dfrac{\sum_{2}^{5} \dfrac{(N^i k - N^5 k)^2}{N^5 k}}{Chi_{0.95,3}}$;
- The values of the critical fraction f_c with (8);

The results are contained in Table 1. The goal is to connect this distance with the values of the critical fraction f_c, calculated with (8), and compare these values, for the quasi- scale free graphs with the result of the (9) formula:

Table 1 Successive graphs' representative characteristics

Graph	Extension	N_k				$(d_{i5})_{R.U.}$	f_c (form (8))
	k	**2**	**3**	**4**	**5**		
G_0		20	3	1	1	0.75	0.3356
G_1	e_{22}	18	5	1	1	0.40	0.3878
G_2	e_{22}	16	7	1	1	0.29	0.4151
G_3	e_{23}	15	7	2	1	0.13	0.4483
G_4	e_{23}	14	7	3	1	0.08	0.4762
G_5	e_{34}	14	6	3	2	0	0.5143

The dependence between the relative distance $(d_{i5})_{R.U.}$ and the value of f_c is represented in fig.3:

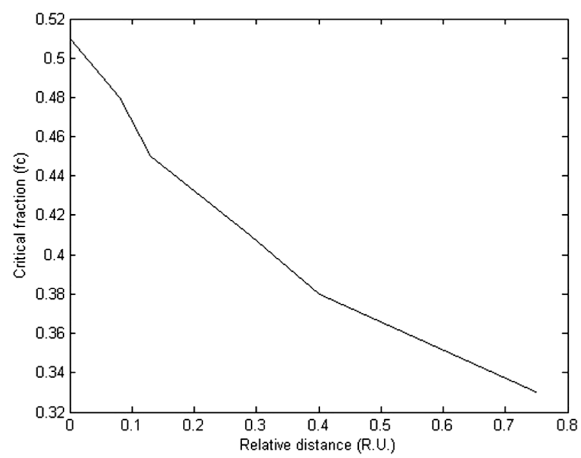

Fig.3 The dependence $(d_{i5})_{RU}$, f_c

CONCLUSIONS

The algorithm presented here offers the possibility to extend step by step one vulnerable network to the nearest robust network. Analyzing the results of the numerical example, we have the possibility to evaluate when a certain network can be considered as quasi-scale free, by comparing the values of the critical fraction values.

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