

# Applying Genetic Algorithms to REI system equivalents

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**Abstract**---This paper describes a new approach to the problem of the REI equivalent design, employing genetic algorithms. The optimum design of the REI equivalent aims to determine the number of REI buses to be used and the aggregation of external buses into the REI buses, using the minimum line power flows absolute errors in the internal system as optimization criterion, over a set of simulated node and branch contingencies. The method was tested using a modified version of the IEEE 57 bus test system.

**Index Terms**---Static network equivalents, REI equivalent, load flow analysis, genetic algorithms

## I. INTRODUCTION

THE actual development of power systems and the continuous growing of power exchanges between remote areas have determined an increasing interconnection degree between power systems, transforming them into wide area power systems (WAPS). The analysis of such complex systems aims particularly at two types of problems: development (off-line) and security assessment and control (on-line), often implying heavy computational efforts. To efficiently approach this analysis, simplifying assumptions must be taken in the way the different parts of the system are represented and interact.

The standard approach of this problem uses static network or system equivalents, especially when the main interest lies in the analysis of a local power system interconnected with other neighboring systems.

The first equivalents are those proposed by Ward in the mid of the 20-th century [1]. Later, in the 1970s, P. Dirmo has defined the REI equivalent [2]. The next type of network equivalent, the so called Ideal Transformers Equivalent, was introduced in 1977 by researchers from EPRI [3].

This paper presents a new approach to the problem of the REI equivalent design optimization based on the sensitivity of the complex power flows in the reduced system to a set of simulated contingencies. Existing approaches in the literature use different grouping methods. For instance, in [4], the buses

are grouped into REI equivalent buses based on the power flow sensitivity in the internal part of the system when the bus loads vary. Similarity indices are computed for all PU-PU, PQ-PQ and PU-PQ pairs of buses in the remote system to assess their sensitivity to the load variation. Then buses are grouped according to the value of the computed indices. The main drawback of this approach consists in the need for a priori specification of thresholds for bus group differentiation.

In this paper, the optimal solution of this problem, for different values of the number of the REI buses used by the equivalent, is determined using a genetic algorithm.

## II. REI EQUIVALENTS

As a general rule, equivalencing methods divide the original power system into three subsystems: (i) the internal power system (IPS), i.e. the part of the power system under analysis; (ii) the external power system (EPS), i.e. the part of the system to be replaced by the equivalent, and (iii) the boundary power system or, simply, boundary nodes (BNs), i.e. the set of nodes which separate IPS from EPS.

The REI equivalent replaces the EPS by one or more fictitious REI buses, that group together different external buses (see Fig. 1). The basic REI model either groups all EPS buses into a single REI bus, or uses 2 REI buses, one for the load buses, and the other for the generator buses. However, multiple REI buses can be used. Moreover, the resulting equivalent network can contain more generator REI buses, more load REI buses or even mixed REI buses, which group together load and generator buses. The grouping procedure is a question which should take into account its influence over the accuracy of the IPS operating conditions computed using the REI equivalents for different contingencies in the IPS. The REI equivalents should be built to fulfill some specific requirements: (i) seen from the boundary nodes, the equivalent should represent accurately the structure and the behavior of the EPS; (ii) the equivalent should describe as accurately as possible the reaction of the EPS to changes in the IPS with respect to the reference operating conditions, and (iii) the REI equivalent should have a minimum number of REI buses.

A distinctive characteristic of the REI equivalent is that it preserves power losses in the initial and equivalent networks through the use of a fictitious, temporary, linear and lossless network which links the buses from the EPS that are to be eliminated to the fictitious REI bus - the so-called Zero Power Balance Network (ZPBN). The procedure of building

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This paper is supported by the National Center for Research Programs Management, through the 22.126/2008 contract, "Intelligent System to Predict and Control Congestions in Large Power Systems and in the High Voltage Distribution Networks"

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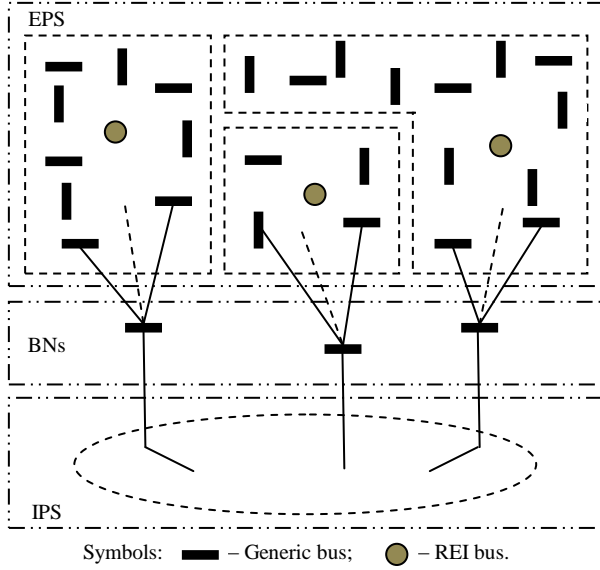


Fig. 1. Grouping EPS buses to create REI equivalent buses

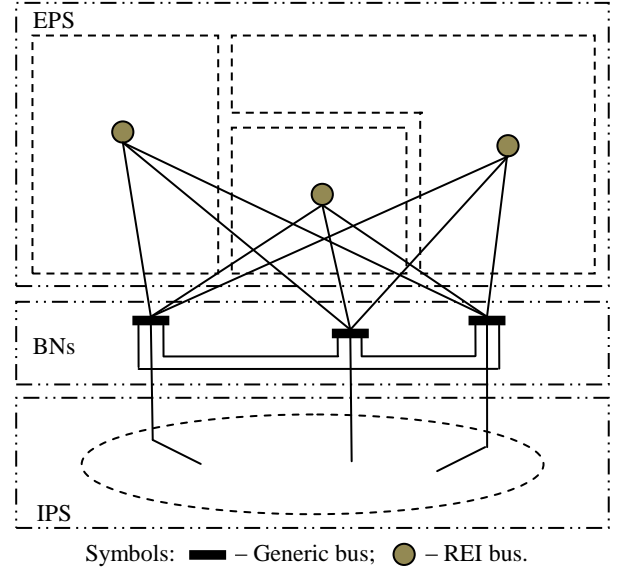


Fig. 2. The REI equivalencing technique

the ZPBN comprises in the following steps:

1. Setup the number of REI buses to be used to build the equivalent of the EPS.
2. Associate the buses from the EPS to the REI buses (each bus from the EPS is associated to a single REI bus).
3. For each group “EPS buses – REI bus” a fictitious ground bus is introduced, which is connected in a radial manner to the REI bus and the reduced buses.
4. The radial networks obtained in the previous step are linearized by replacing bus power injections  $S_p$  with current injection  $I_{p,0}$ , computed using bus voltages  $U_p$  from the reference operating conditions.
5. Admittances from the ZPBN are computed using equations:

$$\underline{y}_{p,0} = \frac{\underline{S}_p^*}{U_p^2} \quad \underline{y}_{R,0} = -\frac{\underline{S}_R^*}{U_R^2} \quad (1)$$

where  $R$  is the REI bus,  $p$  is one of the buses to be eliminated, and  $0$  is the fictitious ground bus. Apparent power injection from the REI bus  $S_R$  is the sum of apparent power injection from the buses associated to the REI bus. REI bus voltage  $U_R$  is computed based on apparent power  $S_R$  and equivalent current injections  $I_{p,0}$ .

The first two steps of this procedure can influence in a great extent the accuracy of the REI equivalent.

After building all ZPBNs associated to the REI buses, the network is reduced applying a traditional Gauss reduction technique, which aims to bring the nodal equation of the system including the newly formed REI nodes:

$$[Y] \cdot [U] = [I] \quad (2)$$

to a partially triangular form.

The application of Gauss reduction ends when all the lower

diagonal elements from the columns of matrix  $[Y]$  corresponding to the external and REI buses, have been zeroed. The submatrix corresponding to the internal and boundary buses is extracted to represent the admittance matrix in the equivalent network [8].

### III. PROBLEM FORMULATION

The problem aims to determine the REI equivalent that best fit the operating conditions of the IPS for a given set of contingencies that occur in this system.

The accuracy of the equivalent network is assessed using the mean absolute percentage error of the complex power flows in the IPS branches, for all contingencies in the input data set:

$$FO = \frac{1}{NC \cdot NI} \cdot \sum_{i=1}^{NI} \sum_{j=1}^{NC} \frac{|S_{ik,j}^{ref} - S_{ik,j}^{eq}|}{|S_{ik,j}^{ref}|} \cdot 100 \quad (3)$$

where:  $S_{ik,j}^{ref}$  - the complex power flow ( $\underline{S} = P + j \cdot Q$ ) on the branch connecting the  $i$  and  $k$  buses, for contingency  $j$ , for the reference operating conditions;  $S_{ik,j}^{eq}$  - the complex power flow on the branch connecting the  $i$  and  $k$  buses, for contingency  $j$  when the EPS is replaced with the REI equivalent;  $NI$  - number of buses in the IPS;  $NC$  - number of contingencies considered in the input data set.

For each contingency  $j = 1, \dots, NC$  the values of the power flows in the IPS branches will depend on the features of the REI equivalent. The basic two features of a REI equivalent that influence these values are:

- The number of REI buses used by the equivalent, denoted by  $N_{REI}$  and
- The set of buses from the EPS associated to each REI bus.

#### IV. GENETIC ALGORITHMS

Genetic algorithms (GAs) are adaptive techniques that determine an optimal or near-optimal solution for an optimization problem using mechanisms specific to genetics and natural selection [5, 6]. Given the advances in terms of computing power in the last years, they are used today with notable results in a wide range of applications in various technical fields.

GAs encode the admissible solutions as strings or chromosomes of fixed (problem dependent) length, initially random generated, which are then are changed in an iterative evolutionary process, towards the optimal solution, by the “fittest survive” principle. The elements of the strings (called genes) frequently use a binary representation (e.g. 0-1; on-off; present-absent), but real number representation is also possible. The degree in which an admissible solution fits to the problem is described by a fitness function. GAs maximize by principle the fitness function, hence, if the optimization problem aims to minimizing the objective function  $F$ , the fitness function  $f$  must be computed as the reciprocal of  $F$  ( $f=1/F$ ).

The iterative process of evolution of the GA uses three main steps:

*Reproduction or selection.* The better adapted chromosomes are selected from the current population, according to their fitness function, to create a crossover pool. Since the population’s size must remain unchanged, this means that several best adapted chromosomes produce multiple copies, which will have the possibility to improve further, while the worst adapted ones are discarded.

*Crossover.* Each chromosome could contain in its structure a part of the optimal solution that can be obtained through recombination. After the reproduction process, the selected chromosomes exchange parts among them, creating new offspring and renewing the population. The most efficient implementations of the GAs apply a stochastic crossover operator using a probabilistic crossover rate.

*Mutation.* In some cases, the crossover operator is not sufficient to drive the evolutionary process to the optimal solution. The mutation operator produces random changes in the chromosome’s structure. One or more genes in the chromosome change value randomly, to generate new combinations, which can lead to possible better solutions.

To simulate more accurately the process of natural selection and as a tool to fine-tune the algorithm’s performance, the crossover and mutation take place with a certain probability modelled through a crossover and a mutation rate.

A way to improve the performances of the GA is to use the elitism technique, which perpetuates the best adapted chromosome from one generation to another, so that the current best solution is never lost during the crossover or mutation stages. This method was used during the proposed case study.

The basic form of the GA is:

- Set the initial population  $P(\text{Gen})$ ,  $\text{Gen}=1$  ;
- Compute fitness functions for the initial population;

**repeat**

- Apply the selection operator to send parent strings in the crossover pool.
- Recombination: apply the crossover operator to form a new population  $P(\text{Gen}+1)$  ;
- Mutation: apply mutations to change the structure of the new population  $P(\text{Gen}+1)$  ;
- Compute fitness function for the new population  $P(\text{Gen}+1)$  ;
- Next generation:  $\text{Gen}=\text{Gen}+1$  ;

**until** {ending condition}

- The optimal solution corresponds to the string with the highest value of the fitness function.

#### V. CASE STUDY

The implementation of the GA to solve the problem of the REI equivalent design optimization was studied on a modified form of the IEEE 57 bus test system.

The inner part of the IEEE 57 bus test system was considered as the IPS, while the marginal buses and branches was considered to form the EPS (see Table I). Buses with bold face in Table I are generators (PU), the one with bold and underline face is the slack bus, while the rest, with normal face are load buses (PQ). One bus was added to the standard IEEE 57 bus system in order to separate the IPS and EPS, and the slack node was changed. Namely, the slack bus was changed from bus #1, which becomes an external bus, to bus #15. In fact, bus #1 was renamed as bus #58 and a new bus named bus #1, which becomes a boundary node (see Table I and Fig. 4), was added between buses #15 and #43, through a fictitious

TABLE I  
THE TYPE OF BUSES FROM THE IEEE 57 BUS TEST SYSTEM

External Power System (EPS)	Internal Power System (IPS)	Boundary nodes (BNs)
2, 3, 4, 5, <b>6, 8, 9</b> , 10, 11, <b>12</b> , 13, 14, 16, 17, 58	<u>1</u> , 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 47, 48, 50, 52, 53, 54, 56, 57	7, 15, 18, 41, 46, 49, 51, 55

TABLE II  
CONTINGENCIES USED IN THE GA IMPLEMENTATION FOR THE REI EQUIVALENT DESIGN OPTIMIZATION PROBLEM

Location and type of contingency	EPS	IPS
Branch	12-13 7-8	38-48, 29-52 28-29, 22-23
Bus	8 (-20%) 12 (-50%)	38 (-100%) 47 (-100%) 50 (-100%) 53 (-50%)

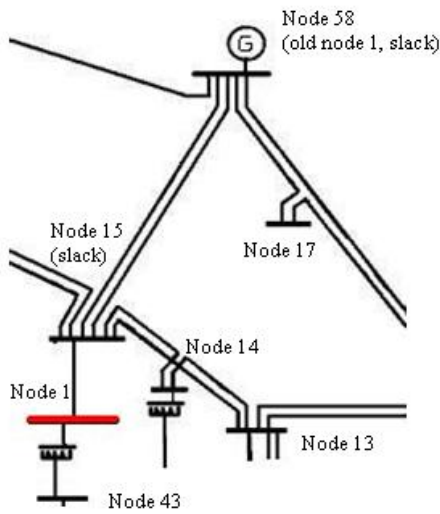


Fig. 4. Changes made to the IEEE 57 bus system.

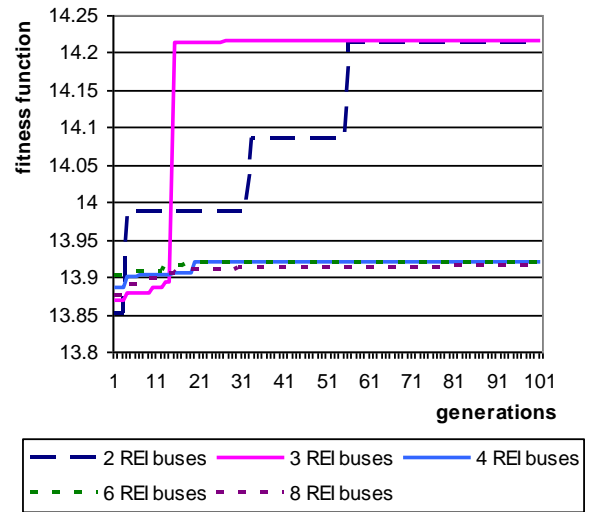


Fig. 5. The evolution of the fitness function for 2, 3, 4, 6 and 8 REI buses

TABLE III  
THE STRUCTURE OF A CHROMOSOME USED BY THE GENETIC ALGORITHM IN THE OPTIMISATION PROBLEM DESIGN

Genes (allocated REI bus)	11	10	7	11	1	3	11	7	5	10	4	1	1	9	11
Buses in the EPS	2	3	4	5	6	8	9	10	11	12	13	14	16	17	58

TABLE IV  
THE SOLUTIONS OBTAINED FOR 2-8 REI BUSES

Buses in the external system															Fitness function
2	3	4	5	6	8	9	10	11	12	13	14	16	17	58	
2 REI buses															14.214
2	1	1	1	2	2	2	2	1	2	1	1	1	2	1	
3 REI buses															14.216
1	3	3	3	1	1	1	1	1	1	2	3	2	1	2	
4 REI buses															13.92
2	2	1	2	3	4	2	1	3	1	3	3	2	3	2	
5 REI buses															13.92
4	4	1	4	2	5	4	3	4	1	2	2	2	4	4	
6 REI buses															14.211
3	6	6	2	3	3	3	3	3	3	1	2	1	3	5	
7 REI buses															13.918
2	2	5	2	1	3	2	1	3	4	1	1	6	1	2	
8 REI buses															13.917
4	4	1	4	6	8	4	6	4	2	6	2	6	4	4	

TABLE V  
THE ABSOLUTE POWER FLOW ERRORS IN THE TEST SYSTEM (IN MVA)  
WHEN USING 2 TO 8 REI BUSES FOR EQUIVALENCING THE EXTERNAL SYSTEM

		contingencies												average
		branch 38-48	branch 29-52	branch 28-29	branch 22-23	branch 12-13	branch 7-8	bus 8 (-20%)	bus 12 (-50%)	bus 47 (-100%)	bus 50 (-100%)	bus 53 (-50%)	bus 38 (-100%)	
REI buses	2	1.740	8.084	4.500	5.248	44.175	244.264	98.431	24.863	10.983	14.617	11.485	6.690	39.590
	3	1.730	8.114	4.445	5.235	44.175	244.264	98.431	24.863	10.976	14.604	11.470	6.689	39.583
	4	3.270	7.037	5.050	7.560	44.175	244.264	98.431	24.863	12.190	15.253	14.795	7.459	40.362
	5	3.270	7.015	5.027	7.563	44.175	244.264	98.431	24.863	12.228	15.347	14.816	7.486	40.374
	6	1.720	8.063	4.430	5.265	44.175	244.264	98.431	24.863	11.006	14.616	11.511	6.703	39.587
	7	3.260	6.974	5.049	7.587	44.175	244.264	98.431	24.863	12.285	15.395	14.864	7.521	40.389
	8	3.300	7.032	5.064	7.596	44.175	244.264	98.431	24.863	12.201	15.365	14.840	7.483	40.385

line of zero-admittance connected to bus #15. The complete one-line diagram of the original IEEE 57 bus test system can be consulted at [7].

The model has considered two types of contingencies (branch and bus contingencies in the EPS and the IPS – see Table II). Although the network equivalent analysis is centered on the IPS, what happens in the EPS can also influence the behavior of the IPS. Thus, a number of contingencies in the EPS have been considered too.

The strings which encode admissible solutions in the GA have a number of 15 genes, equal to the number of buses in the external system. Each gene can have a value less or equal to the number of maximum REI fictitious buses to be used in the current simulation, and this value shows to which REI node is assigned the current bus belonging to the external system. For instance, the string from Table III describes a solution that can use a maximum number of 11 REI buses. Buses 6, 14 and 16 are associated to the first REI bus, bus 8 is allocated to the third REI bus, bus 13 is allocated to the fourth REI bus and so on. In this case, not all the maximum available 11 REI buses were used. The optimal solution (number of used REI buses and which external buses are allocated to them) will be considered the one which gives the highest value of the fitness function for the set of contingencies from Table III.

All tests with the GA have used a population of 100 chromosomes, over a span of 100 generations. In the preliminary stage, tests for different number of REI nodes were undertaken. The best solutions obtained for configurations with a high number of REI buses never used more than 8 REI buses, for instance, for the configuration with maximum 11 REI buses from Table III, only 8 were used, with a low value for the fitness function (13.9025). The best values for the fitness functions were obtained with configurations using 2, 3 and 6 REI nodes.

In the second stage, the configurations using 2 up to 8 REI buses were tested thoroughly, to identify the optimal number of REI buses. Table IV shows the best REI node allocation obtained for each case, out of 10 attempts. The optimal solution is obtained for a configuration of three REI buses, in which buses 11, 5 to 11, and 16 are allocated to the first REI bus, buses 12, 14 and 17 are assigned to the second REI bus, and buses 2, 3, 4, 13 and 58 will be equivalenced with the third REI bus. For this configuration, the fitness function has the maximum value of 14.216. It should also be noted that configurations using only two REI buses are close to this value, with fitness functions above 14. Generally, with the increase of the number of REI buses, the value of the fitness function decreases slightly and the maximum number of REI buses is not always used. Although the solution with 6 REI buses has a high value of the fitness function too, the solution with 3 REI buses was preferred because it uses the minimum number of REI buses. Fig. 5 shows the evolution of the fitness function for some of the cases described in Table IV

As for each contingency's influence in the overall fitness functions, as can be seen in Table V, the most significant difference in power flows in the internal system occurs when branch 7-8 is disconnected or the load from bus 8 is reduced. On the other hand, since the power flow differences for these two contingencies remain unchanged for all REI bus cases, the value of the fitness function (which can be considered inversely proportional to the average value of the absolute power flow errors for all contingencies – the last column from Table V) is mostly differentiated by the contingencies which have a smaller effect on the power flows. Also, the configuration with 3 REI buses determines the lowest average variation of the power flows, which is consistent with the highest value of the fitness function obtained with this configuration.

## VI. CONCLUSIONS

The development of power systems and the growing of power exchanges between systems, have determined an increasing interconnection degree between power systems. The analysis of such large scale networks, with complex operation conditions, requires a heavy computational effort, which can be simplified using system equivalents.

This paper presents a new approach to the problem of the REI equivalent design optimization. The optimization process is based on the sensitivity of the power flows on the branches from the internal power system to a set of simulated contingencies and was conducted using the optimization model of genetic algorithms.

The optimum design of the REI equivalent aimed to determine the number of REI buses to be used and the aggregation of external buses into the REI buses. Test results show that the optimal solutions generated by multiple runs of the genetic algorithm tend to use a moderate number of REI buses. For the IEEE 57 bus test system, the optimal solution consists in using 3 REI buses.

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