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Research on Allocation of Congestion Cost in a Pool Based Market

H. F. Xiao, and W. D. Li

Abstract--**Under a deregulated environment, transmission congestion involves in system security and market efficiency. How to relieve congestion and allocate congestion cost reasonably is one of the important contents. In this paper, a method for congestion cost allocation in a pool based model is proposed. The congestion cost is firstly allocated to congested lines by Shapley-value; then the line costs are allocated to all market participants according to their utilization degree of congested lines. This method decomposes the allocation into two steps and computation speed for Shapley-value is improved greatly for the number of congested lines is far less than that of transactions. Furthermore the method makes the revenue collected from the transmission users be equal to the cost of relieving the congestion and its results provide fair, economic signals.**

***Index Terms*--Cost allocation, electricity market, pool based model, shadow price, Shapley-value, transmission congestion.**

I. Introduction

Transmission congestion occurs due to an operating condition that causes limit violations in the system, such as dynamic stability, thermo stability, power flow and node voltage violations et al. For the reason of system security and market efficiency, necessary and valid measures have to be taken to relieve congestion. Meanwhile congestion cost should be allocated reasonably, fairly among market participants. The importance has been recognized by both regulating bodies and utilities.

Three categories methods can settle congestion cost allocation--uplift tariff methods, nodal pricing methods, and cost allocation methods. Uplift tariff has been used in United Kingdom power market early [1]. This kind of method can be handled and satisfy the break-even requirement, but it dose not provide fair, efficient prices since it uses average cost information. Contrarily nodal prices can provide efficient signals. However, their often subtle and not quite predictable dependence on network and system characteristics make their sure somewhat questionable. Furthermore they create a merchandise surplus (MS). Transmission right helps to deal with the problem, but it relies on market operation mechanism. So it is hard to bring into effect perfect transmission right. Under cost allocation, congestion cost is distributed according to the degree of involvement by each market participant in the congestion. These methods provide efficient signals to some extent and do not create merchandise surplus. In current market phase, there is biggish room for growth for cost allocation methods.

Cost allocation methods are correlated to transaction model. Bilateral transaction is defined by specific contract path, with a pair of nodes--power injection node and extraction node. The allocation can be decomposed into two steps, in which the total congestion cost is first allocated to the congested lines [2], [3] and then to the transactions that cause the lines overloaded [4], [5]. We may also allocate cost directly to transactions connected to the nodes through a sensitivity expressed analytically [6], [7]. Nevertheless a pool transaction has only injection node or extraction node, without specific contract path. Thus the cost is often allocated directly to the node where the transaction located. For example, [8] introduces Aumann-Shapley (AS) method to compute congestion cost, and the customer demand at each node is divided into K small demand quantities of equal size, partitioned to a large number. Then the short-run marginal congestion cost (SRMCC) is determined by solving K constrained scheduling problems with nodal demands. What a customer should pay is computed through accumulating SRMCCs multiplied by small demand quantity. Reference [9] also uses the AS method to allocate the congestion cost but the capacity of congested lines is divided into K small quantities instead of customer demand. The AS method collects the exact required amount to recover congestion cost since it uses iterative computation for nodal prices. However troubles derive from here--there is no certain principle to determine the iterative times. Meanwhile optimal power flow (OPF) calculation is carried out in each iteration. If there were too much iterative times, the calculated amount will be very much; else there will be great error and produce merchandise surplus.

This embarrassment brings us back to the starting point of the problem, that is to say, we have to convert the allocation facing to transactions into the allocation facing to congested lines and then allocate line costs to the transactions that cause the lines overloaded just like in a bilateral model. This paper proposes a method for congestion cost allocation in a pool based model. It decomposes the allocation into two steps and uses Shapley-value to determine congested lines cost. The proposed method can reduce calculated amount greatly besides retaining the advantages of the ordinary Shapley-value method.

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II. Transmission Congestion

A. Congestion Management in a Pool Model

In day-ahead market, generation companies (Gen co.) and distribution companies (Dis co.) or consumers submit bids for energy. Independent system operator (ISO) makes predispatch for the given conditions in equilibrium market. If the analysis indicates some congestion conditions or signs threatening system security, the ISO adjusts operation plan by solving a redispatch problem,

$$\min \; C = \sum_i \Delta F_{G,i}\left(p_{G,i}\right) - \sum_j \Delta F_{B,j}\left(p_{B,j}\right) \tag{1}$$

$$P_{G,k} - P_{L,k} - P_k(\theta) = 0 \tag{2}$$

$$Q_{G,k} - Q_{L,k} - Q_k(\theta) = 0 \tag{3}$$

$$P_{G,i,\min} \le P_{G,i} \le P_{G,i,\max} \qquad i \in N_G \tag{4}$$

$$U_{k,\min} \le U_k \le U_{k,\max} \qquad k \in N_B \tag{5}$$

$$P_l \le P_{l,\max} \qquad l \in N_L \tag{6}$$

where (2), (3) represents the power flow equation; (4)~(6) represent inequality constraints including limits on nodes and branches; $p_{G,i}$ and $p_{B,j}$ are bids for generator i and consumer j respectively; θ is phase angle for node k; $P_{G,i,max}$ and $P_{G,i,min}$ are maximum and minimum output for generator i; $U_{k,max}$ and $U_{k,min}$ are maximum and minimum voltage of node k respectively; N_G, N_L, N_B are total number of generator, line and network node respectively.

B. What Is Congestion Cost

In (1) C is congestion cost during the redispatch. In a pool based model it is the sum of the increment of generation cost and decrement of load benefit, that is,

$$\begin{aligned} C &= W_{unc} - W_{con} \\ &= \left(B_{L,unc} - C_{G,unc}\right) - \left(B_{L,con} - C_{G,con}\right) = \Delta C_G - \Delta B_L \end{aligned} \tag{7}$$

where C_G is total generation cost; B_L is total load benefit, W is social welfare; subscripts *con*, *unc* represent constrained, unconstrained situations respectively.

Here the objective is defined to minimize the congestion cost but not merchandise surplus. We think it important to point out the essential differences between the two charges. Congestion cost is the loss of social welfare caused by transmission congestion in constrained and unconstrained system operation situations. There exists congestion cost as long as transmission overload occurs. While merchandise surplus is the system total revenue of selling power minus total cost of buying power, which is caused by pricing mechanism. Whether congestion occurs or not, merchandise surplus can be eliminated as long as appropriate pricing mechanism is adopted; else merchandise surplus will exist and increase evidently when congestion occurs.

We can compare the two charges through a simple example. Under market open only for Gen co., the congestion cost can be calculated by the increment of generation cost (Fig.1-A). To eliminate the overload in line A-B, G_A increase output and G_B decrease output. Congestion cost is equal to the area of rectangle L-K-p_B-p_A. If nodal pricing is introduced to calculate the congestion charges, there is a merchandise surplus remained, i.e. S_{MNKL}=P_Lp_B-(P_Bp_B+P_Ap_A).

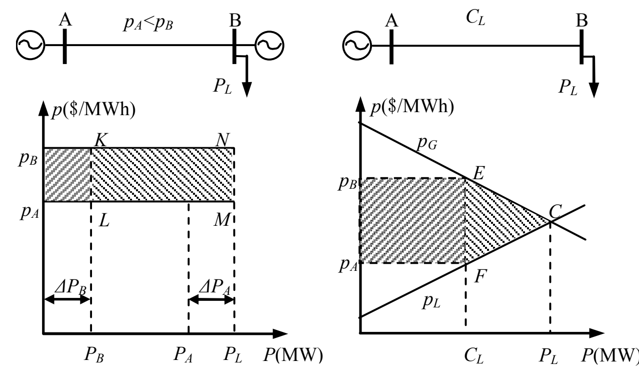

Fig. 1. Congestion cost and merchandise surplus

Under market open for both Gen co. and Dis co., the congestion cost can be calculated by the sum of increment of generation costs and decrement of load benefits. For the situation illustrated in Fig.1-B, congestion cost is equal to the area of triangle C-E-F. If nodal pricing is introduced to calculate the congestion charges, system selling price is p_B, and buying price is p_A. Merchandise surplus is the area of rectangle F-E-p_B-p_A.

Form the point of view of economics, social welfare is an appropriate index to weigh the whole economic property. Therefore congestion cost is a perfect objective for the congestion management problem.

III. Congestion Cost Allocation

A. Shapley-Value Based Congestion Cost Allocation

Power flows in transmission network appear nonlinear, and the relations between nodal injection powers, branch flows, nodal injection powers and branch flows are complex. When flow in some line changes, flows in others change too. For a specific line, whether it is congested caused by overloaded transmission power is related to other lines' transmission capacity.

So it is not easy to determine the exact congestion cost that a line should pay because all the line limits are constrained simultaneously. The allocation method has to be additive, monotonic and defensible. Such attributes are found in cooperative games theory. The solution of the problem how to allocate cost can be supported by the cooperative games theory.

Assume that there are n congested lines in a transmission network, each of which represents a player who takes part in the coalition problem of congestion cost allocation. S represents a subcoalition; v is a profit function derived from cost function C, and it is can be calculate by [10]

$$v(S) = \sum_{i \in S} C(i) - C(S) \tag{8}$$

where $C(i)$ is the congestion cost when each line capacity is constrained independently; $C(S)$ is the system congestion cost when lines' capacities in subcoalition S are constrained together. We use Shapley-value to allocate the $v(n)$ to each congested line,

$$x_i = \sum_{i \in S} \frac{(n-|S|)!(|S|-1)!}{n!}[v(S) - v(S-\{i\})] \tag{9}$$

where $|S|$ is the number of player in subcoalition; S-$\{i\}$ represents the remainder of subcoalition except line i; $v(S)$ is the eigenvalue for the subcoalition S; x_i represents the profit allocated to line i. Then the final congestion cost allocated to line i $C'(i)$ can be calculated by

$$C'(i) = C(i) - x_i \tag{10}$$

In theory, the sum of congestion cost when each line is constrained independently is greater than that when lines in subcoalition S are constrained together. The final cost allocated to each line after coalition is not greater than that before coalition. Therefore the results are reasonable and easy to be accepted for most market participants.

After line congestion costs are determined, they need to be further allocated to transactions. This process is, in essence, transmission facility usage determination. In different market framework congestion cost may be assummed by generation company, or distribution company, or both parts together. This is beyond the scope of the paper.

B. Properties Analysis

The Shapley-value method provides fair, economic signals and maintains a balance between revenue and expenditures. We elaborated on the properties by theory analyses and digital simulations. In this part we compare the proposed method with incremental method, shadow price method, and nodal pricing method through qualitative analysis.

1) : Economic Efficiency

The Shapley-value allocation can consider nonlinearity of network constrains and comply with the economic efficiency. Some methods determine the cost according to the responsibility for congestion. While the foundation of these methods is usually physical, lacking of economic efficiency. Few methods can provide economic signals too, for instance, incremental method [3], [4]. However they compute only two orders to join in a coalition--first and last. So there is larger error remained and the error will increase with the increase of player number. A particular case is when there are two players the results of incremental method and Shapley-value method are completely same (see Appendix).

2) : Balance Of Revenue And Expenditures

The proposed method allocates congestion cost aiming at total power, which makes the revenue collected from the transmission users be equal to the congestion cost. Though some methods provide efficient signals, they aim at marginal power for allocation and can not recover the cost completely. For example, shadow price reflects additional cost increase when the line power increases around the line capacity, and the line congestion cost can be expediently calculated through shadow price multiplied by overloaded power of line, that is , C_l=$\mu_l\Delta P_l$. However the sum of line congested cost is not equal to the actual system congested cost. The reason lies in that the marginal cost caused by incremental power is used to compute the line cost caused by total overloaded power ΔP_l. While there dose not exist linear relation between ΔP_l and C_l for μ_l changes in different line capacity.

Reference [8] ameliorates the shadow price and introduces AS method to compute congestion cost. The method divides overloaded power into K small quantities of equal size, which essentially defines K capacities for a congested line. The cost of a congested line is then computed through accumulating shadow price multiplied by overloaded power of line in K capacities. This method retains the efficient signals of shadow price as well as recovers the congestion cost completely. However an OPF schedule is carried out in each iteration, and there is a heavy computation. If too little iteration times are adopted the error will be remained. The ultimate situation is K=1, which is completely as same as ordinary shadow price method.

3) : Whole Economic Property

The Shapley-value method allocates congestion cost with indicated pricing objective; while some methods can not aim at congestion cost to allocate, for instance, nodal pricing method allocates the merchandise surplus. As illustrated in part II, transmission users will have to pay more for the congestion if the pricing mechanism is not inappropriate. The payment includes extra merchandise surplus besides the necessary congestion cost. Since social welfare is an appropriate index to weigh the whole economic property, congestion management should be dealt with under such objective. The proposed method allocate charges aiming at congestion cost, its results consequently can reflect the transmission constrains' impact on system operation and guide optimized configuration of the resources.

IV. Numerical Example

The proposed approach is demonstrated and compared with other congestion cost allocation approaches with the help of an example of a revised IEEE-5 bus system illustrated in Fig. 2.

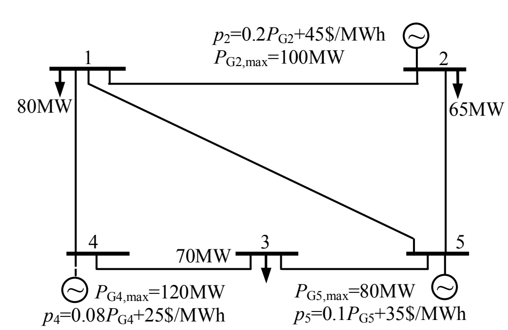

Fig. 2. A revised IEEE-5 bus system

The system consists of five buses and six lines. In pool based model, there are three generators and three loads, whose capacities and bids are shown in Fig. 2. For simplicity, a DC power flow model is adopted.

Under the objective of least operation cost, the unconstrained schedules are given in table I. The system operation cost is 7393.5$/h, and the outputs of G_2, G_4, G_5 are 15 MW、120 MW、80 MW respectively.

TABLE I
CAPACITIES AND POWER FLOWS (MW)

Line	1-2	4-1	4-3	5-1	5-2	5-3
Capacity	30	60	60	35	30	25
Flow	25.79	63.68	56.32	42.10	24.21	13.68

A. Case 1

During actual system operation, there exist capacity limits for lines because of dynamic stability, thermo stability and other security constraints. Given the transmission capacity, line 4-1, 5-1 are overloaded. The constrained schedule is adopted by (1)~(6) to eliminate congestion. In the constrained schedule, the outputs of G_2, G_4, G_5 change to 34.16, 116.25, 64.59 MW respectively; the congestion cost is 176.48$/h, which should be allocated to congested lines 4-1 and 5-1.

Here player 1, 2 represent congested line 4-1 and 5-1 respectively; nonempty coalitions are {1}, {2}, {1,2}. In each coalition, we carry through the OPF calculation taking into account the capacity constrains of the lines involved in the coalition. The congestion costs of each coalitions are $C(1)$=79.42, $C(2)$=108.72, $C(1,2)$=176.48. Then eigenvalues in different coalitions can be obtained through (8): $v(1)$=0, $v(2)$=0, $v(1,2)$=11.66. According to the Shapley-value, the profits each line gets through cooperation are x_1=5.83, x_2=5.83. The final cost allocated to line i is equal to the congestion cost when each line is constrained independently minus the profit each line gets through coalition.

Known from the table II, the results are rational for both individual and integral. For individual, the final allocated cost is not greater than the cost that the line is constrained independently:

$$C'(1) = 73.59 < C(1) = 79.42$$
$$C'(2) = 102.89 < C(2) = 108.72$$

For subcoalition, congestion cost when each line is constrained independently is not greater than that when lines in subcoalition S are constrained together:

$$C(1) \le C(1,2); \quad C(2) \le C(1,2)$$

For integral, the sum of congestion cost allocated to each line is equal to the cost when all lines are constrained together:

$$C'(1) + C'(2) = 176.48 = C(1,2)$$

The results of incremental method and shadow price method are also illustrated in table II. The trend of the proposed method is same to that of shadow price method. However the proposed method balances revenue and expenditures autonomously and dose not create merchandise surplus; while shadow price method produces big errors which need to be corrected to maitain balance of payments. Note that the results of proposed method and incremental method are completely same, which confirms the theory analysis in the part III and appendix.

TABLE II
LINE CONGESTED COST IN DIFFERENT METHODS($/H)

Method	4-1	5-1	MS
Proposed Method	73.59	102.89	0
Incremental Method	73.59	102.89	0
Shadow Price Method	103.15	122.34	49.01

After determinate the congestion cost for each line, we allocate them to transactions. This paper uses power flow tracing to allocate line costs to transactions in three distribution coefficients [11]. The output of G_2 is completely absorbed by L_2, consequently it dose not flow through any line, and allocated cost is zero. The congestion cost for L_3 is zero too, since it only uses lines 1-2, 5-3 which are not overloaded.

TABLE III
CONGESTION COST ALLOCATED TO TRANSACTIONS ($/H)

Coefficient	G_2	G_4	G_5	L_1	L_2	L_3
r=1.0	0	73.59	102.8	0	0	0
r=0.0	0	0	0	148.6	27.86	0
r=0.5	0	36.80	51.45	74.31	13.93	0

B. Case 2

We applied our method in different transmission constrains, generation bids and load levels to verify the validity. In this case the capacity of line 4-3 is set as 50MW; A, B, C and a, b, c represent three generation bids and three load levels:

A: $p_{G2} = 0.2P_2 + 45, p_{G4} = 0.08P_4 + 30, p_{G5} = 0.1P_5 + 40$

B: $p_{G2} = 0.2P_2 + 45, p_{G4} = 0.08P_4 + 25, p_{G5} = 0.1P_5 + 35$

C: $p_{G2} = 0.4P_2 + 45, p_{G4} = 0.08P_4 + 25, p_{G5} = 0.1P_5 + 35$

a: $P_{L1} = 80, P_{L2} = 65, P_{L3} = 60$

b: $P_{L1} = 90, P_{L2} = 65, P_{L3} = 60$

c: $P_{L1} = 80, P_{L2} = 65, P_{L3} = 70$

The test results are summarized into nine groups (table Ⅳ). In each group, three columns represent the proposed method, incremental method and shadow price method; three rows represent line 4-1, 5-1 and 4-3 respectively.

Observe that the results of the proposed method is provided with as same trend as the incremental method, which verifies the consistency of the methods. The differences lie in that Shapley value considers all orders of congestion relief and gets precise balance of payments. However, the incremental method considers two orders of line's joining in a coalition--the first time and the last time. There is calculation error remained in each group. The error would increase with the increase of the number of congested line.

TABLE IV
LINE COSTS IN DIFFERENT GENERATION AND LOADS ($/H)

	a			b			c		
A	54.19	47.17	0.00	74.50	60.93	0.00	24.91	25.71	0.00
	64.14	57.13	131.03	159.20	145.63	395.75	107.10	107.89	214.60
	65.76	58.74	140.46	76.24	62.67	117.71	252.55	253.34	365.62
B	82.44	72.62	0.00	107.18	90.48	0.00	37.77	39.71	0.00
	166.73	156.91	322.28	297.14	280.44	560.71	197.13	199.07	309.28
	94.86	85.04	160.74	104.72	88.02	152.88	349.65	351.59	470.97
C	101.54	84.49	0.0	143.57	115.68	0.00	48.87	49.68	0.00
	262.91	245.86	510.57	496.37	468.48	923.54	327.25	328.07	517.59
	124.00	106.95	227.95	149.09	121.20	230.24	488.40	489.21	702.72

In this case, results of shadow price for 4-1 in nine groups are all zero, since the capacities of 4-3 and 5-1 form strong constrains for 4-1 and it will not exceed its capacity. When OPT is adopted its shadow price is zero, consequently the allocated congestion cost is zero too.

Ordinary Shapley-value method provides many advantages, but the calculated amount would increase sharp when the number of players increase. This restricts the method's application. In power system, there are few lines congested at the same operation time because of strong constrains. The proposed method converts the allocation facing to transactions into the allocation facing to congested lines. Such conversion helps to decrease the calculation for the number of congested lines is far less than that of transactions. The computation speed is increased 2^{T-L} times (T and L are numbers of transactions and lines respectively) compared with the ordinary Shapley-value method facing to transactions. Moreover the proposed method can be applied in bilateral or pool-bilateral model market for the second step is to allocate line costs to transactions, which is, in essence, transmission facility usage determination. There have been enough literature about this point, and we do not introduce details here.

V. CONCLUSION

In this paper a method is proposed to allocate the congestion cost in a pool based model. The congestion cost is firstly allocated to congested lines by Shapley-value; then the line costs are allocated to all market participants according to their utilization degree of congested lines. The method considers all orders of congestion relief and reflects nonlinearity between line capacities, so it complies with the properties of economic efficiency.

The proposed method allocates congestion cost aiming at total power, which makes the revenue collected from the transmission users be equal to the congestion cost. The allocation results are reasonable and easy to be accepted by most market participants.

Differently from methods facing to transactions, the method decomposes the allocation into two steps and Shapley-value is used to determine the line congestion cost. Such conversion retains the advantages of ordinary Shapley-value method as well as improves computation speed. This contributes to it application in actual power system.

VI. APPENDIX

Suppose that two players take part in the coalition problem of congestion cost allocation. When Shapley-value is adopted for calculation, line 1 gets the profit x_1 through the coalition:

$$\begin{aligned} x_1 &= \frac{1}{2}(v(1)-v(0)) + \frac{1}{2}(v(1,2)-v(2)) \\ &= \frac{1}{2}((C(1)-C(1))-0) + \frac{1}{2}((\sum_{i=1}^{2} C(i) - C(1,2)) - 0) \\ &= \frac{1}{2}(C(1)+C(2)-C(1,2)) \end{aligned} \tag{11}$$

The final congestion cost allocated to line 1 is

$$C'(1) = C(1) - x_1 = \frac{1}{2}(C(1) - C(2) + C(1,2)) \tag{12}$$

When incremental method is adopted, the allocation is as followings:

Marginal cost $C_1^{mar} = C(1) - C(0)$

Incremental cost $C_1^{in} = C(1,2) - C(2)$

Final cost $C_1 = \frac{1}{2}(C_1^{mar} + C_1^{in}) = \frac{1}{2}(C(1) + C(1,2) - C(2))$

It can be seen, $C'(1) = C_1$. The same conclusion can be acquired for line 2.

VII. REFERENCES

[1] Y. H. Song, J. X. Sun, "The UK Electricity Market Facing a New Reform," *Power System Technology*, vol. 23, pp. 55-60, Jul. 1999.

[2] S. Harry, S. Y. Hao, and P.Alex, "Transmission congestion management in competitive electricity markets," *IEEE Trans on Power Systems*, vol. 13, pp. 672-680, May. 1998.

[3] M. E. Baran, V. Banunarryanan, and K. E. Garren, "Equitable allocation of congestion relief cost to transactions," *IEEE Trans on Power Systems*, vol. 15, pp. 579-585, May. 2000.

[4] H. F. Xiao, W. D. Li, and L. M. Wei, "Allocation of congestion relief cost based on flow change on the congested facilities," *Proceedings of the CSEE*, vol. 24, pp. 83-87, Feb. 2004.

[5] M. E. Baran, V. Banunarryanan, and K. E. Garren, "A transaction assessment method for allocation of transmission services," *IEEE Trans on Power Systems*, vol. 14, pp. 920-928, Aug. 1999.

[6] H. F. Xiao and W. D. Li, "Fast allocation of congestion relief cost in bilateral transaction framework," *Power System Technology*, vol. 29, pp. 37-42, Mar. 2005.

[7] H. F. Xiao, and W. D. Li, "Fast allocation of congestion relief cost based on flow changes in congested branches," *Proceedings of the CSEE*, vol. 27, pp. 72-78, Feb. 2007.

[8] A. G. Bakirtzis, "Aumann-shapley transmission congestion pricing," *IEEE Power Engineering Review*, vol. 21, pp. 67-69, Mar. 2001.

[9] L. Xu, Y. X. Yu, H. D. Liu, F. Lu, Y. Zhang, and Y. G. Huang, "Double-sided transmission congestion management and quasi-Aumann-Shapley pricing," *Proceedings of the CSEE*, vol. 22, pp. 56-60, Oct. 2002.

[10] Y. Tsukamoto, and I. Iyoda, "Allocation of fixed transmission cost to wheeling transactions by cooperative game theory," *IEEE Trans on Power Systems*, vol. 11, pp. 620-629, May. 1996.

[11] W. D. Li, and L. M. Wei, "Nodal power flows sharing principle during power flows tracing," *Journal of Harbin Institute of Technology*, vol. 35, pp. 491-494, Aug. 2003.

VIII. BIOGRAPHIES

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