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New Method Based on Load Flow with Step Size Optimization for Calculating the Maximum Loading Point

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Abstract-- **In this paper, an improved maximum loading point (MLP) calculation method is proposed. The calculation process is based on the properties of the normal vector to the feasibility boundary computed close to MLP, which is calculated by a load flow method with step size optimization (LFSSO). The process is characterized by obtaining consecutive approximations of the MLP within the infeasible region. Since the feasibility boundary contour in the neighborhood of the MLP may not be smooth, some of the computed approximations may fall within the feasible region. In an earlier paper, a mechanism based on binary search was used to drive the operating point back to the infeasible region. A new load curtailment method to improve the proximity towards feasibility boundary guaranteeing that the next solution will lay within the infeasibility region is the main contribution of this paper. Reactive power generation limits are taken into account. The proposed formulation requires information as the normal vector and power mismatches, and results in a better convergence path towards MLP in comparison with the original version of the method presented earlier. Simulation results for IEEE test systems are shown to validate the proposed method.**

Index Terms-- **Maximum loading point, voltage stability, load flow analysis, step size optimization.**

I. Introduction

In recent years, the increase in peak load demand and power transfers between utilities has led to an increased concern about power systems voltage security. This phenomenon has been deemed responsible for several major disturbances and significant research efforts have been made to further understand voltage phenomena [1]. The voltage instability process is characterized by a monotonic voltage drop, which is slow at first and becomes abrupt after some time. Voltage collapse occurs when the system is unable to meet the demand, and the phenomenon is characterized by the loss of control of the voltages levels. Voltage collapse is generally precipitated by system disturbances, such as load variations, contingencies, or both.

Voltage stability is essentially a dynamic phenomenon, and the system's behavior depends on the models of the loads and other system components. However, analyses based on static approaches present some practical advantages over the dynamical approaches [2]. Analyses based on static approaches have been widely used, since they provide results with acceptable accuracy and little computational effort. These features are desirable in restrictive environments from the computational effort standpoint, such as in a real-time operation environment.

Another difficult problem is related to solving ill-conditioned systems or determining the existence of load flow solutions. Whenever the iterative process diverges or oscillates using the conventional load flow calculation methods, one could not be sure whether the given load flow equations (*i*) have no solution from the initial estimate, or (*ii*) the iterative process did not converge due to numerical problems though some solution exists. In [3], a step size optimization factor is computed at each iteration and is multiplied by the voltage correction vector so as to minimize a quadratic function based on the power mismatches. This method worked very well, however the voltages appeared in rectangular coordinates, which is not a common feature of production grade load flows programs. An approach based on the representation of voltages in polar coordinates was proposed in [4] and its advantages have already been demonstrated [5].

Recently in [6], the authors recommended the implementation of the optimal multiplier modification to the Newton-Raphson load flow method with polar coordinates (rather than rectangular coordinates) to get the fastest, most robust performance, regardless of system solvability or size.

A particular difficulty of voltage stability analysis is the singularity of the Newton-Raphson load flow Jacobian matrix at the steady state voltage stability limit. In fact, this stability limit, also called the critical point or Maximum Loading Point (MLP), is often defined as the point where the load flow Jacobian matrix is singular.

The goal of this work is to use the LFSSO in polar coordinates for calculating the MLP, extending the ideas and load curtailment techniques of [7]. According to method [7], whenever the solution falls inside the feasible region, a mechanism based on binary search is used to drive the operating point back to the infeasible region. A new load curtailment method to improve the proximity towards feasibility boundary guaranteeing that the next solution will lay within the infeasibility region is the main contribution of this paper.

II. Theoretical Concepts

A. Load flow formulation

The load flow equations are formulated as

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$$\mathbf{g}(\mathbf{x},\rho)=\mathbf{0}\ , \quad (1)$$

where $\mathbf{x}\in\Re^{(2nPQ+nPV)}$ is vector of state variables, $\mathbf{x}=[\boldsymbol{\theta}^t\ \mathbf{V}^t]^t$, also $\boldsymbol{\theta}\in\Re^{nPQ+nPV}$ and $\mathbf{V}\in\Re^{nPQ}$ are vectors of bus voltage angles and magnitudes, respectively; $\rho\in\Re$ is the loading factor; *nPQ* and *nPV* are the number of *PQ* and *PV* buses, respectively; $\mathbf{g}(\mathbf{x},\rho)$ is defined as $\mathbf{g}=[\Delta\mathbf{P}^t\ \ \Delta\mathbf{Q}^t]^t$, where $\Delta\mathbf{P}\in\Re^{nPQ+nPV}$ and $\Delta\mathbf{Q}\in\Re^{nPQ}$ are the power mismatches. Equation (1) can also be written as

$$\mathbf{g}(\mathbf{x},\rho)=\begin{bmatrix}\Delta\mathbf{P}(\mathbf{x},\rho)\\ \Delta\mathbf{Q}(\mathbf{x},\rho)\end{bmatrix}=\begin{bmatrix}\mathbf{P}_{sch}(\rho)-\mathbf{P}_{cal}(\mathbf{x})\\ \mathbf{Q}_{sch}(\rho)-\mathbf{Q}_{cal}(\mathbf{x})\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\end{bmatrix}, \quad (2)$$

where subscripts *sch* and *cal* stand for scheduled and calculated terms, respectively. Also, the reactive power generation limits are taken into account, so *PV* buses are switched to *PQ* whenever some limit is reached and can be switched back to *PV* whenever appropriate.

B. Maximum Loading Point

In this paper a constant direction of generation and load increase is considered, which is defined as proportional to the base case, so $\mathbf{P}_{sch}=\rho\mathbf{P}_{sch\text{-}bc}$ and $\mathbf{Q}_{sch}=\rho\mathbf{Q}_{sch\text{-}bc}$ where *bc* is base case (for $\rho_{bc}=1$). Also, $\mathbf{P}_{sch\text{-}bc}=\mathbf{P}_{g\text{-}bc}-\mathbf{P}_{l\text{-}bc}$ and $\mathbf{Q}_{sch\text{-}bc}=\mathbf{Q}_{g\text{-}bc}-\mathbf{Q}_{l\text{-}bc}$ where *l* and *g* are associated to load power and generation, respectively. This load increase direction (constant power factor) is usually adopted by utilities and regulatory agencies for the definition of secure loading margins [8,9].

The loading factor reaches its maximum value $\rho=\rho_{cr}$ (*cr* stands for critical point) on the voltage stability boundary Σ. This point is usually called the maximum loading point (MLP). Boundary Σ divides the space into two regions: (*i*) region where there are two solutions for system (1), or feasible region; and (*ii*) region where (1) cannot be solved, or infeasible region.

C. Load Flow Method with Step Size Optimization

LFSSO was first developed for solving the load flow equations for ill-conditioned power systems. For those, the conventional load flow methods exhibit poorer performance, or simply diverge, although the system indeed operates in a stable equilibrium point. This idea was first presented in [3], where the voltages were represented in rectangular coordinates. In [4], an approach based on the representation of voltages in polar coordinates was proposed.

At the *r*th iteration of the LFSSO (assuming ρ fixed), the state variable vector $\mathbf{x}^{(r+1)}$ is calculated as

$$\begin{aligned}\mathbf{x}^{(r+1)}&=\mathbf{x}^{(r)}+\mu^{(r)}\,\Delta\mathbf{x}^{(r)}\,,\\ \Delta\mathbf{x}^{(r)}&=-\left[\nabla_{\mathbf{x}}\mathbf{g}\right]^{-1}\Big|_{\mathbf{x}=\mathbf{x}^{(r)}}\mathbf{g}(\mathbf{x}^{(r)},\rho)\,,\end{aligned} \quad (3)$$

where $\mu^{(r)}$ is a step size optimization factor that multiplies the state variable deviation vector $\Delta\mathbf{x}^{(r)}$ in each iteration *r*; $\nabla_{\mathbf{x}}\mathbf{g}$ is the Jacobian matrix of $\mathbf{g}$. Also, μ is computed to minimize the following quadratic function based on the power mismatches.

$$\min \quad F(\mu)=\tfrac{1}{2}\left\|\mathbf{g}_{st}\right\|_2^2=\tfrac{1}{2}\sum_{i\in\Omega\mathbf{g}} g_{ts,i}^2\ , \quad (4)$$

where $\mathbf{g}_{ts}$ is $\mathbf{g}$ expanded in Taylor series, considering up to the second-order term, as

$$\mathbf{g}_{ts}(\mu)=\mathbf{g}(\mathbf{x}^{(r)},\rho)+\mu\nabla_{\mathbf{x}}\mathbf{g}(\mathbf{x}^{(r)})^t\Delta\mathbf{x}^{(r)}+\mu^2T(\mathbf{x}^{(r)})\,. \quad (5)$$

Also, $T(\mathbf{x})$ corresponds to second order terms of $\mathbf{g}$, given by

$$T(\mathbf{x})=\frac{1}{2}\left(\sum_{i\in\Omega\mathbf{g}}\Delta x_i\frac{\partial}{\partial x_i}\right)^2\mathbf{g}(\mathbf{x})\,. \quad (6)$$

Substituting (5) in (4) and applying the local minimum condition $\partial F/\partial\mu=0$, a cubic equation is obtained and solved for μ.

For well-conditioned systems, μ assumes values close to one and does not affect the iterative process in a significant way. In the case of ill-conditioned systems, μ assumes values such that the iterative process is smoothed out and the solution is obtained, while the conventional Newton method would have failed.

D. Applications of LFSSO for Voltage Stability Analysis

For the cases outside the feasibility region (either due to an excessive loading or to a contingency), μ assumes very low values (theoretically $\mu\to 0$). Overbye [10] showed that LFSSO leads to a point on the feasibility boundary Σ rather than to simply diverge. Note that the Jacobian matrix is singular on Σ, therefore the step sizes $\Delta\mathbf{x}$ are large in its vicinity. However, the convergence of LFSSO is not affected thanks to μ ($\mu\Delta\mathrm{x}\to 0$). With this information (points on boundary Σ), further applications of the LFSSO (as to calculate the MLP and security margins for voltage stability) can be proposed.

E. Load Curtailment Techniques

In [7], information about the boundary Σ is used for developing applications for the LFSSO (as to calculate the MLP). Fig. 1 shows the general behavior of LFSSO for an excessive ρ (i.e. $\rho>\rho_{cr}$) in load parameter space with $\mathbf{s}_{sch}$ direction, which is a unitary vector in the direction of load increase defined by vector $\mathbf{S}_{sch}$, where $\mathbf{S}_{sch}=[\mathbf{P}_{sch}{}^t\ \mathbf{Q}_{sch}{}^t]^t$.

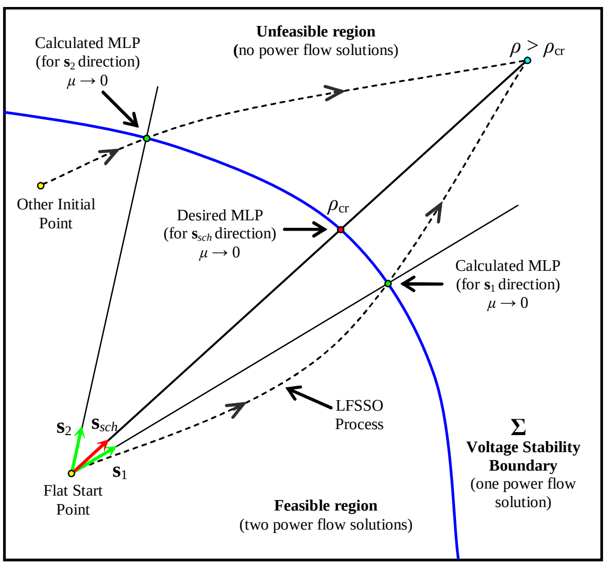

Fig. 1. LFSSO features in load parameter space [7].

Also in [11], the normal vector to boundary Σ is calculated at the last MLP. This normal vector $\mathbf{w}$ is calculated from (1) as

$$\begin{aligned}\nabla_{\mathbf{x}}\mathbf{g}(\mathbf{x}_{mlp})^t\ \mathbf{w}&=\mathbf{0}\\ \left\|\mathbf{w}\right\|_2&=1\end{aligned} \quad (7)$$

where $\mathbf{x}_{mlp}$ is the state variable vector at the last MLP; $\|\mathbf{w}\|_2$

is the Euclidean norm of $\mathbf{w}$, so $\mathbf{w}$ is a unit vector.

According to [7], considering the two-dimension load parameter space shown in Fig. 2, information on the last calculated MLP and (7) can be used to calculate the unit normal vector $\mathbf{w}$ to the boundary Σ at this point. Finally, a load curtailment $\Delta\mathbf{S}_{lc,1}$ is calculated as

$$\Delta\mathbf{S}_{lc,1} = \frac{\langle \Delta\mathbf{S}, \mathbf{w} \rangle}{\cos\beta}\mathbf{s}_{sch}\,, \qquad (8)$$

where $\Delta\mathbf{S} \in \Re^{(2nPQ+nPV)}$ is power mismatches vector, $\Delta\mathbf{S} = [\Delta\mathbf{P}^t\ \Delta\mathbf{Q}^t]^t$; $\langle \Delta\mathbf{S},\mathbf{w}\rangle$ is a dot product of $\Delta\mathbf{S}$ and $\mathbf{w}$; β is the angle between $\mathbf{s}_{sch}$ and $\mathbf{w}$, so $\cos\beta = \langle \mathbf{s}_{sch},\mathbf{w}\rangle$. The loading factor deviation $\Delta\rho \in \Re$ due to load curtailment step is calculated as

$$\Delta\rho = \frac{\left\|\Delta\mathbf{S}_{lc,1}\right\|_2}{\left\|\mathbf{S}_{sch-bc}\right\|_2} = \frac{\langle \Delta\mathbf{S}, \mathbf{w} \rangle}{\cos\beta}\frac{1}{\left\|\mathbf{S}_{sch-bc}\right\|_2} = \frac{\langle \Delta\mathbf{S}, \mathbf{w} \rangle}{\langle \mathbf{S}_{sch-bc}, \mathbf{w} \rangle}\,. \qquad (9)$$

So, the new estimation of ρ will be $\rho_{new} = \rho - \Delta\rho$.

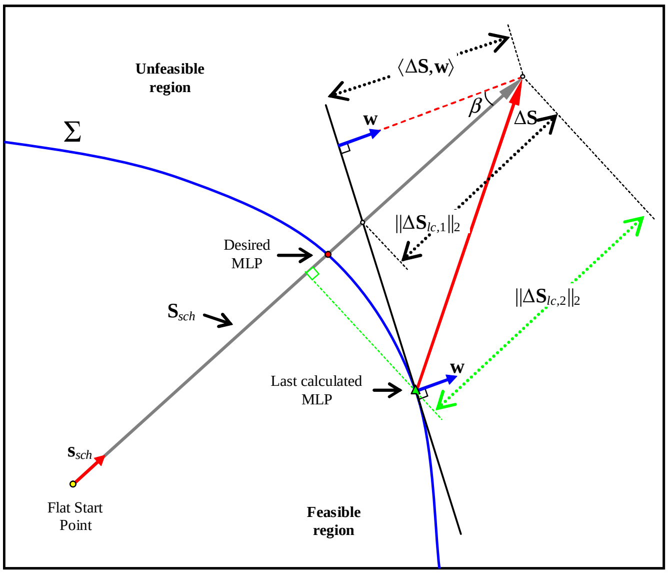

Fig. 2. Load curtailment step to the last calculated MLP.

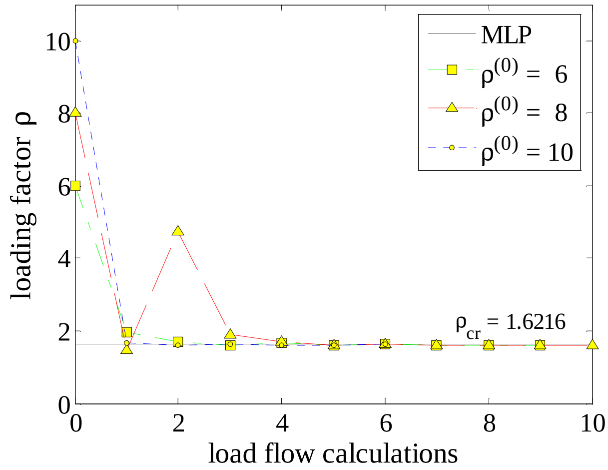

Fig. 3. IEEE 57-bus system – process of computing MLP with method [7].

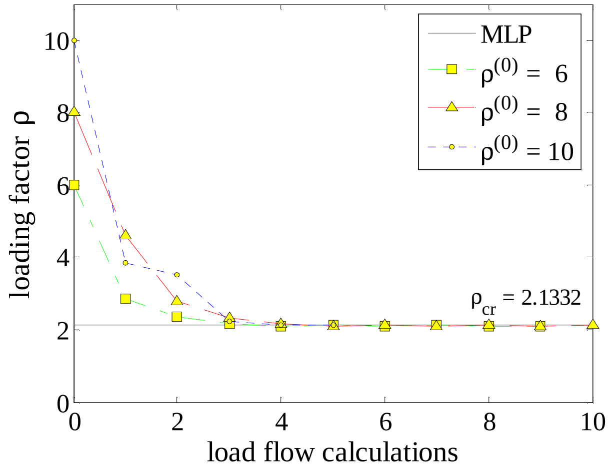

Fig. 4. IEEE 118-bus system – process of computing MLP with method [7].

III. Proposed Method for Computing MLP

Simulation results for IEEE test systems up to 300 buses showed that the method of [7] presents good performance and allows the MLP computation with less iterations, if compared with other methods. However, a detailed analysis of the performance of method [7] showed that it is possible to improve it by extending its idea.

A. *Performance of method [7] within the feasible region*

The main feature of method [7] is that the sequence of operating points is usually within the infeasible region. However, there are situations, especially for robust systems and severe contingencies, where the load curtailment frequently leads to feasible points. The result is an increase in the computational burden, since binary search is used for returning to infeasibility.

Figs. 3 and 4 show some simulations results for the IEEE 57- and 118-bus systems using the method [7].

Note that right in the first iteration a point very close to ρ_{cr} is obtained, showing the efficiency of the method. However, for $\rho^{(0)} = 8$ the point lays within the feasible region in Fig. 3. Fig. 5 shows this situation more clearly, since the region around the first iteration is zoomed in. For $\rho^{(0)} = 8$ the point computed by method [7] lays below the continuous line corresponding to ρ_{cr}, that is, within the feasible region. Analogously, Fig. 6 corresponds to zooming in part of Fig. 4, where a point also lays within the feasible region. According to method [7], whenever a feasible point is obtained, binary search is used in order to search for new infeasible points. The main goal of the proposed method is to deal with such situations more efficiently, since binary seach may result in a larger number of iterations. Therefore, an extended load curtailment technique is proposed here.

B. *Conservative Load Curtailment Technique*

Since boundary Σ may not be smooth, it is possible that a load curtailment $\Delta\rho$ computed by (9) leads to a point into the feasible region. In this case, an extended load curtailment is carried out.

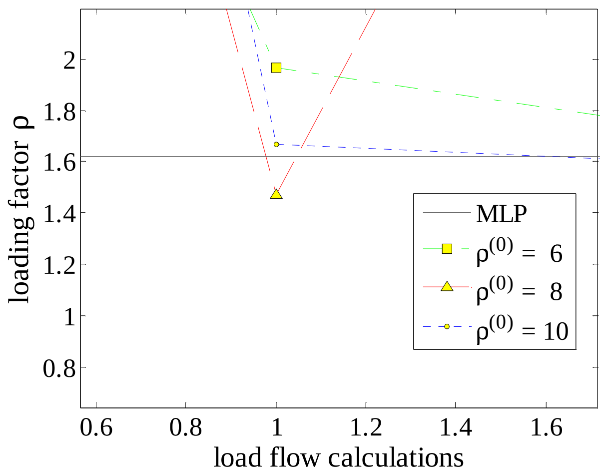

Fig. 5. IEEE 57-bus system – zoom of Fig. 3 around the first iteration.

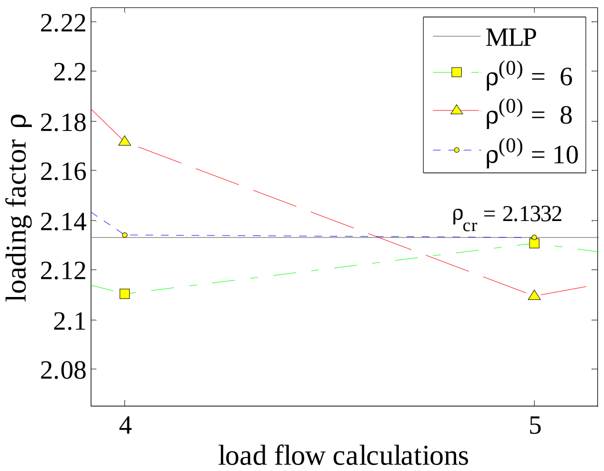

Fig. 6. IEEE 118-bus system – zoom of Fig. 4 around iterations 4 and 5.

As shown in Fig. 2, load curtailment $\Delta\mathbf{S}_{lc,2}$ can be obtained using similar information, and it is calculated as

$$\Delta\mathbf{S}_{lc,2} = \langle \Delta\mathbf{S}, \mathbf{s}_{sch} \rangle \mathbf{s}_{sch}, \qquad (10)$$

where $\langle \Delta\mathbf{S}, \mathbf{s}_{sch} \rangle$ is the dot product of $\Delta\mathbf{S}$ and $\mathbf{s}_{sch}$.

Remarks

Exhaustive simulations with test power systems showed that method [7], with load curtailment $\Delta S_{lc,1}$, presents good performance since it seeks to approximate the solution near to boundary Σ. However, it assumes a risk falling within the feasible region.

Depending on the curvature of Σ, one may have $\|\Delta S_{lc,2}\|_2$ smaller or larger than $\|\Delta S_{lc,1}\|_2$, but for the systems tested in this paper, $\|\Delta S_{lc,2}\|_2 < \|\Delta S_{lc,1}\|_2$ was obtained in most cases.

Additionally, simulations showed that the process using $\Delta S_{lc,2}$ only always results in points within the infeasible region and its performance is poorer than the method [7]. When $\Delta S_{lc,2}$ is applied, it can result in accuracy problems and tend to wrong loading factors to the required tolerance (e.g. off-set close to the solution). Even though $\Delta S_{lc,2}$ is conservative, some characteristics of its performance can be used to improve the numerical stability of method [7].

C. *Extended Load Curtailment Technique*

The extended load curtailment is based on combining (8) and (10), as

$$\begin{aligned}\Delta\mathbf{S}_{lc,new} &= \frac{1}{2}\left(\Delta\mathbf{S}_{lc,1} + \Delta\mathbf{S}_{lc,2}\right)\\ &= \frac{1}{2}\left(\frac{\langle \Delta\mathbf{S}, \mathbf{w} \rangle}{\cos\beta} + \langle \Delta\mathbf{S}, \mathbf{s}_{sch} \rangle\right)\mathbf{s}_{sch}\end{aligned}, \qquad (11)$$

and the resulting loading factor change $\Delta\rho$ due to new load curtailment (11) will be

$$\Delta\rho = \frac{\|\Delta\mathbf{S}_{lc,new}\|_2}{\|\mathbf{S}_{sch-bc}\|_2} = \frac{1}{2}\left(\frac{\langle \Delta\mathbf{S}, \mathbf{w} \rangle}{\langle \mathbf{S}_{sch-bc}, \mathbf{w} \rangle} + \frac{\langle \Delta\mathbf{S}, \mathbf{s}_{sch} \rangle}{\|\mathbf{S}_{sch-bc}\|_2}\right). \qquad (12)$$

In order to guarantee that the load curtailment process will remain in infeasible region, the following modification in (12) is proposed.

$$\Delta\rho = \frac{1}{2}\left(\frac{\langle \Delta\mathbf{S}, \mathbf{w} \rangle}{\langle \mathbf{S}_{sch-bc}, \mathbf{w} \rangle} + \eta\frac{\langle \Delta\mathbf{S}, \mathbf{s}_{sch} \rangle}{\|\mathbf{S}_{sch-bc}\|_2}\right), \qquad (13)$$

where η is a system-dependent parameter.

The implementation of such changes ((11)-(13)) in the original LFSSO algorithm is straightforward.

Remarks

It is important to point out that load curtailment (13) will be applied only when the process falls within the feasible region, thus (13) is used for calculating a new ρ_{IR} (loading factor supposedly within the infeasible region) which replaces the binary search approach.

This effective load adjustment produces an operating point close to the boundary Σ, resulting in a more efficient calculating process of MLP, with a smaller number of iterations when the loading point is located in the feasible region. The performance considering limits on reactive power generation is similar to the one without considering such limits.

IV. Test Results

The proposed method has been tested for several IEEE test systems. The LFSSO convergence tolerance ε_s was set to 0.001 MW/MVAr, the loading factor tolerance ε_ρ was set to 10^{-2} and the step size threshold μ_{min} to 10^{-2} for IEEE 57-bus, μ_{min} to 0.2 for IEEE 118-bus and μ_{min} to 0.5 for IEEE 118-bus with contingence. Reactive power generation limits at generation buses were considered. Parameter η was set to one, except mentioned otherwise.

For simulation purposes, different infeasible initial points $\rho^{(0)}$ were chosen as 6, 8, and 10. Simulations with IEEE 57- and 118-bus systems were carried out, since for these systems the method of [7] frequently led to points within the feasible region. Also, the IEEE 118-bus system was tested under a contingency situation. It is important to stress out that the efficiency of method [7] relies heavily on obtaining infeasible points. The idea of the proposed method is to keep the points within the infeasible region as much as possible, in order to obtain an overall computational effort reduction.

Figs. 7 and 8 shows the process of obtaining the MLP for the IEEE 57- and 118-bus systems, respectively (data obtained from [12]) using the proposed method with the extended load curtailment technique. The gain is clear, since the number of feasible points (below the straight line) is smaller, and so is the number of iterations, when the proposed method is used. The precision of the results was satisfactory when compared to the results from method [13], even though the latter resulted in a heavier computational effort.

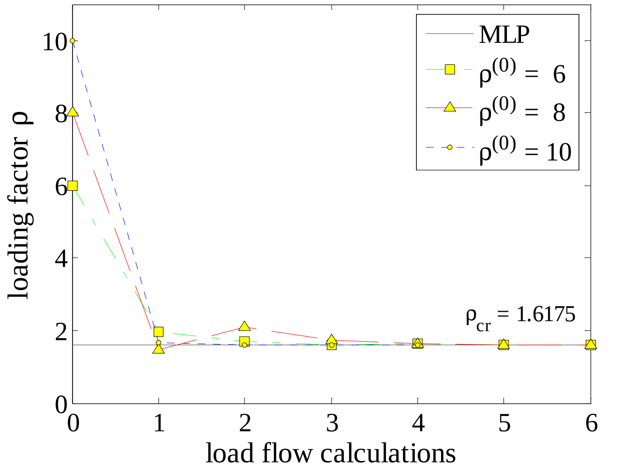

Fig. 7. IEEE 57-bus system – process of computing MLP with proposed method.

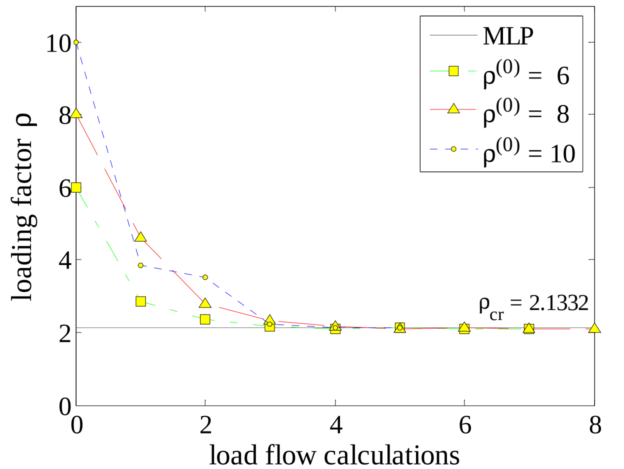

Fig. 8. IEEE 118-bus system – process of computing MLP with proposed method.

Tables I and II show the performance of the proposed method and a comparison with the method [7] for the IEEE 57- and 118-bus systems, respectively. Also, the results obtained by the method [13] (continuation load flow - CLF) are shown for precision comparison purposes. Note that the closer $\rho^{(0)}$ is from ρ_{cr}, the better is the performance of the proposed method. The number of points in the feasible region (FR) and infeasible region (IR) were also shown.

TABLE I
SIMULATION RESULTS FOR THE IEEE 57-BUS SYSTEM

Initial ρ	Number of LF								CLF ρ
	Method [7]				Proposed Method				
	FR	IR	Total	Computed ρ	FR	IR	Total	Computed ρ	
6	4	5	9	1.6173	2	4	6	1.6025	
8	4	6	10	1.6097	2	4	6	1.6075	1.6168
10	3	3	6	1.6216	2	2	4	1.6175	

TABLE II
SIMULATION RESULTS FOR THE IEEE 118-BUS SYSTEM

Initial ρ	Number of LF								CLF ρ
	Method [7]				Proposed Method				
	FR	IR	Total	Computed ρ	FR	IR	Total	Computed ρ	
6	3	6	9	2.1134	2	5	7	2.1116	
8	3	7	10	2.1153	2	6	8	2.112	2.11
10	0	5	5	2.1332	0	5	5	2.1332	

Note in Table II that for $\rho^{(0)}$ = 10 the number of iterations was the same for both methods, since the points obtained by method [7] were all infeasible. Therefore, the proposed load curtailment was not used.

Comparing Figs. 3 and 7 for $\rho^{(0)}$ = 8, note that the proposed method leads to a better point in the second iteration. This is the detail the implies in a better performance of the proposed method.

Figs. 9 and 10 shows some results for the IEEE 118-bus system after the outage of transformer 8-5 using method [7] and the proposed method.

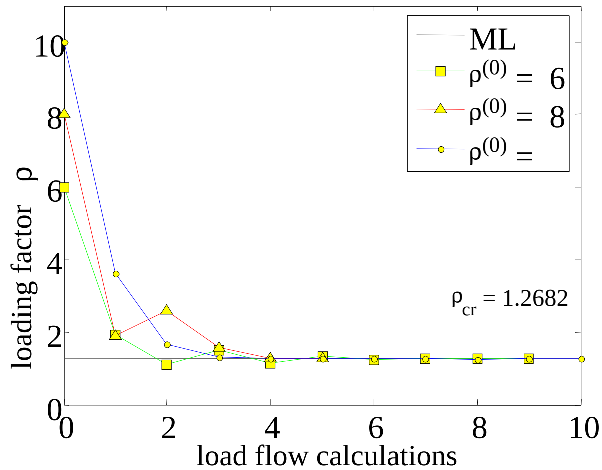

Fig. 9. IEEE 118-bus system – process of computing MLP after the outage of transformer 8-5 using [7].

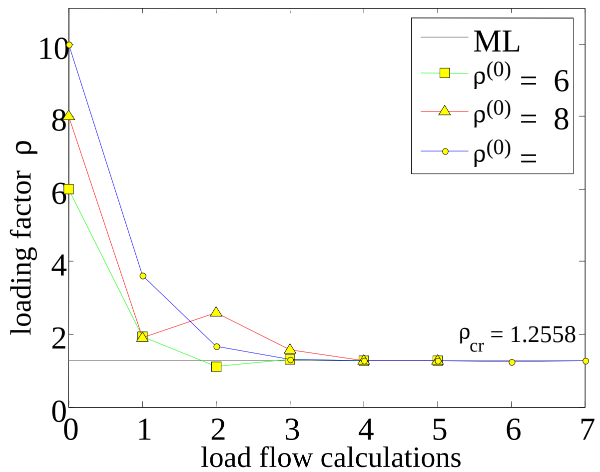

Fig. 10. IEEE 118-bus system – process of computing MLP after the outage of transformer 8-5 using the proposed method.

Table III summarizes the results. The better performance of the proposed method is clear. Method [7] applied to contingency cases tended to drive the loading factor into the feasibility region more often. As mentioned earlier, this leads to a larger number of iterations. The proposed method resulted in less iterations. Figs, 11 and 12 show details of Figs. 9 and 10 by zooming in the images for some iterations. The gain provided by the proposed method is clear after the second iteration, where the feasibility region is reached. The proposed load curtailment is more efficient since it leads to an infeasible point, closer to the feasibility boundary.

TABLE III
SIMULATION RESULTS FOR THE IEEE 118-BUS SYSTEM WITH A CONTINGENCE IN 8-5

Initial ρ	Number of LF							
	Method [7]				Proposed Method			
	FR	IR	Total	Computed ρ	FR	IR	Total	Computed ρ
6	4	5	9	1.2717	2	3	5	1.2702
8	0	5	5	1.2629	0	5	5	1.2629
10	4	6	10	1.2682	2	5	7	1.2558

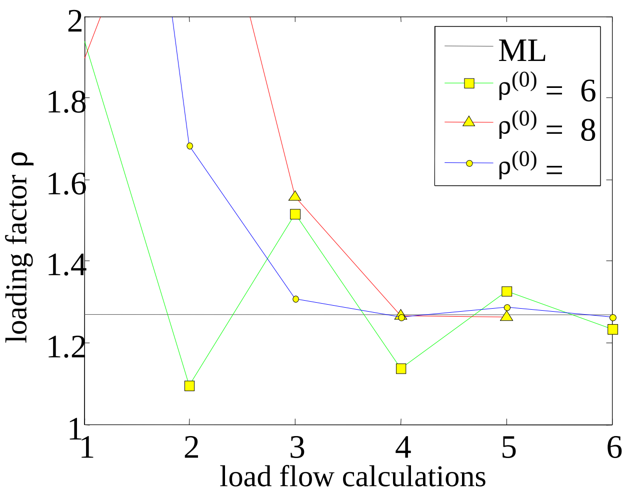

Fig. 11. IEEE 118-bus system – zooming in the process of computing MLP after the outage of transformer 8-5, using [7].

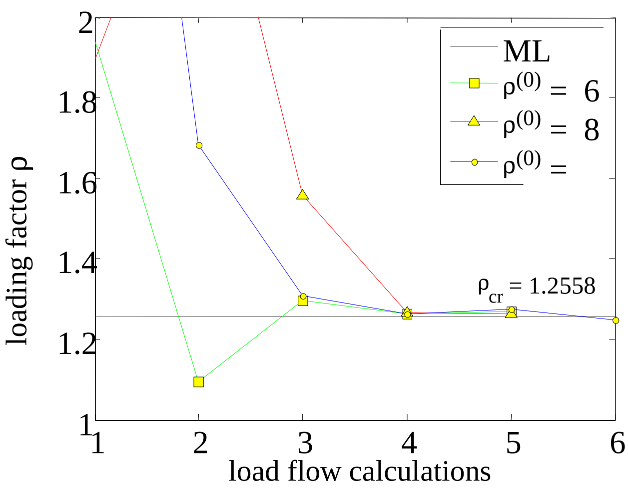

Fig. 12. IEEE 118-bus system – zooming in the process of computing MLP after the outage of transformer 8-5 using the proposed method

Method [7] leads to points very close to the feasibility boundary, however, there is always the risk of a feasible point be obtained during the process. Rather than using binary search, the proposed algorithm uses an extended load curtailment that uses a more conservative measure ($\|\Delta S_{lc,2}\|_2$) in addition to $\|\Delta S_{lc,1}\|_2$, resulting in a better estimate for the next point. This contribution resulted very important for the efficiency of the proposed method.

In general, the proposed method is more efficient than the method [7], resulting in a smaller number of iterations. This is especially true for larger systems and severe contingency situations.

V. CONCLUSIONS

The main idea of this research work was to propose an improvement of the method proposed in [7], by changing the load curtailment process and preserving the main feature of method [7], which relies on obtaining a sequence of points in the infeasible region.

Information on the last MLP computed by the LFSSO and the normal vector to the feasibility boundary are used to define a load curtailment. Usually the next point is also in the infeasible region, which makes the method very efficient. Whenever a point falls within the feasible region, an extended load curtailment is proposed to improve the overall efficiency of the process. The result is a very efficient calculation process, with a very small number of iterations. The proposed method is even better for larger systems and severe contingency situations.

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VII. Biographies

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