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Risk Assessment of Wind Power Generation Project Investments based on Real Options

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Abstract--**This paper presents a decision-making tool for investment in a wind energy plant using a real options approach.**

In the first part of the work, the volatilities of market prices and wind regimes are obtained from Geometric Brownian motion with Mean Reversion (GBM-MR) and Weibull models, respectively. From these and other values, such as investment and maintenance costs, the Net Present Value (NPV) curve (made up of different values of NPV in different periods) of the investment is calculated, as well as its average volatility.

In the second part, a real options valuation method is applied to calculate the value of the option to invest. The volatility of the NPV curve reflecting different periods is inserted into a trinomial investment option valuation tree. In this way, it's possible to calculate the probabilities of investing right now, deferring the investment, or not investing at all. This powerful decision tool allows wind energy investors to decide whether to invest in many different scenarios. Several realistic case studies are presented to illustrate the decision-making method.

Index Terms--**Risk management, wind energy, real options, Monte Carlo.**

I. Introduction

THE world is affected by the incessant demand increase of all types of energy, where prices of raw materials (oil in particular) are in a constant state of turmoil. Due to the instability of fossil fuel-based energy prices over the last years, the need to reduce dependence on conventional energy sources and to protect the environment has fomented the use of clean sources of energy.

Renewable energy sources have been supported and subsidized by different countries. Among them, wind energy seems to be the most successful in terms of market penetration. Increasing generation capacity in liberalized power markets requires the producers to account for future long-term uncertainties. In particular, for wind producers, the inherent intermittency of its energy production, especially due to the wind regime, makes it necessary to use stochastic models to decide upon future investments. Due to the considerable lifetime of these plants and the mentioned uncertainties, traditional discounted cash flow methods used to obtain the Net Present Value (NPV) of the investment are not suitable for these projects.

To take into account uncertainty, real options models can be used to develop flexible investment strategies. Depending on the present value in a certain time, the decision maker can decide to execute, wait, or abandon the construction project of a power plant.

This work focuses on a detailed decision making tool to evaluate a project a wind power plant building project under uncertainty using the real options methodology.

First, the developed tool allows an investor to estimate future cash flows and NPVs derived from the construction of the plant and its future operation in the market. To obtain the cash flows there are several steps that are required. First, the wind speed is estimated by the parameterization of a Weibull function; in addition, the production curve of a wind turbine generates the energy as a function of the wind speed. The electricity prices to remunerate the wind producer are modeled using Geometric Brownian Motion with Mean Reversion (GBM-MR). It's assumed that the wind generations are paid at the spot prices, but any incentive scheme can be easily added to the model. With all this, plus the investment cost estimation, the stochastic cash flows and NPVs are obtained. By shifting the initial time for investment, the model allows to estimate the NPV curves. These curves represent the evolution of the NPVs that would result if the investment were done at different times.

Secondly, the real options methodology is applied using the estimated parameters of the stochastic NPV curves. A trinomial investment decision tree is built, where the values of the parameters are only valid for specific time intervals, since a unique GBM-MR model only holds for prices (not NPVs) during the entire lifetime of the project. To evaluate the attractiveness of the investment, the American option of the project (that allows investing before the expiration of the option takes place) is formulated, as well as the probabilities of three possible alternatives: execute, wait, or abandon. In short, the real options method provides the investor with the capacity to evaluate the price of the option to invest in a project.

The remaining sections of the paper are organized as follows. Section II revises the current literature on real options applications to electricity, and, in particular, to generation investment and renewable energy. Section III outlines the stochastic modeling of the wind plant production, the GBM-MR model of the electricity price and its parameter estimation.

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Section IV introduces the concept of NPV curve and the piecewise estimation of its parameters and shows the effect of risk aversion in the model. Section V shows the formation of the trinomial decision tree from the estimated parameters of the GBM-MR model of the NPV curves. Additionally, it depicts the value of the American option to invest and the calculation of the probabilities to invest now, to wait or to abandon. Section VI provides an illustrative example and several results, and Section VII expounds the conclusions.

II. Literature review

Distributed generation has become an important source of energy in liberalized electricity markets due to its benefits, such as loss reduction, deferment of grid investments and its ability to reduce emissions to comply with the Kyoto protocol [1]. The impact of renewable energy is traditionally measured from the societal perspective and focuses on social welfare and other overall system benefits [2]. Another approach is to assume that the builder of a new wind plant wants to maximize its own profits and has to decide whether to build or not and when to do it. This requires careful evaluation of the financial aspects involved [3] and adequate analytical decision tools are needed.

Real options analysis is widely used as a tool to help decision making in many fields [4], [5]. It is successfully applied both in operation and planning of power systems: i) to perform a generation asset valuation and ii) to find optimal timing for new generation investments under uncertainty.

As regard to the first application, the operation of a power plant can be studied from a generation asset perspective, where fuel and electricity prices are modeled using mean reverting processes [6], [7]. Financial option theory is used to value the plant using the concept of Value at Risk. Sophisticated models add operational constraints, as in [8], or define multi-stage stochastic models of the operational decisions, with two-factor lattices describing fuel and electricity prices, such as in [9]. Another multi-stage investment under market uncertainty model, where the authors use a stochastic process to estimate the power plant's present value, is shown in [10].

Secondly, the problem of investment is addressed using the real options approach. In [11], spark spread and emission costs are used to value a gas-fired plant. The same authors study the upgrade of a base-load plant in [12] into a peak-load plant. The evaluation of an investment for two inter-related plant projects is described in [13], where a quadrinomial tree represents the market value of the units. In [14], a real options method to evaluate investments in renewable generation, including wind generation, makes extensive use of price volatility to decide upon investment timing. In addition, the NPV break-even price is determined under price uncertainty.

III. Stochastic modeling of wind speed and electricity price

This section is devoted to explaining the main stochastic models that are used in the investment valuation of a wind plant. First, the wind speed estimation model and the wind energy production curves are estimated. Secondly, the electricity prices resulting from a GBM-MR model are shown.

A. Wind speed estimation and wind energy production curve

The probability function that most closely resembles the wind speed regime is the Weibull distribution function as a result of the orthogonal composition of two correlated Gaussian functions. The Weibull density function is given by:

$$f(x,k,\lambda) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{\left(x/\lambda\right)^k}, \quad (1)$$

where λ and k are the scale and shape factors, respectively. The λ parameter refers to the maximum speed and the k parameter indicates the degree of dispersion of the samples.

To estimate wind energy production it is necessary to evaluate the amount of power that goes through the rotor with the formula:

$$P = N_t \cdot 0.5 \cdot C_P \cdot \rho \cdot V_c^3 \cdot \pi \cdot r^2, \quad (2)$$

where the power produced depends on the number of turbines, N_t, the Betz coefficient, C_p, the air density, ρ, the wind speed, V_c, and the blade radius, r. The hourly energy production is obtained by using the stochastic wind speed values as inputs in (2). Fig. 1 shows the hourly production curve of a wind turbine whose maximum power is 10 MW, for a 1-month period.

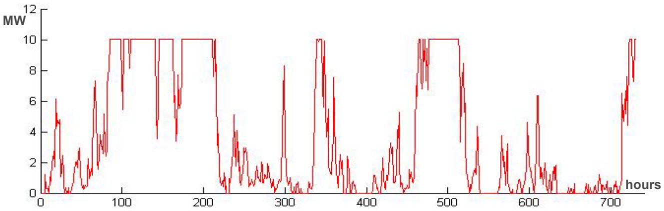

Fig. 1. 10 MW wind plant production curve over 1 month.

To make this curve useful for a planning horizon of 20 years, which is the estimated lifetime of a wind plant, the hourly curve in Fig. 1 is transformed into a monthly curve, as shown in Fig. 2.

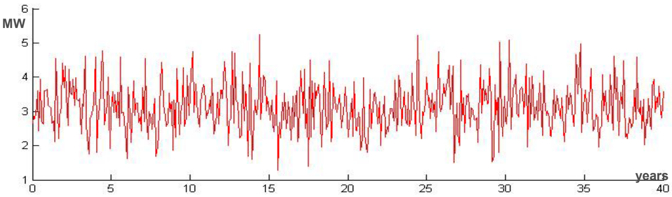

Fig. 2. Estimated monthly wind power production curve over 40 years.

B. Electricity price model and parameter estimation

Due to the inherent uncertainty of the parameters involved in the estimation of the NPV of a project, it is advisable to reproduce its behavior by means of stochastic processes. One of the most commonly used models is the Geometric Brownian Motion with Mean Reversion (GBM-MR) model.

$$\frac{dx}{x} = \lambda\left[\phi - \ln(x)\right]dt + \sigma dz, \quad (3)$$

where the price $x(t)$ follows a stochastic process, λ corresponds to the speed of adjustment of the reversion, ϕ is the average long-term value, σ is the standard deviation of the process, and z defines a Wiener process.

To facilitate the calculation of the value of the parameters of the GBM-MR process, a transformation $y = \ln(x)$ is needed to apply Itō's lemma to obtain the Ornstein-Uhlenbeck model, whose mathematical expression is given by:

$$dy = \lambda[\Omega - y]dt + \sigma dz \ , \quad (4)$$

where:

$$\Omega = \phi - \frac{\sigma^2}{2\lambda} . \quad (5)$$

This is equivalent to the Ornstein-Uhlenbeck process whose mean and variance values are given by the expressions:

$$E[y(t)] \equiv \mu_y(t) = y(t_0)e^{-\lambda(t-t_0)} + \Omega[1 - e^{-\lambda(t-t_0)}] , \quad (6)$$

$$Var\,[y(t)] \equiv \sigma_y^2(t) = \frac{\sigma^2}{2\lambda}[1 - e^{-2\lambda(t-t_0)}] . \quad (7)$$

Using the transformed values, and knowing that $x = e^y$, the mean and variance of the original function are obtained:

$$E[x(t)] \equiv \mu_x(t) = e^{\left[\mu_y(t) + \frac{\sigma_y^2(t)}{2}\right]} , \quad (8)$$

$$Var\,[x(t)] \equiv \sigma_x^2(t) = e^{[2\mu_y(t) + \sigma_y^2(t)]}\left(e^{\sigma_y^2(t)} - 1\right) . \quad (9)$$

Next, the transformed function in (8) is discretized taking discrete time steps δt as follows:

$$y_k = y_{k-1}e^{-\lambda\delta t} + \Omega\left(1 - e^{-\lambda\delta t}\right) + \varepsilon_k , \quad (10)$$

where the uncorrelated independent residual error term, ε_k, has a standard distribution of the form:

$$\varepsilon \approx N\left(0, \frac{\sigma^2}{2\lambda}\left(1 - e^{-2\lambda\delta t}\right)\right) , \quad (11)$$

or, equivalently,

$$\mu_\varepsilon = 0 \quad \text{and} \quad \sigma_\varepsilon^2 = \frac{\sigma^2}{2\lambda}\left(1 - e^{-2\lambda\delta t}\right) . \quad (12)$$

Taking differences of adjacent elements of the process in (10), the following expression is obtained:

$$y_k - y_{k-1} = \left(e^{-\lambda\delta t} - 1\right)y_{k-1} + \Omega\left(1 - e^{-\lambda\delta t}\right) + \varepsilon_k . \quad (13)$$

This expression can be assimilated to a first-order autoregressive process AR(1) defined as:

$$y_k - y_{k-1} = \beta_0 + \beta_1 y_{k-1} + \varepsilon_k . \quad (14)$$

Equalizing (13) and (14):

$$\beta_0 = \Omega\left(1 - e^{-\lambda\delta t}\right) , \quad (15)$$

$$\beta_1 = e^{-\lambda\delta t} - 1 . \quad (16)$$

The Cholesky decomposition expressed as a product of matrices is used to estimate the values of β_0 and β_1:

$$\hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} = \left(A' \cdot A\right)^{-1} \cdot A' \cdot Z . \quad (17)$$

The matrices used in (17) are composed of the transformed (ln) values of the GBM-MR process, and they are generated by Monte Carlo simulation:

$$A = \begin{bmatrix} 1 & y_2 \\ 1 & y_2 \\ \dots & \dots \\ \dots & \dots \\ 1 & y_n \end{bmatrix} ; \quad Z = \begin{bmatrix} y_2 - y_1 \\ y_3 - y_2 \\ \dots \\ \dots \\ y_n - y_{n-1} \end{bmatrix} . \quad (18)$$

Clearing λ in (16), the expression for the first estimated parameter is attained:

$$\hat{\lambda} = -\frac{\ln\left(1 + \hat{\beta}_1\right)}{\delta t} . \quad (19)$$

On the other hand, from (15), the estimated value of Ω is:

$$\hat{\Omega} = -\frac{\hat{\beta}_0}{\hat{\beta}_1} , \quad (20)$$

and from (5), the estimated value of ϕ is:

$$\hat{\phi} = \frac{\hat{\sigma}^2}{2\hat{\lambda}} - \frac{\hat{\beta}_0}{\hat{\beta}_1} . \quad (21)$$

To calculate the volatility it's necessary to have residual terms resulting from the difference between the values of the matrix Z and the ones in the estimated matrix $\hat{Z}$:

$$\hat{Z} = \hat{\beta} \cdot Y = \hat{\beta}_0 + \hat{\beta}_1 \cdot Y , \quad (22)$$

where,

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ \dots \\ y_{n-1} \end{bmatrix} , \quad (23)$$

and

$$\varepsilon = \hat{Z} - Z . \quad (24)$$

The standard deviation of the residual terms, $\hat{\sigma}_\varepsilon$, is calculated and, from (12), the expression of the estimated variance is attained:

$$\hat{\sigma}^2 = \frac{2\hat{\lambda}\hat{\sigma}_\varepsilon^2}{1 - e^{-2\hat{\lambda}\delta t}} . \quad (25)$$

From (25) and (19), the value of the estimated volatility is obtained:

$$\hat{\sigma} = \hat{\sigma}_\varepsilon \sqrt{\frac{2\ln\left(1 + \hat{\beta}_1\right)}{\delta t\left[\left(1 + \hat{\beta}_1\right)^2 - 1\right]}} . \quad (26)$$

IV. NPV Curve and Its Piecewise Parameter Estimation. Risk Profile Characterization

A. NPV curve calculation

In this work it is assumed that the income resulting from the energy produced is the product of two stochastic processes: the energy produced, as shown in Fig. 2, and the price, represented by a GBM-MR model. The price is estimated for a period of more than 1 year, since the NPV calculation requires long periods. The model could also be enriched by using long-term contract values, but this is outside the scope of this paper.

The yearly cash flows resulting from the income model, subtracting the investment, maintenance costs, depreciation and interests, and applying corporate taxes, can be used to estimate the NPV of the wind investment. This is the standard calculation of the value of an investment used in Economics and Finance textbooks. However, a more interesting approach is to study the evolution of the NPV for long periods of time. The NPV curve represents the construction of the trajectory of the resulting NPVs when the investment takes place at successive times, updating the investment costs and cash flow values to the corresponding reference periods. In this paper, the NPV curve is calculated on a monthly basis and the annual value corresponds to the average value of the 12 months. Fig. 3 presents an example of how to obtain the different NPVs and Fig. 4 shows the resulting NPV curve.

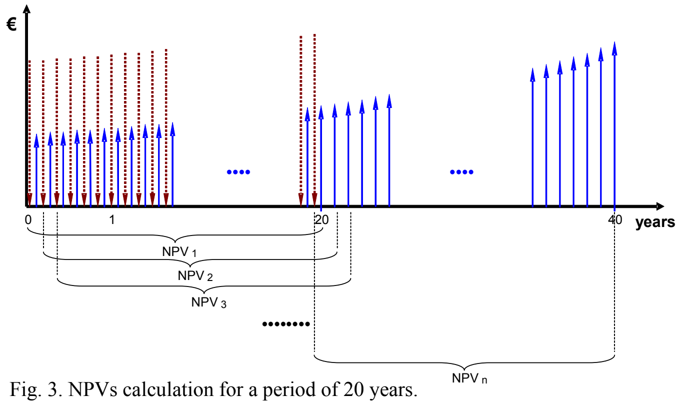

Fig. 3. NPVs calculation for a period of 20 years.

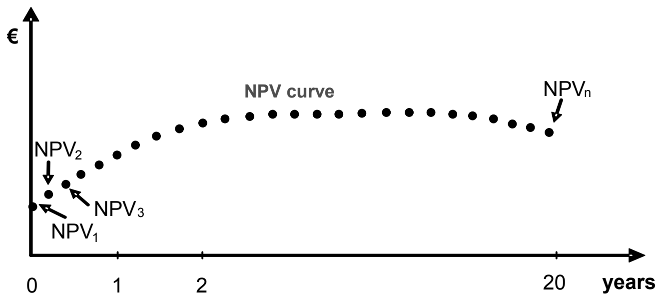

Fig. 4. Example of construction of an NPV curve for a 20-year period.

B. NPV curve piecewise parameter estimation

A similar derivation to the one in section III.B is possible to characterize the stochastic process that represents the NPV. However, as mentioned in the introduction, it cannot be assumed that a NPV curve follows a GBM-MR process with constant parameters. The values of the parameters change, in particular ϕ and λ, with the σ parameter remaining approximately constant showing a slightly increasing trend. To solve this problem, the estimation of the parameters is done piecewise. In this way, the matrices in (18) are created with the information of each of the processes in a piecewise fashion so that the parameters, ϕ and λ, are optimally adjusted to the actual trajectory. Fig. 5 shows the piecewise estimation of the parameters of the NPV curves.

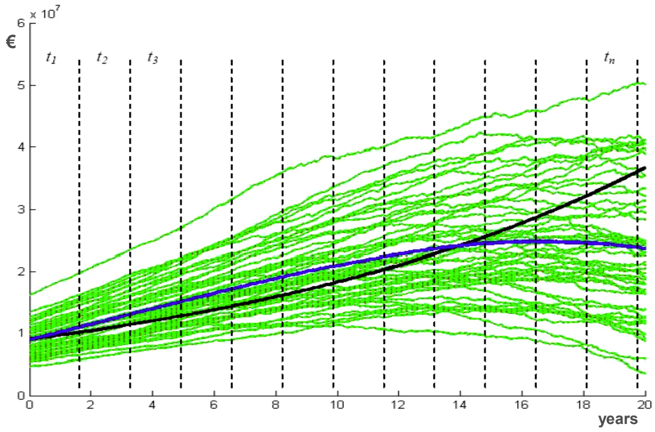

Fig. 5. Piecewise estimation of the NPV curve parameters.

On the other hand, due to the stochasticity of the model and, in order to obtain estimators as accurate as possible, many paths are needed to calculate representative values of the estimated parameters. This is done by means of Monte Carlo simulation.

C. Risk profile characterization

To compare the risk involved in a wind plant investment with the yield derived from a risk-free asset, a new parameter τ is introduced. This parameter, that expresses the return that can be obtained investing in an risk-free asset (Treasury Bonds), is now included in the model from Section III.B, combined with the average long-term parameter ϕ. In this way, the evolution trend of the estimated value of the NPV curve is corrected as a function of the risk aversion of the investor. This means that the more averse the investor, the more he will choose risk-free assets with lower yield; because of that, the correction to the model will be greater to achieve risk neutrality. Assuming that the risk-free asset yield is given by τ and that the expected yield is given by η, the average trajectory of the NPV curve is given by $\alpha = \lambda[\phi - \ln(x)]$. Therefore, the expected value of a risk-neutral investor is equal to $\delta = \eta - \alpha$. To evaluate the risk aversion of an investor, the difference between a risk-free yield and the expected value is obtained: $\tau - \delta$. As a consequence, the estimated average long-term parameter $\hat{\phi}$ becomes $\hat{\phi}_r = \hat{\phi} - \left(\frac{\eta - \tau}{\lambda}\right)$.

V. Trinomial Decision Tree Construction. Calculation of the Value of an American Option and its Probabilities

Once obtained the parameters of the NPV curve, they are used to construct the trinomial decision tree to decide upon a future wind plant investment. In this section, an introduction to the trinomial decision trees is included, as well as the calculation of the probabilities derived to either invest, wait, or abandon the project.

A. Construction of the trinomial decision tree from the estimated parameters of a GBM-MR process

The numerical procedure to construct trinomial trees assumes that for each node of the dominion of the solution there are three possible paths to follow: go up, maintain the value, or go down, therefore three transition probabilities must be defined: p_u, p_m and p_d, respectively. Moreover, if a mean reversion process is modeled, three alternative branching structures are generated.

In [15] a general procedure to construct binomial decision trees is proposed to represent processes with a stochastic factor with mean reversion of the Ornstein-Uhlenbeck type. The general procedure of the algorithm consists of two stages:

First stage: Creation of a symmetric tree of the process

Starting from an estimated GBM-MR process as in (3): $\frac{dx}{x} = \hat{\lambda}\left[\hat{\phi} - \ln(x)\right]dt + \hat{\sigma}\, dz$ and making the transformation $y = \ln(x)$, the following expressions result for the estimated process:

$$dy = \hat{\lambda}\left[\hat{\Omega} - y\right]dt + \hat{\sigma}\, dz, \tag{27}$$

$$dy^* = -\hat{\lambda}\, y^* dt + \hat{\sigma}\, dt \text{ , where} \tag{28}$$

$$\Delta y = \hat{\sigma}\sqrt{3\delta t}\ . \tag{29}$$

These expressions are necessary to define the first stage of construction of the trinomial tree corresponding to a GBM-MR process. The following probabilities are obtained:

Node with no bounds (a):

$$p_u = \frac{1}{6} + \frac{1}{2}\left(\hat{\lambda}^2\, j^2 \delta t^2 - \hat{\lambda}\, j\delta t\right), \tag{30}$$

$$p_m = \frac{2}{3} - \left(\hat{\lambda}^2\, j^2 \delta t^2\right), \tag{31}$$

$$p_d = \frac{1}{6} + \frac{1}{2}\left(\hat{\lambda}^2\, j^2 \delta t^2 + \hat{\lambda}\, j\delta t\right). \tag{32}$$

Node with an upper bound (b):

$$p_u = \frac{1}{6} + \frac{1}{2}\left(\hat{\lambda}^2\, j^2 \delta t^2 + \hat{\lambda}\, j\delta t\right), \tag{33}$$

$$p_m = -\frac{1}{3} - \left(\hat{\lambda}^2\, j^2 \delta t^2 + 2\hat{\lambda}\, j\delta t\right), \tag{34}$$

$$p_d = \frac{7}{6} + \frac{1}{2}\left(\hat{\lambda}^2\, j^2 \delta t^2 + 3\hat{\lambda}\, j\delta t\right). \tag{35}$$

Node with a lower bound (c):

$$p_u = \frac{7}{6} + \frac{1}{2}\left(\hat{\lambda}^2\, j^2 \delta t^2 - 3\hat{\lambda}\, j\delta t\right), \tag{36}$$

$$p_m = -\frac{1}{3} - \left(\hat{\lambda}^2\, j^2 \delta t^2 - 2\hat{\lambda}\, j\delta t\right), \tag{37}$$

$$p_d = \frac{1}{6} + \frac{1}{2}\left(\hat{\lambda}^2\, j^2 \delta t^2 - \hat{\lambda}\, j\delta t\right). \tag{38}$$

Note that the expressions are entirely equivalent to the ones in (36)-(44), since we are using the same modeling approach.

Second stage: Definition of the price dynamics

The transformations below are needed for the second stage of the construction of the trinomial tree:

$$\alpha(t) = y(t) - y^*(t), \tag{39}$$

$$d\alpha = \hat{\lambda}\left[\hat{\Omega} - \alpha(t)\right]dt, \tag{40}$$

$$\alpha(t) = y(0,t) + \frac{\hat{\sigma}^2}{2\hat{\lambda}^2}\left(1 - e^{-\hat{\lambda}\,\delta t}\right)^2 . \tag{41}$$

The final values of the trinomial tree are obtained as: $y(t) = y^*(t) + \alpha(t)$.

In a discrete form, the evolution of α is a function of the parameters of a Brownian motion: $\alpha_i = \alpha_{i-1} + \hat{\lambda}\left(\hat{\Omega} - \alpha_{i-1}\right)\delta t$, where i is the column depicting the period.

Once the trinomial tree is constructed with the estimated parameters, the expected value of the set of nodes of each period is given by:

$$V_{esp}(i) = \sum_{j=j_{\min}}^{j=j_{\max}} p(i,j)\cdot y(i.j), \tag{42}$$

where the periods i can be assimilated with the time axis, i.e.: $V_{\exp}(i) = V_{\exp}(t)$.

It must be noted that the parameters change at each interval of the NPV curve depending on the section of the curve where the nodes of the tree are located. In this way, the expected value of the states at the nodes follows the initial trajectory without significant errors.

B. Calculation of the value of an American option and its probabilities

To calculate the value of an option from a tree (binomial, trinomial, etc.) that is already built for a specific process, it is necessary to subtract the project investment cost from the NPV of the project once the last period is reached.

Once the investment costs are discounted from the NPV in the last period, the real option tree is built backwards up to the initial node. Mathematically this can be modeled by dynamic programming.

The possibility to exercise an American option is available at any time at each node, unlike the European option, that only depends on the values of previous nodes and their associated probabilities. The value of the option is the maximum of three possible choices: invest now (a), wait (b), or abandon (c), as shown in Fig. 6.

$$Vnode_{(i,j)} = \max(a;b;c)$$

a) $NPV_{(i,j)} - Investment_{(i)}$

b) $NPV_{(i+1,j+1)}\cdot p_{u(i,j)} + NPV_{(i+1,j)}\cdot p_{m(i,j)} + NPV_{(i+1,j-1)}\cdot p_{d(i,j)}$

c) 0

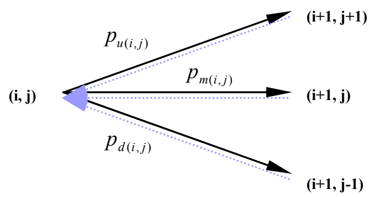

Fig. 6. Valuation of an American option to invest.

Fig. 7 shows graphically the three different alternatives: exercise the option, wait, and abandon. It also depicts the probability distribution of each alternative throughout the life of the real option. The probability to exercise the option is marked in red, the waiting option in green, and the abandoning option in black. The visual check corresponding to the sum of the three probabilities is marked in blue.

In parallel to the construction of the real options decision tree there must be an estimation of the probability of each possible investment situation. Due to the probability distribution of the nodes, the central zone of the tree has the highest weight in the estimation.

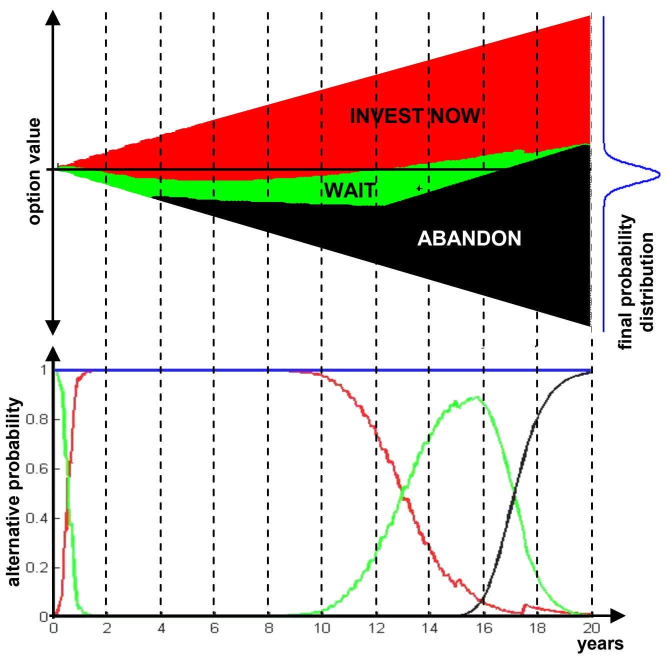

Fig. 7. Graphical depiction of the alternative probabilities: to invest, to wait, or to abandon.

VI. Case Studies

This section contains several case studies where the investment model described before is applied. A sensitivity analysis is applied with respect to key parameters of the model. The base case assumes a wind farm of 10 MW, composed of 5 generators of 2 MW each. The parameters are as follows:

General parameters:

- NPV calculation period: 20 years.
- Estimation period for the cash flows: 40 years.
- Time step duration: For the wind the step is 1 h and for the energy produced, cash flows and NPV curves is 1 month.
- NPV discount rate: 7%.
- Number of trajectories of the Monte Carlo simulation: 200.

Wind parameters:

- Shape factor: k_w = 1.8, indicates the dispersion with respect to the average value of the wind speed.
- Scale factor: λ_w = 5.3, indicates the average value of the wind speed in a certain location.

Production parameters:

- Maximum power: 10 MW.
- Minimum, Rotor and Maximum rotor speed: 4, 11 and 25 m/s.

Electricity spot price parameters:

- Initial price: €60 MWh, based upon OMEL data [17].
- Volatility: $\sigma = 0.05$.
- Strength of reversion: $\lambda = 0.05$.
- Spot price trend increase: s_t: 7% annual.
- Trend: with $\phi = \ln\left(x_0 \cdot (1+s_t)^{years}\right)$, the base value is 16.95.

Risk parameters:

- Estimated expected yield: 10%.
- Risk-free asset yield: 5%, i.e., Treasury Bonds.

Investment cost parameters:

- Price of the kW installed: €1,000 kW.
- Investment cost increase: 7% annual.

A. Case study 1: Parameterization of the project financing percentage

This first case study studies the effect produced in the model by changing the external financing % of the required investment. The base value of the financing parameter is 80% to mimic what is already in use in Spain (see [18]). The parameter values oscillate between 0 and 100% of external financing, applying a 10% increase for each of the 11 scenarios analyzed. To solve this case study there are 4 steps to follow:

- Step 1: Construction of the NPV curves: Fig. 8 shows the evolution of the NPV curves for each of the scenarios for a period of 20 years. It can be observed that the curves that represent the evolution of the shareholders' equity increase linearly. In the NPV curve of scenario 1 the percentage is 100%, so there is no need for equity, but at the other extreme, scenario 11, the percentage is 0%, where the investment and the equity are the same.

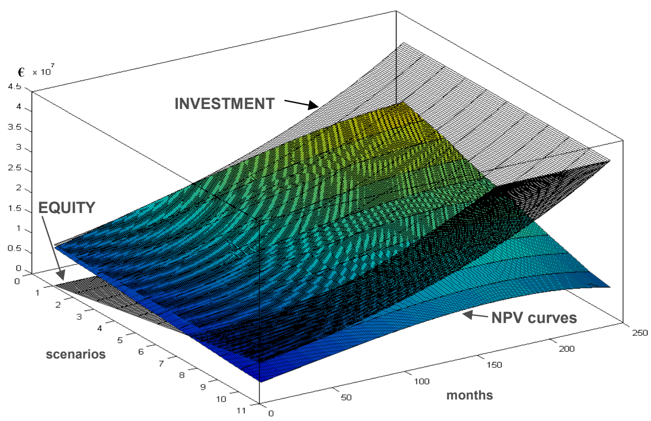

Fig. 8. NPV curves for each scenario.

- Step 2: Estimation of the GBM-MR parameters: From the different NPV curves the parameters of their piecewise GBM-MR associated parameters are obtained. According to the speed of change of the NPV curves, the optimal number of blocks whose parameters remain constant is found. See Fig. 5.
- Step 3: Construction of the trinomial trees for the real options: Once tested that the parameters fit with the original data the different values of the American option to invest are calculated. Fig. 9 shows their values for a specific NPV curve: wait, execute or abandon. In the figure, the first scenarios, in which the % of external financing is high, the "invest now" alternative is present throughout the entire lifetime (20 years). When the external financing decreases, there is more need of equity and the "wait" alternative has more weight. Finally, the "abandon" alternative only shows up in scenarios with more equity and in the latest moments of the lifetime.

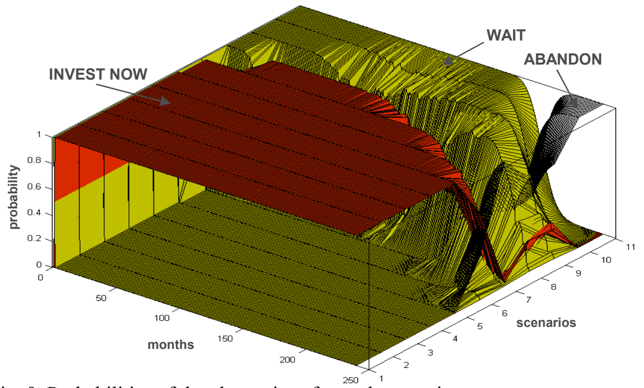

Fig. 9. Probabilities of the alternatives for each scenario.

- Step 4: Calculation of the option to invest: As a result of the above calculations, the value of the real option is attained. As observed in Fig. 10, more equity implies that the "invest now" alternative loses value, up to scenarios where the value of the option is zero, therefore, an investment is not advised.

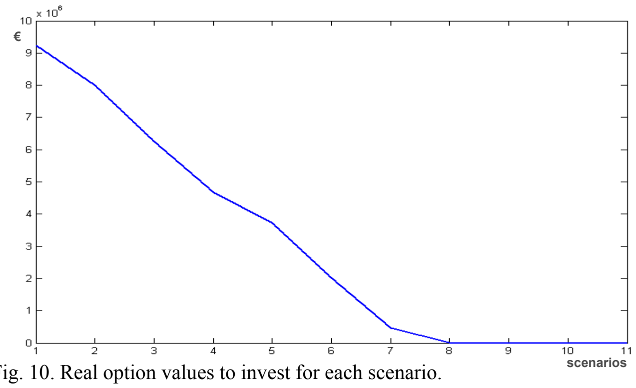

Fig. 10. Real option values to invest for each scenario.

These results can be easily compared to the ones resulting from a more traditional analysis. In particular, a classic investment valuation method would estimate the value of the NPV at the beginning of the lifetime (year 0) and depending on its value would decide to invest or not. Fig. 11 shows how these NPV values correspond to the first point of the NPV curves. For this case, it's observed that in the initial scenarios, up to the seventh, the NPV is above the equity and, little by little, the difference is diminishing until it becomes negative in scenario 8.

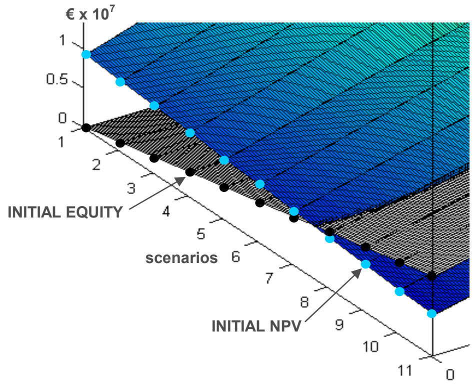

Fig. 11. Initial NPV vs. equity.

As a conclusion for this case, the "invest now" alternative is present from the beginning of the option in all scenarios except the last ones.

Table 1 presents the descriptions of all the cases developed in this work.

TABLE I
CASE STUDIES DESCRIPTION

Case	Parameters	Range (step)
1	Project financing	0 to 100% (10)
2	Investment cost (100% of equity)	750 to 1,250 €/kW (50)
3	Investment cost (50% of equity)	750 to 1,250 €/kW (50)
4	Annual investment cost rise	5 to 10% (0.5)
5	Risk aversion	0 to 10% (1)
6	Spot price volatility	0 to 0.01 (0.001)
7	Spot price trend (annual increase)	5 to 10% (0.5)
8	Spot prices' strength of reversion	0.025 to 0.075 (0.005)
9	Wind speed's shape parameter	1.5 to 2.5 (0.1)
10	Wind speed's scale parameter	5 to 6 (0.1)

VII. CONCLUSIONS

This paper has developed a decision-making model to evaluate wind energy investments based on two concepts: a stochastic model of the parameters that affect the NPV, such as wind production and electricity prices, and a real options model derived from a trinomial tree that evaluates numerically the probabilities to invest now, wait or abandon. The hypothesis of GBM-MR model that is applied to the prices is valid for the NPV, as long as the parameters are estimated piecewise.

A new concept, the NPV curve, which is made up of points that correspond to the values of the NPV at different successive times, is introduced and compared to the traditional NPV method. Several case studies, where the most important parameters have a variable range, are analyzed to illustrate the applicability of the method proposed.

As regards risk valuation, the model does not explicitly produce a specific risk associated with the investment, but adapts the results as a function of the risk aversion profile of the investor.

With regard to the real options model used, it allows:

- Numerical valuation of alternatives to undertake a project.

- Estimation of the best time within the project's lifetime to execute the investment seeking the maximum profit.
- Estimation of the probabilities of investment alternatives.

In relation to the results of the cases, the following conclusions can be extracted:

- It's necessary to have at least a 40% project financing. Below this value, the project is not profitable.
- As long as the minimum project financing requirement is fulfilled, the project will be possible if the price of the kW installed is less than € 1100.
- Due to the integration that takes place in the calculation of the NPV curves, the annual investment cost rise has a small effect for parameter values close to the base value, 7%. From this value onwards the investment increments are clearly reflected in the result, where the option can reach a value of M€ 3.5.
- The variation in the option price is almost linear with respect to the risk aversion. For high risk aversion values (risk-free parameter between 0 and 3%) the "abandon" alternative is shifted to the middle of the option lifetime (10 years). For low risk aversion values (risk-free parameter between 7 and 10%) the "abandon" alternative does not take place until the end of the option lifetime.
- In relation to the spot price, its volatility does not affect the results significantly, due to the aforementioned integration effect. However, an increase in the spot price trend or the strength of reversion has considerable effects, with the option reaching values higher than M€ 8. On the contrary, due to the great sensibility of the model to these two parameters, for annual increases in the spot price trend below 6% or with a strength of reversion below 0.04, the project is not profitable.
- Finally, referring to the wind speed, the shape and scale parameters have opposite effects. When the shape parameter increases, the dispersion of the wind speed decreases, and the value of the option decreases up to zero for parameter values higher than 2.2. If the scale parameter increases, the profitability of the project increases. In spite of this, due to the extreme sensitivity of the model with respect to this parameter, for parameter values below the base value, 5.3, the profitability is highly reduced.

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