Paper accepted for presentation at 2009 IEEE Bucharest Power Tech Conference, June 28th - July 2nd, Bucharest, Romania

# Dynamic Voltage Stability Assessment of an Electric Power System using Trajectory Sensitivity Analysis

R. M. Monteiro Pereira, C. M. Machado Ferreira and F. P. Maciel Barbosa

***Abstract*--Dynamic voltage stability has been an important problem in electric power systems. The dynamic behavior of modern power networks has been experiencing significant changes mainly due to the restructuring of electric industry, the introducing of open market environment and competition. In this paper it is presented a methodology to assess the dynamic voltage stability of an electric power system using trajectory sensitivity analysis. Trajectory sensitivity analysis can provide valuable insights into the security of an electric power system. In this study it was simulated a significant load demand disturbance in a test power network. The Automatic Voltage Regulator of the generating units, the Under Load Tap Changer and loads models were taken into account. The simulation results were obtained using EUROSTAG program and post-processing module developed using the Matlab software package.**

***Index Terms*-- Electric power systems, Trajectory sensitivity analysis, Voltage Stability.**

## I. INTRODUCTION

In recent years, there has been an increasing interest for the analysis of dynamic voltage stability on power systems. Modern electric power networks are large and widely interconnected and overall considerably more complex [1]. Increasing demand for electricity requires increasing transmission capabilities. A variety of factors, including financial and environmental issues, constrain the construction of new power lines. Electric utilities are forced to operate the system in a way that makes the best use of existing transmission facilities. Under such increasingly complex conditions, the maintenance of power systems stability has become a very important issue. Furthermore, there have been observed in several networks some weak damping phenomena where power swings are unable to be controlled by conventional power system stabilizers.

The dynamic phenomena causing voltage instability, occurring in electric power systems subjected to strong load demands, lead to a progressive decreasing of the voltage magnitude at one or more buses, resulting sometimes in network islanding, thus leading to local or global blackouts. Oscillatory instability and voltage instability are two major problems. Consequently, dynamic voltage stability poses a primary threat to system security and reliability [2].

Dynamic voltage stability plays a very important role during the planning and design stages of an electric power network as well as during the system operation. In the last years in various countries worldwide, several power network collapses caused by voltage problems have been reported. This can be produced by a lack of sufficient reactive power reserve during heavy load or by the occurrence of severe contingencies. The collapse effect can be characterized by a continuous decrease of the power system voltage. In the initial stage the decrease of the system voltage starts gradually and then decreases rapidly. Voltage stability has become one of the most important power system research areas. A lot of effort has been focused upon the cause and mechanism of voltage instability and corresponding counter measures [3]. Trajectory sensitivities have a potential for both preventive and emergency control. Trajectory sensitivities provide an insight into the behavior of a dynamic system, which would not be otherwise obvious only from its nominal trajectory [4]. An impact of initial conditions and/or parameters on the system trajectory can be analyzed [5]. Moreover, the computational cost of evaluate the sensitivities and perturbed trajectories is minimum. These capabilities of trajectory sensitivities have been used so mainly for post-mortem analysis [6]. More recent applications have included stability assessment of power systems [4].

It has been observed and can be mathematically justified, that as the stability margin decreases the trajectory sensitivities undergo larger excursions [5]. For unstable situations, trajectory sensitivities increase much more rapidly than the nominal system trajectory. A fast increase in trajectory sensitivities can be linked with an underlying stability problem. Consequently, sensitivities can be used as an early indicator of imminent instability [7].

This paper is organized as follows. In Section II it is described succinctly the applied software package. Section III is devoted to the formulation of the problem using dynamic voltage stability assessment combined with a trajectory

---

R. M. Monteiro Pereira is with the Instituto Superior de Engenharia of the Polytechnic Institute of Coimbra, Portugal, Quinta da Nora, Rua Pedro Nunes, 3030-199 Coimbra, Portugal, (e-mail: rmfm@isec.pt).

C. M. Machado Ferreira is with the Instituto Superior de Engenharia of the Polytechnic Institute of Coimbra and INESC Coimbra, Portugal, Quinta da Nora, R. Pedro Nunes, 3030-199 Coimbra, Portugal, (e-mail: cmacfer@isec.pt).

F. P. Maciel Barbosa is with FEUP, Faculdade de Engenharia da Universidade do Porto and INESC Porto, Campus da FEUP, Rua Dr. Roberto Frias, 4200-465 Porto Portugal, (e-mail: fmb@fe.up.pt).

sensitivity approach. In section IV it is presented the test power network and two cases that were analyzed. Section V shows the results obtained using the proposed methodology. Finally, in section VI, some conclusions that provide a valuable contribution to the understanding of the dynamic voltage stability assessment of a power system are pointed out.

## II. APPLIED SOFTWARE

The advanced time-domain simulations provide a realistic picture about voltage collapse phenomena. In this paper the simulations were carried out using the professional grade time domain simulation software package EUROSTAG, developed by Electricité de France (EDF) and Tractebel Energy Engineering [8]. The main feature of these computer programs is to propose a unique solution to various issues with a high degree of performance [9], [10]. This single integrated program dedicated to the dynamic simulation of electric power systems is able to simulate the full range of electrical phenomena, from transient to long term stability and gives continuous display for fast and slow events. Therefore, it is oriented towards all domains of power system security, and is particularly adapted to the new conditions of the competitive electricity market, in which it is essential to accurately know the technical operating limits of the electric power system.

The numerical integration scheme is performed using a mixed Adams–BDF implicit integration method with automatic variable step size [11]. This integration technique, making use of an automatically and continuously varying integration step-size, is the cornerstone allowing to use a single program and a single model to the whole range of application. It provides accuracy and robustness to the simulation, even for large deviations from normal conditions.

The results were exported into Matlab software package, since it is not possible to impose the same time step in two distinct time domain simulations. The trajectory sensitivity solutions were produced using an interpolation technique in order to have a uniform time step in all simulations. It was developed a post processing unit that allows displaying and analyzing the trajectory sensitivity results. The post-processing unit is designed to take advantages of the advanced user interface features of the Matlab environment. This unit enhances the user ability to analyze a large amount of output data and to produce visually appealing graphic representations of the results.

## III. FORMULATION OF THE PROBLEM

Trajectory sensitivity analysis can provide valuable insights into the security of an electric power systems that otherwise would not be clear from its nominal trajectory [12]. This approach is based upon linearizing the system around a nominal trajectory rather than around an equilibrium point [4], [12].

Analysis of electric power system dynamics requires a computationally efficient non-restrictive model formulation capable of capturing the full range of events. The systems dynamics can be modeled, taking into account their hybrid nature –combination of continuous and discrete dynamics – as a set of differential algebraic (DA) equations [4]:

$$\dot{\underline{x}} = \underline{f}\left(\underline{x}, y\right) \tag{1}$$

$$0 = \begin{cases} g^{-}\left(\underline{x}, y\right) & s\left(\underline{x}, y\right) < 0 \\ g^{+}\left(\underline{x}, y\right) & s\left(\underline{x}, y\right) > 0 \end{cases} \tag{2}$$

with the vectors

$$\underline{x} = \begin{bmatrix} x \\ \lambda \end{bmatrix} \text{ and } \underline{f} = \begin{bmatrix} f \\ 0 \end{bmatrix} \tag{3}$$

where $x$ are the dynamic state variables, $y$ are the algebraic variables and $\lambda$ are the system parameters. A switching occurs when $s\left(\underline{x}, y\right) = 0.$ The initial conditions are given by:

$$\underline{x}\left(t_0\right) = \underline{x}_0 \text{ and } y\left(t_0\right) = y_0 \tag{4}$$

Trajectory sensitivity analysis studies the variations of the system variables with respect to the small variations in initial conditions and parameters [13], [14]. If the discontinuities are not taken into account, the system dynamics evolve according to the DA system:

$$\dot{\underline{x}} = \underline{f}\left(\underline{x}, y\right) \tag{5}$$

$$0 = g\left(\underline{x}, y\right) \tag{6}$$

The flows of the system can be defined as:

$$\phi\left(\underline{x}_0, t\right) = \begin{bmatrix} \phi_{\underline{x}}\left(\underline{x}_0, t\right) \\ \phi_y\left(\underline{x}_0, t\right) \end{bmatrix} = \begin{bmatrix} \underline{x}\left(t\right) \\ y\left(t\right) \end{bmatrix} \tag{7}$$

Differentiating equations (5) and (6) with respect to the initial conditions and parameters yields:

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}\left(t\right)\underline{x}_{\underline{x}_0} + \underline{f}_y\left(t\right) y_{\underline{x}_0} \tag{8}$$

$$0 = g_{\underline{x}}\left(t\right)\underline{x}_{\underline{x}_0} + g_y\left(t\right) y_{\underline{x}_0} \tag{9}$$

where $\underline{f}_{\underline{x}}$, $\underline{f}_y$, $g_{\underline{x}}$ and $g_y$ are time varying matrices and are evaluated along the system trajectories; $\underline{x}_{\underline{x}_0}(t)$ and $y_{\underline{x}_0}(t)$ are the trajectory sensitivities.

Initial trajectory sensitivities can be computed from:

$$\underline{x}_{\underline{x}_0}\left(t\right) = \mathbf{I} \tag{10}$$

$$y_{\underline{x}_0}\left(t_0\right) = -\left[g_y\left(t_0\right)\right]^{-1} g_{\underline{x}}\left(t_0\right) \tag{11}$$

where $\mathbf{I}$ is the identity matrix. It is assumed that $g_y\left(t_0\right)$ is non-singular along the trajectories.

The trajectory sensitivities can be obtained by solving equations (8) and (9) simultaneously with (5) and (6) using (4), (10) and (11) as the initial conditions. The sensitivities can also be evaluated using a Taylor series expansion of the system flows (7). The expansion for $\phi_{\underline{x}}$ and $\phi_y$ can be expressed as [15]:

$$\Delta \underline{x}(t) = \frac{\partial \phi_{\underline{x}}(\underline{x}_0, t)}{\partial \underline{x}_0} \Delta \underline{x}_0 + \text{higher order terms} \tag{12}$$

$$\Delta y(t) = \frac{\partial \phi_y(\underline{x}_0, t)}{\partial \underline{x}_0} \Delta \underline{x}_0 + \text{higher order terms} \tag{13}$$

Neglecting the higher order terms and taken into account (7):

$$\Delta \underline{x}(t) \approx \frac{\partial \underline{x}(t)}{\partial \underline{x}_0} \Delta \underline{x}_0 \tag{14}$$

$$\Delta y(t) \approx \frac{\partial y(t)}{\partial \underline{x}_0} \Delta \underline{x}_0 \tag{15}$$

If the trajectory sensitivities are known, the sensitivity of the system dynamic behavior to small changes in the initial conditions and parameters can be evaluated from the following relation:

$$\Delta \phi(\underline{x}_0, t) = \begin{bmatrix} \Delta \underline{x}(t) \\ \Delta y(t) \end{bmatrix} = \begin{bmatrix} \underline{x}_{\underline{x}_0}(t) \\ y_{\underline{x}_0}(t) \end{bmatrix} \Delta \underline{x}_0 \tag{16}$$

Trajectory sensitivities can be obtained as a by-product of implicit numerical integration techniques and require a little additional computational effort [2].

The dynamic voltage stability margin for a particular disturbance can be defined as the smallest distance between the system trajectory and the stability boundary. For a specified stability scenario a large margin indicates that the system is very stable, while a zero margin corresponds to a borderline situation. When the stability margin assumes a negative value the system is unstable.

The dynamic voltage stability studies allow evaluating the stability margin. The sensitivity coefficient of the stability margin, $\eta$, with respect to parameter, $\lambda$, can be expressed as:

$$S_\lambda^\eta = \frac{\eta(k) - \eta(k-1)}{\lambda(k) - \lambda(k-1)} \tag{17}$$

where $k$ stands for the $k$th simulation

Therefore, the value of parameter, $\lambda$, which cancels the stability margin is given by:

$$\lambda\big|_{\eta = 0} = \lambda(k) - \frac{\eta(k)}{S_\lambda^\eta} \tag{18}$$

In equations (17) and (18), parameter, $\lambda$, could be the generators power level or any other quantity accountable to influence the system voltage stability. So, it could be used to evaluate the stability limits.

## IV. ELECTRIC POWER SYSTEM

In Fig. 1 it is shown the Electric Power Network that will be used in this study. The simulations were carried out considering the network data presented in [10], [16]. The operating point assumed in this study corresponds to a 1600 MW and 850 MVAr load level.

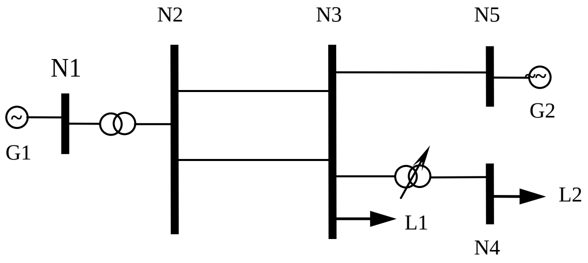

Fig. 1. Test power network single line diagram.

In busbar N4 the load was assumed as constant impedance, whereas in busbar N3 the load was modelled as constant power. Generator G2 is considered as an infinite busbar and generator G1 is modelled in detail. The automatic voltage regulators (AVR) of the generating units and the turbine speed governors (SG) were taken into account. The Under Load Tap Changer (ULTC) actions of the power transformer between busbars N3 and N4 are represented considering a time delay and a deadband. Time delays for ULTC operations are assumed to be 30 seconds for the first tap movement and 5 seconds for subsequent tap movements.

Generators generally represent one of the most important sources of reactive power and voltage support to a power network. AVR action attempts to maintain the generator terminal voltage at its pre-set reference value by continuously adjusting the field voltage and the field current. However, an AVR is able to control generator terminal voltage only if the field current is within its steady-sate continuous limit. Once the current becomes higher than its limit, field current limiter action starts to reduce the field current to within its limit [17].

Two scenarios were analyzed. In the first one (case I) a significant load demand disturbance was simulated in the busbar N3 at the time equal to 50 seconds corresponding to an increase of 50% of the total active and reactive power. In order to avoid the negative effects of this disturbance there were used several values of series compensation applied to the transmission line between busbars N3 and N5. This preventive control measure can be applied successfully to restore the system voltage stability.

In case II it was simulated the tripping of the 380 kV overhead transmission line between busbars N3 and N5 at the time equal to 20 seconds. It was not considered the switch on of the tripped transmission line. The load shedding was performed in busbar N3 at the time equal to 40 seconds. As a corrective control measure there were used different values of load shedding for the active as well as for the reactive power. Applying a bisection algorithm it is possible to find the value that avoids the system voltage collapse.

## V. RESULTS

For a better understanding of the simulation results obtained this section is organized as follows: part A is devoted to case I and part B shows the solutions produced in case II. In order to compare the results produced by the developed formulation with the solutions obtained with the time-domain simulation scheme it is presented the voltage variation curves and the generator's field current for the same scenarios.

### *A. Case I*

The study of the trajectory sensitivity analysis was performed considering four values for the series compensation applied to the transmission line between buses N3 and N5: 0.1%, 2.5%, 5% and 10%. Fig. 2 presents the voltage in busbar N4 trajectory sensitivities produced by the post processing unit.

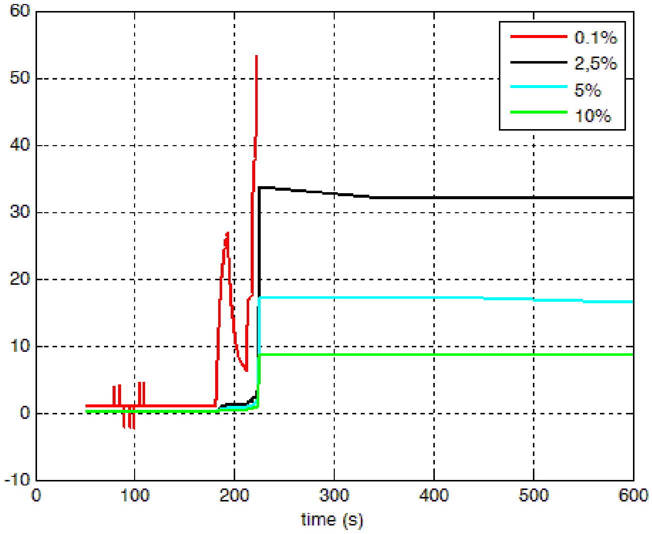

Fig. 2. Voltage in busbar N4 trajectory sensitivities.

In Fig. 3 it is shown the variation of the voltage in busbar N4 for the different series compensation applied in the transmission line between busbars N3 and N5. The simulation results were obtained using the EUROSTAG.

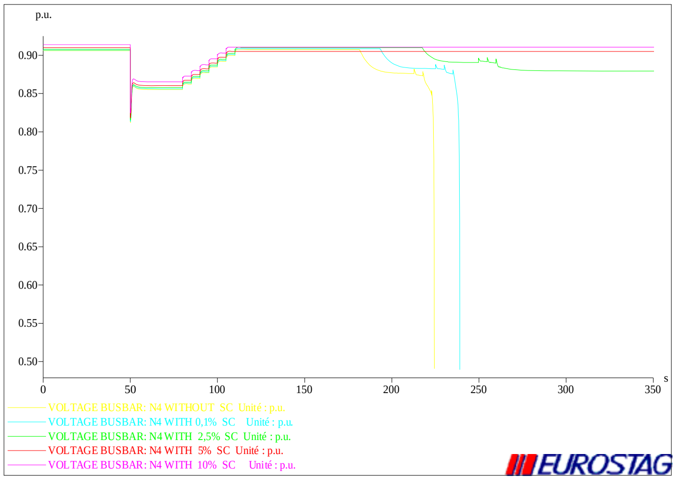

Fig. 3. Voltage variation in busbar N4.

Fig. 4 presents the field current of generator G1 trajectory sensitivities for the different series compensation applied in the transmission line between busbars N3 and N5 produced by the post processing unit.

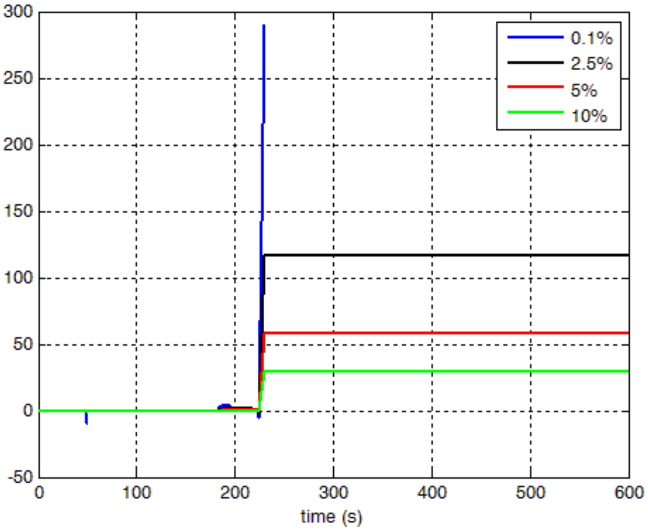

Fig. 4. Field current of generator G1 trajectory sensitivities.

Fig. 5 shows the excitation field current variation of the generator G1. The OvereXcitation Limiter (OXL) works in all situations except where 10% of series compensation was applied. The OXL leads the field current changes to its maximum value of 2.9 per unit. When there is no series compensation or when series compensation is equal to 0.1%, the OXL acts more rapidly, 180 and 195 seconds respectively. In these situations the OXL also contributes to the voltage instability in the system.

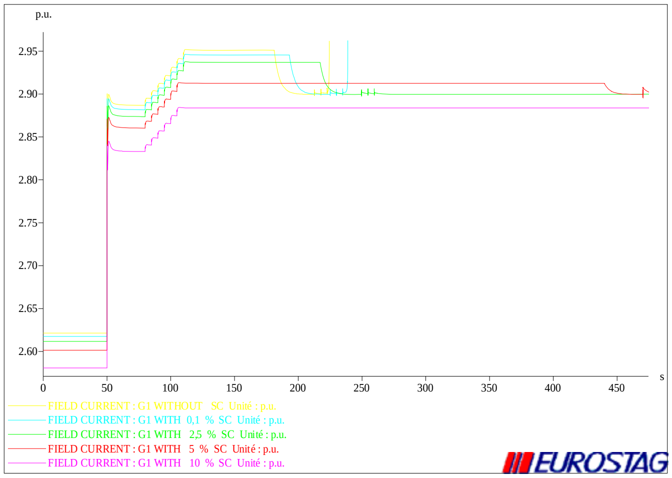

Fig. 5. Field current of generator G1

### *B. Case II*

In this case there were considered five values of load shedding at busbar N3 at the time equal to 40 seconds: 2.5%, 7.5%, 10%, 20% and 30%. Fig. 6 presents the voltage in busbar N4 trajectory sensitivities produced by the post-processing unit.

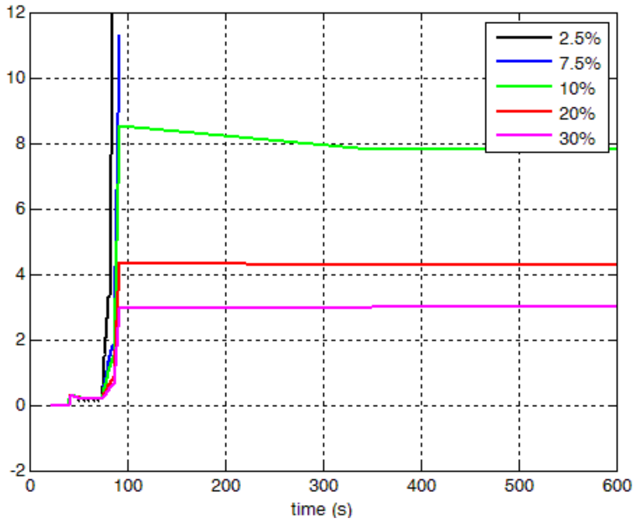

Fig. 6. Voltage in busbar N4 trajectory sensitivities.

In Fig. 7 it is shown the variation of the voltage in busbar N4 for different values of load shedding.

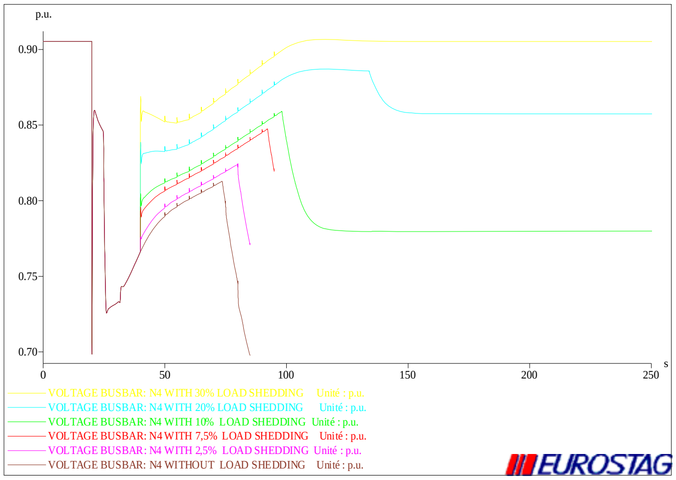

Fig. 7. Voltage variation in busbar N4.

Fig. 8 presents the field current of generator G1 trajectory sensitivities for different values of load shedding produced by the post processing unit.

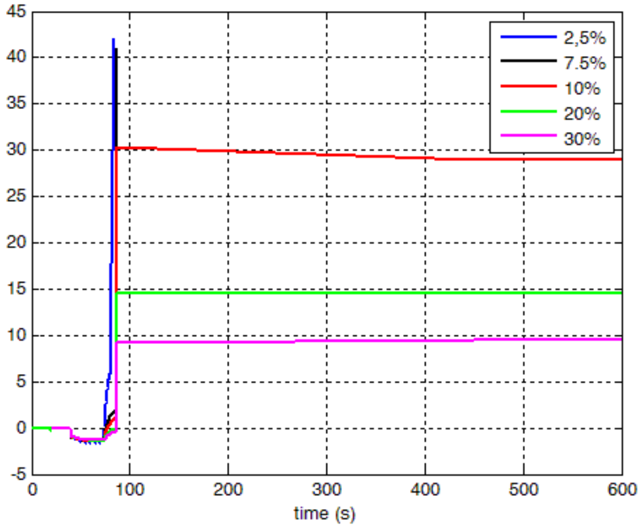

Fig. 8. Field current of generator G1 trajectory sensitivities.

As it is shown in Fig. 9, the OXL only acts when the load shedding corresponds to 30%. In all other situations the OXL works leading to the excitation current to its maximum value of 2.9 per unit. When the load shedding is less than or equal to 7.5%, the OXL acts faster not allowing to restore the system voltage stability.

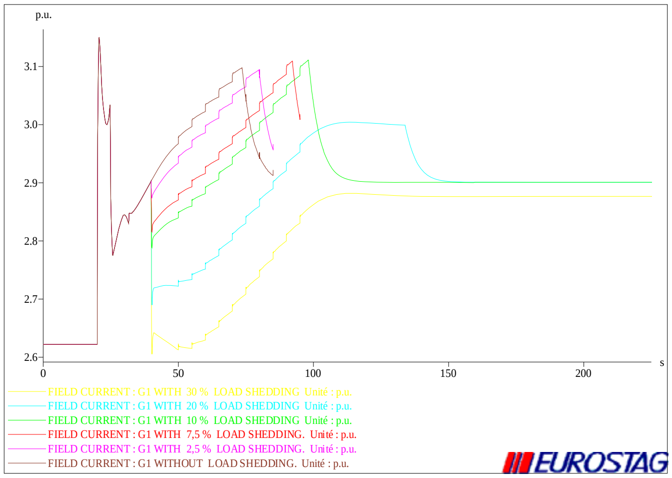

Fig. 9. Field current of generator G1

## VI. CONCLUSIONS

This paper presents a study of the dynamic voltage stability of an electric power system using trajectory sensitivity analysis. Trajectory sensitivities were effectively obtained as a sub-product of simulating the nominal time-domain trajectory. These sensitivities offer a way of ranking the relative influence of system parameters.

In case I, the trajectory sensitivity analysis indicates that there is a collapse situation for the series compensation of 0.1%. For the other values of the series compensation the system remains stable. For the series compensation of 2.5% and 5% the sensitivity values are higher than in the situation of 10% of series compensation. This situation corresponds to the most stable scenario. The same conclusion can be drawn from observation of figure 3 where it is shown that the variation of the voltage on the busbar N4 for 10% of series compensation is the most stable. In case II, trajectory sensitivity analysis proves that when the load shedding is 2.5% and 7.5% the system voltage collapses. However, only using a load shedding of 30% (lower value of sensitivity) it is possible to restore the system stability closer to the initial voltage level.

From the obtained results it was proved that the developed technique is feasible and provides a deeper insight into the influence of parameters on system performance. The solutions obtained are in accordance with the results calculated using the time domain simulation program. The accurate modeling of the OXL is an important factor in the simulation of voltage instability. The trajectory sensitivities of the system variables to different parameters can be used successfully to avoid the power system instability. Due to the output information supplied by the proposed approach it is possible to implement preventive control and corrective actions in order to avoid power system dynamic voltage instability.

## VII. ACKNOWLEDGMENT

The first author would like to thank Fundação para a Ciência e Tecnologia, FCT, that partially funded this research work through the PhD grant nº SFRH/BD/38846/2007.

## VIII. REFERENCES

[1] Gomez-Exposito, A., Conejo, A. J., Canizares, C., "Electric Energy Systems: Analysis and Operation (Electric Power Engineering)", CRC Press, 2008.

[2] Chen, H., Wang, Y., and Zhou, R., "Transient and voltage stability enhancement via coordinated excitation and UPFC control", IEE Proc. Gener. Transm. Distrib., vol. 148, nº.3, pp. 201–208, May, 2001.

[3] Van Cutsem, T., "Voltage Instability: Phenomena, Counter-measures and Analysis Methods", in Proc. of the IEEE special issue on the Technology of Power System Competition, Vol. 88, No. 2, pp. 208-227, Feb. 2000.

[4] Hiskens, I. A., and Pai, M. A, "Trajectory Sensitivity Analysis of Hybrid Systems," IEEE Transactions on Circuits and Systems, part I: Fundamental Theory and Applications, vol. 47, pp. 204-220, February 2000.

[5] Hiskens, I. A., and Akke M., "Analysis of the Nordel Power Grid Disturbance of January 1, 1997 Using Trajectory Sensitivities," IEEE Trans. Power Systems, vol. 14, nº. 3, pp. 987-994, Aug. 1999.

[6] Rehtanz, C., and Bertsch, J., "Wide Area Measurement and Protection System for Emergency Voltage Stability Control," IEEE Power Engineering Society Winter Meeting, vol. 2, pp. 842-847, 2002.

[7] Hiskens, I. A., and Pai, M. A., Power system applications of trajectory sensitivities, (Invited paper), Proc. IEEE Power Engineering Society Winter Meeting, New York, USA, vol. 2, pp. 1200-1205, Jan. 2002.

[8] Fillâtre, O., Evrard, C., et al., "A powerful tool for dynamic simulation of unbalanced phenomena", Proc. of the International Conference on Advances in Power System Control, Operation and Management, Hong Kong, vol. 2, pp. 526-531, Nov. 1997.

[9] Vernotte, J.-F., Pantiatici, P., Meyer, B., Antoine, J.-P., Deuse, J., and Stubbe, M., "High-Fidelity simulation of power system dynamics", IEEE Computer Applications in Power, vol. 8, pp. 37-41, Jan. 1995, Online. Available: www.eurostag.be.

[10] Eurostag Tutorial, Tractebel Energy Engineering and Réseau de Transport d'Electricité, Aug. 2005.

[11] Astic, J. Y., and Jerosolimski, M., "The mixed Adams BDF variable step size algorithm to simulate transient and long-term phenomena in power systems", IEEE Transactions on Power Systems, vol. 9, pp. 929-935, May 1994.

[12] Zima, M., and Andersson, G., "Stability assessment and emergency control method using trajectory sensitivities", Proc. IEEE Bologna PowerTech, CD-ROM, June 2003.

[13] Laufenberg, M. J., and Pai, M. A., "A new approach to dynamic security assessment using trajectory sensitivities", IEEE Transactions on Power Systems, vol. 13, no. 3, pp. 953-958, August 1998.

[14] Hiskens, I. A. and Pai, M. A., "Sensitivity analysis of power system trajectories: recent results", Proc. of IEEE International Symposium on Circuits and Systems, vol. 3, pp. 439-443, June 1998.

[15] Hiskens, I. A., Pai, M. A., and Sauer, P. W., "An iterative approach to calculating dynamic ATC, Proc. Bulk Power Systems Dynamics and Control IV", Santorini, Greece, pp. 585-590, August 1998.

[16] Monteiro Pereira, R. M., C. Pereira, Adelino J., Machado Ferreira, C., Dias Pinto, J. A., and Maciel Barbosa, F. P., "Reactive Power Support in the Dynamic Voltage Stability Assessment of a Restructured Electric Power System", Proc. of the 41th Universities Power Engineering Conference, UPEC'06, Newcastle, UK, CD-ROM, Sep. 6-8, 2006.

[17] Taylor, C. W., Power System Voltage Stability, McGraw-Hill, New York, 1994.

## BIOGRAPHIES

**R. M. Monteiro Pereira** was born in Cabaços, Portugal in 1974. She received his diploma and M.Sc. degrees in Electrical Engineering and Computers from the Faculdade de Engenharia da Universidade do Porto, FEUP, Portugal, in 1998 and 2003, respectively. In 1998 she joined the Coimbra Polytechnic Institute (ISEC) where she is currently "Equiparada" to Adjunct Professor. She is a Ph.D. student at FEUP and his main research interest includes dynamic voltage stability, influence of a wind farm in the dynamic voltage stability of a power network.

**C. M. Machado Ferreira** received his diploma, M.Sc. and Ph.D. degrees in Electrical Engineering and Computers from the Faculty of Engineering of Oporto University (FEUP), Portugal, in 1991, 1996 and 2006 respectively.

In 1993 he joined the Coimbra Polytechnic Institute (ISEC) and currently holds the position of Coordinator Professor. His main research interest includes electric power system security analysis, control and optimization. He has published several research papers in national and international conferences.

**F. P. Maciel Barbosa** received the "licenciatura" degree (a five years course) in Electrical Engineering from FEUP (Porto University) in 1971 and the M.Sc. and the Ph. D. degrees in Power Systems from UMIST in 1977 and 1979 respectively. His main research interest areas include Power System Reliability and Power System Analysis. He is a full Professor of Electrical and Computer Engineering with FEUP, where he has been since 1971.