Comparison of Different Methods for Optimal Placement of PMUs

A. M. Almutairi, *Student Member, IEEE* and J. V. Milanović, *Senior Member, IEEE*

***Abstract*— The paper compares three different methods for optimal placement of PMUs. The objective of the placement methods is to provide the maximum observability information of the electromechanical modes of interest. The first method is based on the observability factor analysis, the second on the sequential orthogonalization algorithm and the third combines coherency identification technique with the observability factor analysis. The methods are illustrated on the New England test system and assessed by applying a wide-area controller (WAC) for damping electromechanical oscillations in the system. The WAC is designed in each case based on using different PMU placement method. The effectiveness of the controller in damping critical electromechanical modes in the system is assessed using both, small disturbance and transient stability analysis.**

***Index Terms*— coherency, phasor measurement unit (PMU), optimal PMU placement, observability, orthogonalization, wide-area control.**

I. INTRODUCTION

It has been shown in the past e.g., [1], that when using remote (global) signals for a wide-area controller (WAC) the damping of the interarea modes can be highly improved. For such controllers, synchronized measurements at system buses are taken and supplied by Phasor Measurement Units (PMUs) . Due to the costs associated with the installations of PMUs, e.g., communication infrastructure costs, unit cost, installation cost, etc., their number should be minimized. In addition, by minimizing the number of the supplied measurements (input signals) to the WAC the complexity of controller is also reduced. The installed PMUs nevertheless should provide maximum observability of the system modes of interest, e.g. lightly damped or interarea modes. Therefore, the candidate locations for the PMU placement should be optimally selected to minimize the number of units installed to ensure sufficient supply of synchronous information about the modes of interest.

In this paper three methods for optimal placement of PMUs are reviewed and compared. The objective of the three methods is to minimize number of placed PMUs while maximizing amount of information supplied by them. The information of concern here is the observability of the system electromechanical modes of interest. The approach of the first compared method is based on the observability factor (OF) analysis [2]. The second method is based on the Sequential Orthogonalization (SO) algorithm introduced in [3]. The third method is a coherency identification technique combined with the OFs analysis. The methods are illustrated on the New England test system. Results of each placement method are used to design a wide-area controller. The closed-loop systems formed using the wide-area controllers are assessed using small disturbance and transient stability analysis.

II. CONSIDERED METHODS FOR OPTIMAL PLACEMENT OF PMUS

A. *Observability Factor Analysis*

Consider the state space representation of a linearized power system [4]

$$\Delta\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\Delta\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}\Delta\boldsymbol{u}(\boldsymbol{t}) \tag{1}$$

$$\Delta\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{C}\Delta\boldsymbol{x}(\boldsymbol{t}) \tag{2}$$

The observability factors for a linearized power system are computed as follows

$$\boldsymbol{f}_{oj}(\boldsymbol{k}) = \boldsymbol{c}_j\boldsymbol{\phi}_k \tag{3}$$

where $\boldsymbol{c}_j$ is the *j*th row vector of the system output matrix $\boldsymbol{C}$ and $\boldsymbol{\phi}_k$ is the *k*th right (column) vector.

The geometric measure of observability [5] is the dimensionless alternative to the OF in (3). Both measures lead to the same results when one type of outputs is considered, as is the case in this paper. The geometric measure of observability however, can be used, without loss of generality, instead of the OF.

Output locations having the highest OFs for a given mode(s) of interest are selected as the system outputs,i.e., PMU locations.

There are two possible ways of selecting a set of PMU locations which give the maximum observability of the modes of interest. The first is to select locations which have the maximum individual OF for each individual mode of interest, i.e., the individual approach. The second one is to select output locations which have the maximum cumulative sum of OFs for the set of modes of interest, i.e., the cumulative approach.

In the individual approach, number of PMU placements should be at least the same as the number of modes of interest. When only one output location is selected for each mode of interest, the redundancy of information is reduced. For large systems with too many electromechanical modes considered, however, the number of placements (i.e., locations, and therefore required PMUs) will increase. On the other hand,

---

The authors are with the School of Electrical and Electronic Engineering, The University of Manchester, PO Box 88, Manchester, M60 1QD, UK. (e-mail: a.almutairi@postgrad.manchester.ac.uk, milanovic@manchester.ac.uk).

the number of required PMUs can be reduced if the cumulative approach is ussed. The number of placed PMUs in this case however, is set arbitrarily. In addition, redundancy of information cannot be considered.

The method based on the observability factor analysis is introduced first duet to its simplicity and to demonstrate the need to reduce redundancy of observability information while minimizing the number of PMU placements.

B. The SO algorithm

The optimal placement of PMUs using the SO method [3] is based on constructing first the system mode observability matrix, denoted by $\boldsymbol{H}$. The mode observability matrix [4] composed of all the observability factors, for each output of the system, is then reduced by eliminating columns corresponding to the non-electromechanical modes. Then columns corresponding to the modes of interest n_I are selected to form a matrix $\boldsymbol{H_I}$ and similarly the columns corresponding to the other electromechanical modes n_L are selected to form a matrix $\boldsymbol{H_L}$. A weighting factor is then computed for each of the m outputs as follows

$$\boldsymbol{w_i} = \varepsilon + \frac{\left\| \boldsymbol{h_{Li}} \right\|_2}{\left\| \boldsymbol{h_{Ii}} \right\|_2} \tag{4}$$

where $\boldsymbol{h_{Li}}$ is the ith row of $\boldsymbol{H_L}$ and $\boldsymbol{h_{Ii}}$ is the ith row of $\boldsymbol{H_I}$, $\boldsymbol{\varepsilon}$ is a constant determining the sensitivity to the other modes, and $\| \;\|_2$ is the Euclidean norm of a vector. The higher the value of $\boldsymbol{\varepsilon}$, the higher the toleration of the effects of other modes.

Each row of $\boldsymbol{H_I}$ when divided by the corresponding weighting factor will yield the weighted modal observability matrix, $\boldsymbol{Q}$ of the modes of interest for each output. Then values of the norms for each row in $\boldsymbol{Q}$, $\left\| \mathbf{q_i} \right\|_2$, are ranked and the corresponding output having the largest value will be the first selected location, denoted as the reference location. The subsequent locations are then selected based on orthogonalizing the corresponding rows of $\boldsymbol{Q}$ to the reference location.

The main feature of the SO algorithm is the consideration of the sensitivity to the other modes in the selection procedure. In addition, the orthogonalization procedure ensures the non-redundancy in the information contained by the subsequent selections. Each subsequent selection is based on adding sufficient new information, weighted modal observability, to the set of previous selections. The algorithm stops the selection procedure when no new and sufficient information can be added by the rest of subsequent candidate locations. Therefore the minimum number of placed PMUs is determined as well as their locations while having the maximum non-redundant observability information of the modes of interest.

C. Combined Coherency and Observability Factor Analysis

The combined coherency identification and observability factors analysis (COFA) placement method partitions the network generators into coherent groups and then select one (or more) representative generator(s). The coherency identification method used here is the PCA-based cluster analysis [6]. The method processes the generators' time responses following a large disturbance and then transforms it into a linear combination of uncorrelated (orthogonal) variables called the principal components. The coefficients of these principal components are then used to construct a proximity matrix, using the Euclidean distance (norm), between their coordinates. A cluster analysis method is then applied to this proximity measure to cluster generators into coherent groups.

The OF analysis method is then applied to select location of representative generator (arbitrarily limited to one) for each coherent group of generators. The selection of the representative generator for each coherent group is based on the maximum cumulative sum of OFs corresponding to the modes of interest. In this way, the minimization of the placed PMUs is dependent on the coherency partitioning of the network. The observability information will be ensured to be non-redundant as output measurements are supplied from different coherent regions.

III. COMPARISON OF CONSIDERED METHODS

The three PMU placement methods considered in this study are illustrated on the New England Test System (NETS) shown in Fig. 1. Full details about NETS can be found in [7, 8]. Each generator in the network is equipped with PSS. Models and parameters of the used AVRs and PSSs are listed in the Appendix. The NETS system was modeled and linearized in Matlab and Simulink environment. The system outputs (p=10) are generators' speeds. The system eigenvalues corresponding to the electromechanical modes are listed in Table I. Modes 7-9 are considered (arbitrarily for the purpose of illustrating proposed methodologies) as the modes of interest.

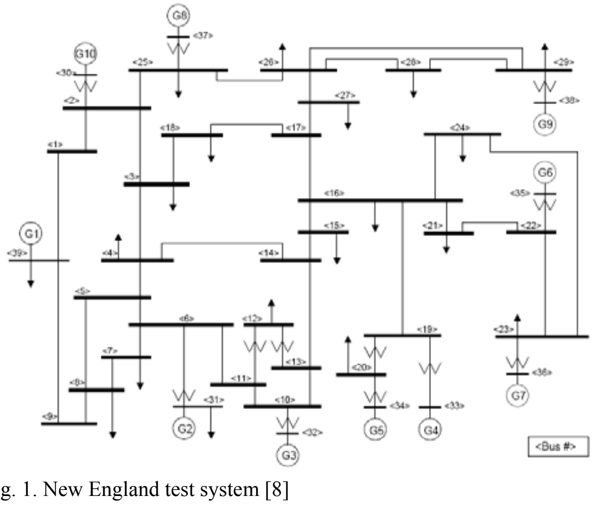

Fig. 1. New England test system [8]

A. Observability Factor Analysis

The OF magnitudes corresponding to each individual electromechanical mode are listed in Table II. The minimized set of placed PMUs is selected such that it provides the maximum observability of the modes of interest. Generators with the highest OFs (underlined in Table II) for each individual mode of interest are selected (arbitrarily limited to one location for each mode of interest) for PMU placements. The selected set of locations is listed in Table III. It should be noted that there is no selection sequence (order) for this chosen set of PMU locations.

The cumulative sum of OF magnitudes, corresponding to the modes of interest for each output location in the system, are shown in the last row (in bold font) of Table II. The generators with highest cumulative sum, shown underlined in Table II, are selected for the PMU placement as shown in Table III. It can be seen from the table that the chosen locations (arbitrarily limited to 3) are ordered according to the ranking of their cumulative sums of OFs corresponding to the modes of interest. Therefore, the use of cumulative sum of the OFs for the modes of interest has the advantage of ordering the selected locations.

B. The SO algorithm

The SO algorithm is applied to the NETS system and the optimal selected locations for the PMU placement are listed in Table III. The number of selected PMUs is minimized to 3 and they are located far away from each other. In should be noted here that the optimal placement of PMUs is highly influenced by the chosen set of the modes of interest. The same influence is noticed when applying the OF analysis method.

TABLE I
EIGENVALUES LOCATIONS CORRESPONDING TO THE ELECTROMECHANICAL MODES

Mode no.	Eigenvalues [1/s +j rad/s]	Frequency [Hz]
1	-2.67 ± j 9.38	1.49
2	-1.51 ± j 8.96	1.43
3	-1.78 ± j 9.03	1.44
4	-2.49 ± j 8.14	1.30
5	-0.53 ± j 7.42	1.18
6	-1.99 ± j 7.55	1.20
7	-1.52 ± j6.89	1.10
8	-1.68 ± j6.79	1.08
9	-0.44 ± j2.55	0.41

TABLE II
THE OBSERVABILITY FACTORS MAGNITUDES (NORMALIZED TO THE HIGHEST) CORRESPONDING TO THE ELECTROMECHANICAL MODES

Output Location	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
Mode#1	0.01	0.17	0.13	0.10	0.15	0.24	0.21	1.00	0.16	0.27
Mode#2	0.01	1.00	0.23	0.05	0.04	0.03	0.13	0.08	0.03	0.10
Mode#3	0.00	0.14	0.05	0.03	0.04	0.80	1.00	0.08	0.03	0.05
Mode#4	0.01	0.21	0.26	0.18	0.55	1.00	0.81	0.80	0.38	0.68
Mode#5	0.00	0.02	0.08	1.00	0.27	0.10	0.13	0.02	0.07	0.04
Mode#6	0.01	0.01	0.05	0.48	1.00	0.19	0.23	0.10	0.11	0.15
Mode#7	0.01	0.04	0.32	0.15	0.27	0.09	0.07	0.15	1.00	0.17
Mode#8	0.02	0.17	1.00	0.15	0.26	0.10	0.10	0.12	0.54	0.13
Mode#9	0.57	0.34	0.43	0.62	1.00	0.85	0.84	0.51	0.59	0.42
Modes 7-9	**0.60**	**0.55**	**1.75**	**0.92**	**1.53**	**1.03**	**1.01**	**0.78**	**2.14**	**0.71**

TABLE III
SELECTED SITES FOR PMU PLACEMENT USING THE THREE COMPARED METHODS

Placement Method	(1) Observability Factors Analysis		(2) The SO algorithm	(3) The Combined Method of Coherency Identification and Observability Factors Analysis
	Individual Observability Factors	Cumulative Sum of Observability Factors		
Selected Locations	G9	1st. G9	1st. G1	G4
	G3	2nd. G3	2nd. G3	G1
	G5	3rd. G5	3rd. G9	G9

C. Combined Coherency and Observability Factor Analysis

A three phase fault at bus#16 is simulated and all generators' speeds are processed by the PCA-based cluster analysis method. The fault location was selected to be in the middle of the network to reduce the coherency method dependency on the disturbance location.For further discussion of this dependency please see [6].

The results of the clustering procedure are illustrated using a multilevel hierarchical tree shown in Fig. 2. According to the interpretation of the clustering results discussed in [6] the vertical heights of links determine the coherent groups. As shown in Fig. 2 generators of the NETS system are partitioned into 3 coherent groups (shown by ellipses) and are listed in Table IV. The OF analysis is then applied to select the location of representative generator (arbitrarily limited to one) for each coherent group of generators. The selection is based on the maximum cumulative sum of OFs corresponding to the modes of interest, shown in Table II. The selected locations (representative generators) are listed in Table III and Table IV. It can be seen that the placed PMUs using this approach are located in different geographical regions. The number of chosen locations was limited to one location for each coherent group of generators. Therefore the optimal number of PMUs cannot be achieved using this combined method, as was the case with the SO method. In addition a selection sequence in this case cannot be achieved neither.

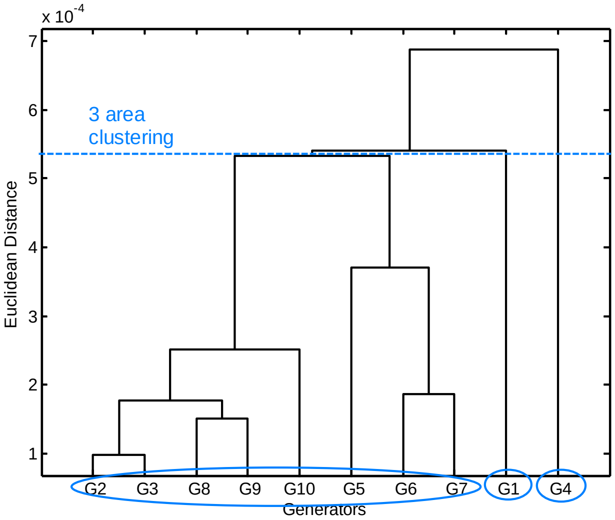

Fig. 2. Clustering tree (top view) of the NETS system for a three phase fault at bus#16, the horizontal dashed line shows the clustering into 3 groups

TABLE IV
SELECTED SITES FOR PMU PLACEMENT USING THE COMBINED METHOD OF COHERENCY IDENTIFICATION AND THE OBSERVABILITY FACTORS ANALYSIS

	PCA-based Cluster Analysis	Observability Factors Analysis
Group no.	Group Members	Representative Generator
1	G4	G4
2	G1	G1
3	G2, G3, G8, G9, G10, G5, G6, and G7	G9

IV. ASSESSMENT OF CONSIDERED METHODS

A. *Application of Placement Methods to WAC*

The compared three placement methods were assessed by applying a wide-area controller to the NETS system. The configuration of the multivariable power system, i.e. Multi-Input Multi-Output (MIMO), control is shown in Fig.3. Note that input reference signals are omitted in the figure for simplicity. The PMU measurements (outputs of the system) are supplied to the wide-area controller (controller inputs) through the feedback control loop. The placement method chooses a subset of measurement locations p_1 out of p candidate locations. The wide-area controller then sends back the control signals (inputs) to the system. The number of control signals should, ideally, be reduced similarly to the output measurements from m candidate signals to m_1 signals. The selection of these control input locations is however out of the scope of this paper.

The reduced control input signals locations can be considered from the same locations chosen by the placement method. In this way the communication costs will be reduced as existing communication channels, i.e., channels for sending PMU measurements to the wide-area controller, are used to for sending back control input signals. The PMU placement methods considered mode observability for the optimal locations instead of mode controllability which would be needed in case of deciding on optimal control inputs. As a result, the assessment of the effectiveness of PMU placement methods may not be entirely correct in this case. To avoid this, the control signals are sent to all generators in the system. In this way the comparison will be based entirely on the difference of output measurement locations based on mode observability. It should be noted that communication time delays, which naturally accompany application of WACs in the system, are not considered here as the aim of the paper is to compare only the PMU placement methods based on electromechanical mode observability.

The wide-area control configuration is illustrated in Fig. 4. The WAC provides a supplementary control signal V_{WAC} through the Automatic Voltage Regulator (AVR) added together with the existing PSS signal V_{PSS}. The inputs to the WAC are coming from a selected sub-set of generators specified by the PMU placement method. The wide-area control signals on the other hand are sent to all generators in the network.

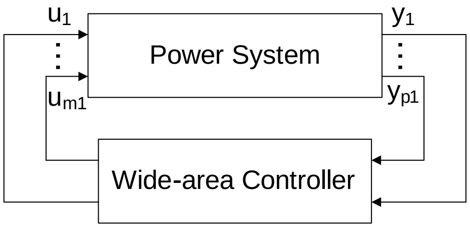

Fig. 3. Configuration of multivariable power system control

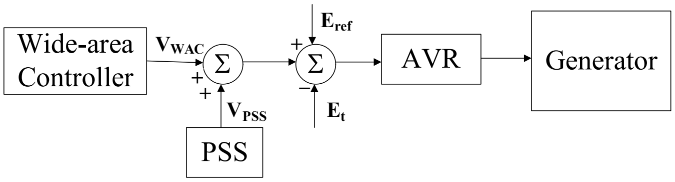

Fig. 4. Wide-area control configuration

The WAC is designed based on the Linear Quadratic Gaussian (LQG) control [9]. Outputs of the closed-loop system, i.e. controller inputs, are active power, terminal voltage, and speed deviation of generators chosen by the PMU placement method. The terminal voltage and active power measured signals were added, in addition to speed deviation, to increase the accuracy of the estimation process by the Kalman filter and hence the robustness of the LQG controller. The tuning parameters of the LQG controller were kept fixed for all compared closed-loop systems in order to make a fair comparison and assessment of the considered PMU placement methods.

The compared closed-loop systems are the closed-loop system based on placement results of the OF analysis method (CL-Method 1, in Figures 5 to 7), closed-loop system based on placement results of the SO algorithm (CL-Method 2, in Figures 5 to 7), closed-loop system based on placement results of the combined coherency and OF analysis (CL-Method 3, in Figures 5 to 7).

B. Small Disturbance Stability Assessment

The improvement of the damping ratio of each electromechanical mode is computed for the three closed-loop systems formed using different sets of output signals, and with all input signals, supplied by the placed PMUs. The results are listed in Table V. Improvement is computed as the difference between the damping ratio of the mode in the open-loop system (without WAC) and the damping ratio of the same mode in the closed-loop system (with WAC).

TABLE V
DAMPING RATIOS OF THE ELECTROMECHANICAL MODES OF THE OPEN-LOOP AND CLOSED LOOP SYSTEMS

Mode	CL-Method 1	CL-Method 2	CL-Method 3
Mode#1	0.70	0.65	0.71
Mode#2	1.52	1.52	1.40
Mode#3	0.00	0.21	0.52
Mode#4	1.26	0.88	1.31
Mode#5	1.13	0.47	0.98
Mode#6	0.99	0.26	0.77
Mode#7	**11.78**	**17.70**	**16.89**
Mode#8	**0.87**	**2.12**	**1.67**
Mode#9	**3.15**	**5.55**	**5.76**

It can be seen from Table V that the damping ratios of the modes of interest, modes 7-9 (shown in bold font), in general have been improved in all of the closed-loop systems. It can be seen also that the closed-loop systems formed using outputs measurements supplied from PMU locations placed by both method 2 and method 3 improve damping of modes of interest significantly and much better than in the case of method 1. The overall improvement achieved by method 2 is slightly better than that achieved with of the method 3.

C. Large Disturbance Stability Assessment

The large disturbance (transient) stability assessment of the closed-loop systems was also performed. A three phase self-clearing fault, lasting 4 cycles, was simulated at bus 16 in the open-loop and the closed-loop systems. The resulting speed responses of generators 1, 3 and 9 are shown in Fig. 5, 6, and 7, respectively.

It can be seen from these figures that the WAC that uses output measurements supplied by PMUs placed by method 2 and method 3 enhances the damping in the system and performs better than in the case when method 1 is used for PMU placement. This coincides with the results of small disturbance analysis.

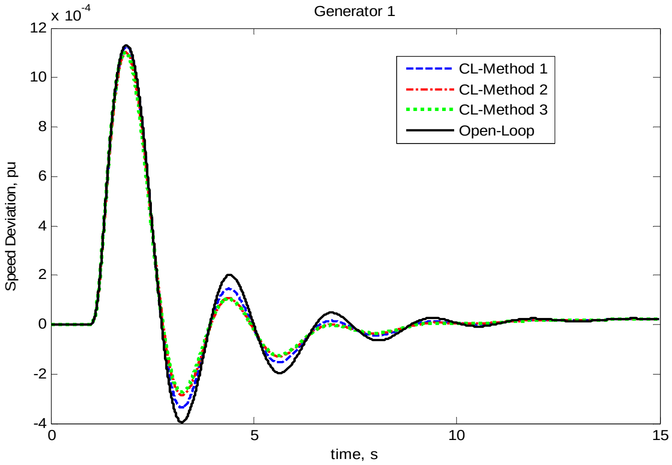

Fig. 5. Speed deviation responses of generator 1 for a three phase fault at bus#16

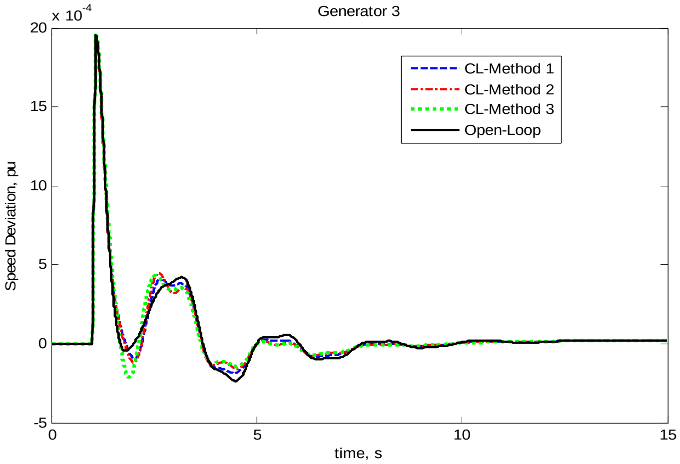

Fig. 6. Speed deviation responses of generator 3 for a three phase fault at bus#16

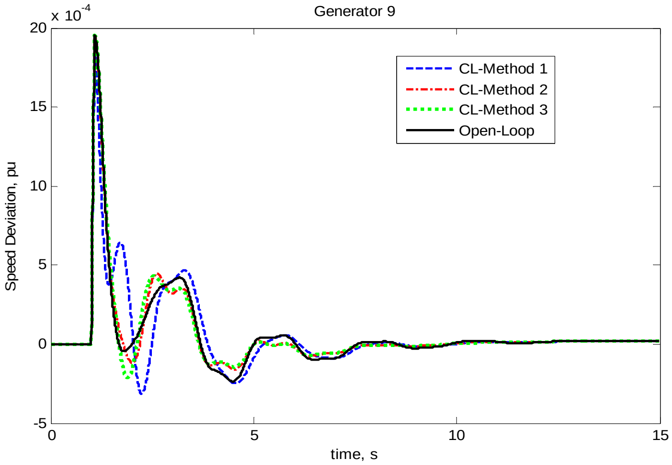

Fig. 7. Speed deviation responses of generator 9 for a three phase fault at bus#16

V. CONCLUSIONS

The paper compared and discussed three different methods for optimal placement of PMUs. The methods discussed include approach based on the observability factor analysis, on the sequential orthogonalization algorithm and on the combination of coherency identification technique and the observability factor analysis.

The aim of the paper was not to propose the best method for the optimal PMU placement but to illustrate and critically asses different possible approaches. The particular features and limitations of the considered methods, in the context of number of PMUs required and redundancy of information, were also addressed.

The signals coming from the PMUs placed using the three compared methods were then used as the inputs to the multi-input multi-output wide area supplementary controller. The controller is designed using Linear Quadratic Gaussian control method. It is applied in the New England test system and its effectiveness in damping critical electromechanical modes is assessed using both small disturbance and transient stability analysis.

Designed WAC using signals derived based on the Sequential Orthogonalization algorithm and the combined coherency and observability factor analysis performed similarly in case of both small and large disturbance analysis. In both these cases WAC performed better than in the case when its input signals were derived using the method based only on the observability factor analysis.

APPENDIX

TABLE
PSS PARAMETERS

Generator	K_{PSS}	T_1	T_2	T_3	T_4
G1	20	0.8685	0.44689	0	0
G2	17	0.4742	0.1179	0.4742	0.1179
G3	8	0.4559	0.13156	0.4559	0.13156
G4	26	0.4247	0.09687	0.4247	1.09687
G5	10	0.5538	0.1291	0.5538	0.1291
G6	9	0.5056	0.1007	0.5056	0.1007
G7	8	0.4264	0.1071	0.4264	0.1071
G8	8	0.4594	0.08113	0.4594	0.08113
G9	10	0.7820	0.0925	0	0
G10	25	0.5339	0.1037	0.5339	0.1037

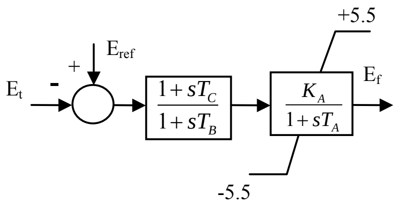

Fig. 8. AVR model (IEEE type AC4A excitation system model)

The parameters of the AVRs are: T_A=0.055, T_B=10, T_C=2, K_A=198.

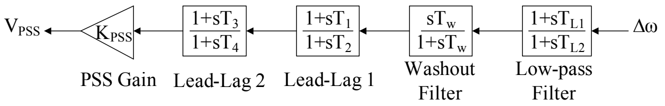

Fig. 9. PSS model

The parameters of the local PSSs are: T_{L1}=0.0563, T_{L2}=0.1125, T_w=10.

REFERENCES

[1] M. E. Aboul-Ela, A. A. Sallam, J. D. McCalley, and A. A. Fouad, "Damping controller design for power system oscillations using global signals," *IEEE Trans. Power Syst.*, vol. 11, pp. 767-773, 1996.

[2] N. Martins and L. T. G. Lima, "Determination of suitable locations for power system stabilizers and static VAR compensators for damping electromechanical oscillations in large scale power systems," *IEEE Trans. Power Syst.*, vol. 5, pp. 1455-1469, 1990.

[3] E. W. Palmer and G. Ledwich, "Optimal placement of angle transducers in power systems," *IEEE Trans. Power Syst.*, vol. 11, pp. 788-793, 1996.

[4] P. Kundur, "*Power System Stability and Control*": McGraw Hill, 1993.

[5] A. M. A. Hamdan and A. M. Elabdalla, "Geometric measures of modal controllability and observability of power system models," *Elect. Power Syst. Research,* vol. 15, pp. 147-155, 1988.

[6] A. M. Almutairi, S. K. Yee, and J.V.Milanovic, "Identification of coherent generators using PCA and cluster Analysis," in *Proc. Power Systems Computation Conference -PSCC2008*, Glasgow, 2008.

[7] R. D. Zimmerman, C. E. Murillo-Sánchez, and D. Gan, "MATPOWER", http://www.pserc.cornell.edu/matpower/.

[8] http://psdyn.ece.wisc.edu/IEEE_benchmarks/index.htm.

[9] J. M. Maciejowski, "*Multivariable Feedback Design*", UK: Addison-Wesley, 1989.

Abdulaziz M. Almutairi (S'2006) received the B.Sc. degree from Kuwait University, Kuwait, and the M.Sc. degree form the University of North Carolina at Charlotte, NC, USA.

He is currently a Ph.D. student in the School of Electrical and Electronic Engineering at The University of Manchester, UK.

Jovica V. Milanović (M'95, SM'98) received his Dipl.Ing. and his M.Sc. degrees from the University of Belgrade, Yugoslavia, his Ph.D. degree from the University of Newcastle, Australia, and his D.Sc. degree from The University of Manchester, UK, all in Electrical Engineering.

Currently, he is a Professor of electrical power engineering and Deputy Head of School (Research) in the School of Electrical and Electronics Engineering at The University of Manchester (formerly UMIST), UK.