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# Distributed System for Dispatching of Generation in Large-Scale Electrical Power Systems

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*Abstract*—This paper presents the principles and foundations on which construction and functioning of control system destined for optimal control of large integrated electrical power systems are based. These large interconnections consist of power systems of independent countries and independent markets of electricity. The most important problem, which should be solved in creation of such system is the necessity to combine in it on the one hand the ability to achieve global optimality of interconnection as a whole and on the other hand the ability to maintain self-determination of system operators in power systems of independent countries and restrictions on access to their internal information.

It is shown in this paper that this problem can be solved in distributed control system having hierarchical structure and containing computers, belonging to different hierarchical levels of the system. In this paper general structure and functioning of this system, based on the algorithms of functional modelling (FM), are presented.

*Index Terms*— Distributed algorithms, distributed control, hierarchical systems, interconnections, optimal control, optimization methods, power generation dispatch, functional modeling method.

#### I. INTRODUCTION

Initial representation of a power system, i. e. mathematical model of a system, is the basic factor determining the structure of the algorithms intended for calculation of steadystate and transient operating conditions as well as solution Of operational planning problems in these systems. The well known and commonly used representation of a system, on which these algorithms are based, has the form of system of equations pertaining to a system as a whole. Solution of a technical problem in this case can be reduced to solution of this system of equations. All data representing this system of equations are usually stored on one computer and all calculations necessary for solution of this system are executed in this case on this computer. Algorithms based on such representation are usually implemented as serial algorithms on one computer systems.

Another type of representation, more efficient for large power systems, has the form of several systems of equations, each of them pertaining to one of parts of large power systems called subsystems. In the algorithms, based on such decomposition of original system, solution of a technical problem can be performed on several computers, each of them charged with solution of system of equations for one of of subsystems. Accordingly all data pertaining to each subsystem are stored in one of these computers and all internal variables of this subsystem are calculated in it. Algorithms based on this representation are usually implemented as parallel and distributed algorithms on parallel and distributed computer systems.

The main difficulty in application of this concept lies in calculation of boundary variables, i. e. state variables, pertaining to borders between subsystems. This difficulty is caused by the fact that the values of these variables should on the one hand satisfy equations of two subsystems adjacent to boundary nodes and on the other hand satisfy matching conditions for the values of these variables when they are computed in adjacent subsystems.

General approach of relaxation algorithms is applied in order to satisfy these conditions in the decomposition methods. Due to it solutions, obtained in different subsystems should be coordinated in order to compensate mismatches in values of boundary variables found in adjacent subsystems. Therefore convergence properties of the algorithms of this type ( they can be called decomposition – coordination algorithms ) are inferior to those of the basic serial algorithms from which they are derived [6], [7].

In this paper we present algorithms based on hierarchical representation of a power system. This representation has the form of the system of systems of equations, belonging to different levels of hierarchical model. In the simplest case this model includes N systems of equations of lower level and one

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system of equations of upper level. Internal and boundary variables of subsystems are present in the systems of equations of the lower level whereas only boundary variables are present in the system of the higher level.

Composition of the hierarchical models is based on the functional modeling (FM) method presented in [1], [2], [3]. Key element in the FM method is the concept of functional characteristic (FC), that is mathematical representation of subsystem on the higher level of model as a black box. In FC only boundary variables (and in some cases integral variables) of subsystem are present.

Hierarchical algorithms based on this method are intended mainly for solution of power system problems on distributed and parallel computer systems. Important advantage of these algorithms is that numerical results obtained by them on each iteration are identical to those of the basic serial algorithms from which they are derived. Consequently convergence properties of these algorithms are exactly the same as those of the basic serial algorithms.

By this time hierarchical FM algorithms have been developed for solution of power flow and state estimation problems as well as for optimal operation and dynamic simulation problems in large power systems.

#### II. OUTLINE OF THE FUNCTIONAL MODELLING METHOD

The main principles of the FM method, presented in [1], [2] [3] are as follows.

1. Representation of a technical system as a set of subsystems, adjoining each other in boundary nodes.

2. Building of a model as a hierarchical structure, consisting of interconnected systems of equations. In this structure subsystems are represented by lower level systems of equations. A higher level system of equations represents borders between subsystems (boundary nodes).

3. Representation of subsystems on the higher level of model by functional characteristics (FCs). FCs are inputoutput characteristics in which vectors of boundary variables of one kind are considered as input variables and boundary variables of another kind as output variables. These characteristics are obtained while meeting all constraints within subsystem.

4. Determination of the values of boundary variables on the higher level of the model through formation and solution of the system of connection equations (SCE), obtained from general expressions for boundary variables, pertaining to all boundary nodes.

Solution of modeling and optimization problems in accordance with FM method consists of the following steps:

1. Model formation in which structure and parameters of a hierarchical model are determined so that the solution process would be optimal.

2. Model functioning, which includes in it's turn following main steps:

a) determination of the FC's of subsystems up to the highest level of the model- upward move;

b) formation and solution of the SCE on the highest level of the hierarchical model, finding in this way the values of boundary variables of this level;

c) calculation of the internal variables of subsystems down to the level of the model- downward move.

Subject to optimization in formation of a hierarchical model in this method are the number of levels of analysis, the number of subsystems on each level, organization of interaction between the levels and other parameters. In this paper we consider only the case of two levels of analysis in the hierarchical model and one level of subsystems.

In accordance with representation of a technical system as a set of subsystems, adjoining each other, all variables in a hierarchical model can be divided into two types: boundary and internal variables.

Boundary variables are the variables pertaining to boundary nodes, e.g. currents and powers crossing the boundary nodes, voltages in the boundary nodes. All other variables in the model are internal variables of subsystems. The variables of these two types are shown in Fig. 1.

According to the general definition given above the FC of subsystem in steady state problems can be presented as follows

$$V = F(X) \tag{1}$$

where X is the vector of input variables of subsystem and V is the vector of output variables. It is assumed in the FM method that in steady state problems these vectors consist of boundary variables.

It is implied according to the definition of FC that expression (1) should be obtained under condition that the whole set of equality and inequality constraints within subsystem is observed.

If these sets of internal constraints are written together for all subsystems, composing an electric system, it yields

$$G_J(W, V, X) = 0$$
  $J = 1,.., M$  (2)

where W is the vector of internal variables in the subsystem J, V and X are vectors of output and input variables in this subsystem, M is the total number of subsystems in system.

FCs of subsystems should be found from the sets of internal constraints, entering the system (2).

Complete description of electrical system as a set of subsystems can be obtained if the system (2) is supplemented with the system of equations for boundary variables

$$\sum_{J \subset J_i} V_i^J = 0 \qquad i = 1, ..., k$$

$$x_i^J = x_i \qquad J \subset J_i \qquad i = 1, ..., k$$
(3)

following from Kirchhoff law written for these variables.

It is assumed in (3) that the output variables of subsystems are currents or powers and the input variables are voltages.

For any variable v in the subsystem J we have also from (1)

$$\boldsymbol{v}_i^J = \boldsymbol{F}_i^J(\boldsymbol{X}_J) \tag{4}$$

This expression can be considered as FC of subsystem J presented in explicit form.

Formation of the SCE in the FM method is based on substitution of expressions for boundary variables in the right hand side of FCs (4) into equations (3). It results in the following system of equations

$$\sum_{J \subset J_i} F_i^J(X_J) = 0 \qquad i = 1, ..., k \quad (5)$$

where  $J_i$  is a set of subsystems adjacent to the i - th boundary node.

The dimension of the SCE (5) is equal to the number of boundary variables considered as input variables for subsystems.

Solution of (5) yields the vector X of boundary variables.

Vectors of internal variables W in all subsystems should be found after that by back substitution of subvectors of the vector X, pertaining to all subsystems, into equations (2) and solution of these systems.

In case of optimization problems solved by the FM method the FCs of subsystems may take the following form

$$\lambda_J = F(X_J) \tag{6}$$

where  $\lambda_J$  is the Lagrange multiplier for the internal balance of power equation in subsystem *J*,  $X_J$  is the vector of input variables in this subsystem.

These FCs should be found from the sets of internal constraints in subsystems, constituting the following system similar to (2)

$$E_J(\lambda, W, X) = 0$$
  $J = 1,.., M$  (7)

where W is the vector of internal control variables in the subsystem J.

Each of the systems in (7) includes optimality equations for one of subsystems in the hierarchical model of power system.

Vectors of input variables of subsystems in (7) together form the vector of boundary variables of the hierarchical model. These variables are considered in optimization problems as control variables of the higher level in this model.

Original equations for determination of optimal values of boundary variables can be obtained taking the first derivatives of Lagrange function for hierarchical model with respect to each of the boundary variables and setting these derivatives equal to zero. It results in the following set of equations

$$\lambda_{lb} - \lambda_{lb} = 0 \tag{8}$$

where index  $I_b$  denotes for one of subsystems adjacent to the boundary node *b*, and  $J_b$  denotes for another.

Substitution of expressions for boundary variables in the right hand side of FCs (6) into equations (8) gives the SCE

$$F_{lb}(X_{lb}) - F_{lb}(X_{lb}) = 0 \qquad b = 1, \dots n_b.$$
(9)

Vector of optimal values of boundary variables should be found after it from solution of (9). In optimal power flow problems these boundary variables are power flows between subsystems.

Then back-substituting the values of power flows on the borders of subsystems into (6) we find the value of  $\lambda_i$  and then back-substituting into equations (7) the values of optimal internal variables in subsystems.

The FM method presented above on general lines is applicable both for solution of linear and non-linear steadystate problems in electrical systems. The FCs which are used in algorithms solving these problems can be linear and nonlinear.

In case if linear FCs are used for representation of subsystems in hierarchical FM algorithms several iterations are necessary for solution of nonlinear steady-state problems. On each of these iterations a linear problem is solved with one upward and downward move in hierarchical structure of algorithm.

Final and intermediate results (on each iteration) obtained by these algorithms are identical to those of basic algorithms from which these hierarchical algorithms are derived. By a basic algorithm we mean an algorithm intended for solution of a system of equations pertaining to electrical system as a whole.

# III. HIERARCHICAL FM ALGORITHMS FOR SOLUTION OF ECONOMIC DISPATCH PROBLEMS

If the FM method is applied to solution of economic dispatch problem in large power system this system should be represented as a set of subsystems adjoining each other [2], [4]. If power flows through boundary nodes of subsystems are considered as boundary variables and there is only one boundary node between each pair of adjoining subsystems Lagrange function for this model can be constructed as a sum of Lagrange functions of subsystems in following form

$$L_{s} = \sum_{I} \sum_{i} F(P_{iI}) + \sum_{I} \left[ \lambda_{I} \left( P_{LI} - \sum_{iI} P_{iI} + \sum_{bI} \pm P_{bI} \right) \right]$$
(10)

where  $P_{iI}$  is the power of the station *i* in the subsystem *I*,  $P_{bI}$  is the power flow through the boundary node *b* adjacent to

Note that  $P_{bi}$  enters this function with sign + for one subsystem and sign – for adjacent subsystem.

Necessary condition for an extreme value of the objective function (10) can be obtained taking the first derivative of (10) with respect to each of the boundary variables and setting these derivatives equal to zero. It results in the following set of equations

$$\lambda_{Ib} - \lambda_{Jb} = 0 \qquad b = I, \dots, n_b \tag{11}$$

where index  $I_b$  denotes for one of subsystems adjacent to the boundary node *b*, and  $J_b$  denotes for another.

Each of equations in (11) applies to one of the boundary nodes between subsystems in this model and total number of these equations is equal to the number of these nodes.

The systems of internal equations, representing the minimum cost operating conditions for subsystem I if network losses are not taken into account look as follows

$$\frac{dF_{iI}}{dP_{iI}} - \lambda_I = 0 \tag{12}$$

$$P_{LI} - \sum_{iI} P_{iI} + \sum_{bI} \pm P_{bI} = 0$$
(13)

Upper equations in this system are obtained from the condition that the first derivatives of the function (1) with respect to internal variables  $P_{iI}$  should be equal to zero. Eq. (13) is the constraint equation of subsystem *I* (balance of power equation).

Finding expressions for the first derivatives of the cost functions of stations we can present Eq. (12) in explicit form

$$a_{iI}P_{iI} + b_{iI} - \lambda_I = 0 \tag{12a}$$

In this case it is assumed for simplicity that the cost function has the form of a quadratic function.

Solution of the economic dispatch problem by the hierarchical FM algorithm consists of the following steps:

*l* Formation of Eqs. (3), (4) and determination of the FCs of subsystems. Applying the Gaussian elimination of internal variables (i.e.  $P_{il}$ ) to the system (4), (4a) for each of the subsystems the FCs of subsystems should be determined, having the following form

$$\lambda_I = a_I \sum_{bI} \pm P_{bI} + c_I \tag{14}$$

2 **Formation and solution of the SCE.** Substituting for  $\lambda_{lb}$  and  $\lambda_{Jb}$  from right-hand side of Eq.(14) for FCs of the subsystems I and J (adjoining to boundary node *b*) into Eq. (11) yields one of the equations, forming the SCE. In this

way equations for all boundary nodes should be obtained. Set of these equations forms the SCE shown below

$$A P_b = b \tag{15}$$

Solution of this linear system gives optimal values of power flows between subsystems.

3 **Determination of power outputs of stations in subsystems.** Substituting the values of optimal power flows on the borders of subsystems into Eq. (14) gives the values of Lagrange multipliers  $\lambda_I$  in subsystems. Then back substituting  $\lambda_I$  into Eq. (12) optimal power outputs of stations in subsystems should be found. These values are exactly the same that can be obtained by basic one level algorithms for solution of this problem.

It is important to note that in the systems of equations of subsystems values of boundary variables (power flows between subsystems) formed on step 1 in this method are considered as unknown variables.

Algorithm presented above illustrates general structure of the hierarchical FM algorithms. Detailed description of these algorithms is given in [2] and [5].

#### IV. DESCRIPTION OF DISTRIBUTED SYSTEM FOR DISPATCHING OF GENERATION IN POWER SYSTEMS

General structure of the FM algorithms and principles of the FM method constitute a basis on which construction and functioning of distributed control systems intended for optimal control of large integrated electrical power systems can be founded. These large power systems (interconnections) consist of power systems of independent countries and independent markets of electricity. The most important problem, which should be solved in creation of such system is the necessity to combine in it on the one hand the ability to achieve global optimality of interconnection as a whole and on the other hand the ability to maintain self-determination and local optimality of power systems of independent countries and restrictions on access to their internal information.

This problem can be solved in distributed control system having hierarchical structure and containing computers, belonging to different hierarchical levels of the system [8].

Figure 1 shows the configuration of this system.

As can be seen from the Figure 1 distributed system for dispatching and controlling of generation in electric power system consisting of a plurality of subsystems, comprises a central computer that is a higher-layer computer.

This system comprises also a specified dispatching optimization module, solving the problem of optimal interchange of power and energy between subsystems. This module is located in the higher-layer computer.

This controlling system further comprises a plurality of computers according to a number of subsystems, said computers being lower-layer computers each comprising a specified subsystem dispatch optimization module designed to determine parameters for an optimal dispatch of generation between power plants within a subsystem, and a module for computation of functional characteristics for each subsystem.

Each lower-layer computer is connected by lower-layer communications means to respective power plants of respective subsystem.

Said controlling system also comprises higher-layer communications means, wherein the lower-layer computers are connected to a higher-layer computer via the higher-layer communications means.

In solution of optimal power flow problems (as well as other optimization problems) this system operates in following way.

1. All information relating to one of subsystems and being necessary to solve a problem of computing an optimum dispatch of generation in the electric power system is transmitted through the lower-layer communications means to a respective lower-layer computer. This information stream is denoted by InI (Figure 1). This information contains data *on* input-output characteristics of power generation units included in respective subsystems of the EPS for systems operating under *regulation* or information of electrical energy tariffs for plants in a system operating under regulated market conditions, or information about bids for selling electric energy of individual plants within a system operating under competitive market conditions.

2. Then computation of a functional characteristic corresponding to a system of equations internal optimal dispatch of generation computed in the optimization module is performed for each subsystem.

3. Functional characteristic data for each subsystem are supplied through the higher-layer communications means in information streams denoted by FC to the higher-layer computer.

4. This computer builds up and solves the system of connection equations (SCE) on the basis of the obtained data. As a result of solving this system of equations, a vector of optimal values for boundary variables in a specified iteration, that is inter-subsystem power flows, corresponding to parameters of functional characteristics of subsystems on this iteration, is determined.

5. A sub-vector of a complete vector of values taken by boundary variables, pertaining to each subsystem, is then transmitted through the higher-layer communications means in information stream BV and is directed to that of lowerlayer computers, which is located in this subsystem.

6. The optimization modules then compute again values of the internal variables that meet values of the boundary variables computed in the higher-layer computer. The units then compute again functional characteristics of each subsystem for computed values of internal variables, and information of said characteristics is transmitted in information streams to the higher-layer computer that newly builds up and solves the system of connection equations.

Power flow values in individual iterations can be computed as complete values and as increments to the flow values at a specified iteration. In the latter case, said values compensate for residuals in equations that represent optimality conditions for power flow values. If residual values  $\Delta S$  in the optimality equations reach values not higher than specified values, that is, E, the iteration process stops, and a flow vector resulted from the final iteration is considered as a vector of optimal power flow values.

The respective message and the resulted vector are transmitted to the lower-layer computers. In this case, the values of the internal variables computed once more for subsystems in the resulted optimal dispatch of generation are transmitted through the lower-layer communications means to each subsystem for execution. When the optimality conditions are not met in the higher-layer computer , the iteration process continues.

Important features of this control system are the following.

1 All operations on the lower level of the system (i. e. formation of systems of equations for subsystems, determination of their FCs and calculation of internal variables) are executed concurrently by computers of lower level, placed within the limits of all subsystems.

2. Convergence of iteration processes in the distributed algorithms working in this system is the same as in the corresponding basic algorithms.

3. Data delivery between computers in this system is limited to delivery of data on FCs and boundary variables. No data on internal parameters and internal state of subsystems of power system should be transferred from subsystems.]

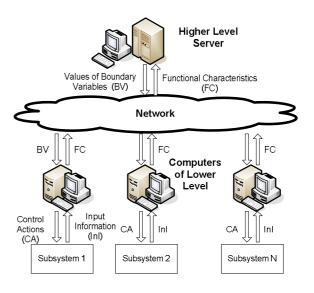


Fig. 1 Configuration of the distributed control system

Optimal operation of large interconnections is based in this control system on coordinated solution of optimization problems of two levels: optimization problems of subsystems on lower level and optimization problem of upper level determination of optimal power flows between subsystems.

In solution of optimization problems for this class of power systems Lagrange multipliers in subsystems can be interpreted as marginal costs of production of energy and hence as the prices of energy in subsystems. The FCs of subsystems represent in this model relationships between prices of exported (imported) energy and the export (import) volumes of energy for subsystems. In this case the problem of upper level, represented by SCE, can be considered as a problem of market of upper level, i.e. market of energy interchanges between national power systems (markets of lower level). The values of power flows (boundary variables in the SCE) calculated on the computer of upper level are optimal from the point of view of global objective function, assumed for integrated power system. The problems of lower level in this distributed system are the problems of local markets, in which optimal interchange of energy between local markets is taken into account.

### V. CONCLUSION

The following important results can be obtained by implementation of the distributed control system presented in this paper:

optimal global operation of large power interconnections or integrated electricity markets, consisting of several national electricity markets, while retaining authorities and selfdetermination of national power system operators;

efficient organization of computation and data transfer processes in solution of optimal control problems in which calculations in lower layer computers are executed concurrently and no information about internal state of subsystems should be transferred outside their borders.

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#### VII. BIOGRAPHIES





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