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Simple Method for Computing Power Systems Maximum Loading Conditions

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***Abstract*--A very simple and fast method for computing power systems maximum loading points is proposed in this paper. These points are simply computed through repeated load flow solutions. The main contribution resides in the appropriate use of a special load flow with step size optimization and the extraction of useful information from it, which guides the search for the desired maximum loading point. The simplicity and robustness of the proposed method are verified through simulations involving test and realistic systems.**

***Index Terms*--Load flow analysis; Voltage control; voltage stability.**

I. Introduction

VOLTAGE stability has been widely recognized as one of the most important problems related to power systems secure operation. Many blackouts that occurred in the last years were caused by instabilities and are clear examples of the importance of this subject. Voltage collapse and energy rationing occurrences have been reported worldwide, particularly in Brazil [1], [2] and the USA [3]. Those occurrences are mainly due to the lack of investments in the power area, leading systems to operate very close to their physical limits. The power industry restructuring process has also introduced a number of factors that have increased the number of possible sources for system disturbances, leading to a less robust, more unpredictable system as far as the operation is concerned [4]. Among these factors are the lack of new transmission facilities, cutbacks in system maintenance, workforce downsizing, power flow patterns different from those for which systems were designed, just to name a few. Special care should be given to transmission expansion and to the development of efficient operation techniques to best use the equipments' capabilities.

The concern with voltage stability has led many utilities and regulatory agencies to establish guidelines for keeping systems operating within a secure region. One important measure of the system's security degree regarding voltage stability is the voltage stability margin, related to the distance from the current operating point (base case) to the maximum loading point (MLP, point corresponding to the maximum admissible load for stable operation). According to the Brazilian National System Operator (ONS) [5], the minimum voltage stability margin required for single contingencies is 6%. Under normal operating conditions, the minimum margin must be a bit larger, depending on the demand. The Western Electricity Coordinating Council (WECC) adopts a 5% margin for normal operating conditions [6].

Estimates of the MLP can be obtained through several different methods proposed in the literature, such as the continuation power flow [7], direct methods [8], sensitivity based methods [9], non linear programming based methods [10], voltage stability index based methods [11].

Currently, there is a clear need of including voltage stability aspects into the analysis of real time operation and operation planning, in special **(a)** in the system's monitoring, for providing the voltage stability security conditions, **(b)** in contingency analysis, for determining the contingencies which significantly impact the voltage stability margin, and **(c)** in the preventive/corrective analysis, for defining fast and adequate control actions in cases where a voltage stability margin increase is needed.

In this paper a method for determining the MLP is proposed. It is specially suited to be used in situation **(a)** mentioned above, even though it can be also used in situations **(b)** and **(c)**. The idea of the proposed method is to obtain the MLP using load flows only, in an efficient way. One difficulty of this kind of approach resides in the singularity of the load flow Jacobian matrix at the MLP. Load flow computations in the vicinity of the MLP may lead to slow convergence, errors, and even divergence. This problem is dealt with by using a special load flow method with step size optimization.

II. Some basic aspects

A. Power system model

The power system is represented by

$$\mathbf{g}(\theta, \mathbf{V}, \lambda) = 0 \,, \tag{1}$$

or

$$\begin{cases} \Delta \mathbf{P} = \lambda \mathbf{P}^{\mathbf{sch}} - \mathbf{P}^{\mathbf{cal}}(\theta, \mathbf{V}) = 0 \quad \text{for PQ and PV buses} \\ \Delta \mathbf{Q} = \lambda \mathbf{Q}^{\mathbf{sch}} - \mathbf{Q}^{\mathbf{cal}}(\theta, \mathbf{V}) = 0 \quad \text{for PQ buses} \end{cases} , \tag{2}$$

where θ and $\mathbf{V}$ are the vectors of voltage phase angles and magnitudes, λ is the loading factor ($\lambda = 1$ corresponds to the base case), $\mathbf{g} = [\Delta \mathbf{P} \quad \Delta \mathbf{Q}]^T$ is the set of load flow equations comprised by the real and reactive power mismatches, and

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subscripts **sch** and **cal** stand for scheduled and calculated powers. Consider the two-bus example system shown in Fig. 1.

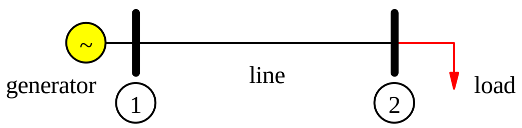

Fig. 1. Two-bus example system.

Fig. 2 shows the two-dimensional parameter (load) space corresponding to the example system.

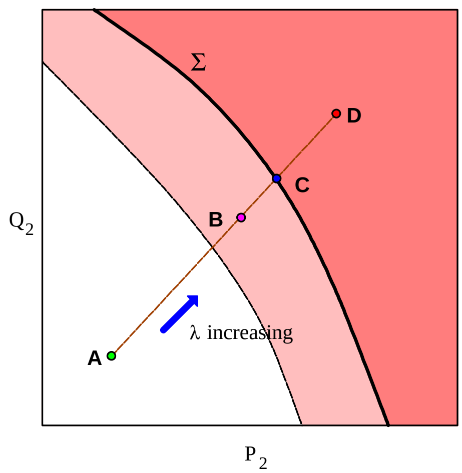

Fig. 2. Parameter (load) space for the two-bus example system.

Points **A** and **B** are feasible, and the load flow equations (1) have a stable solution. At point **A**, the system operates without any violated operational limit. At point **B**, some limits are violated. Point **C** corresponds to the MLP (for $\lambda = \lambda^*$), and point **D** is infeasible, that is, the load flow equations have no solutions. Σ corresponds to the feasibility boundary, which divides the parameter space into two regions, namely the feasible (for which the load flow equations present stable solutions) and infeasible (no solutions exist) regions. Therefore, point **C** is located on Σ.

B. Maximum loading point and voltage stability margin

Fig. 3 shows a typical PV curve for a system such as the one of Fig. 1. Note the correspondence between points **A** (base case) and **C** (MLP) from Figs. 2 and 3. The distance from **A** to **C** defines the voltage stability margin (VSM), which can be given by

$$\mathrm{VSM} = \left(\mathrm{P}_2^{\mathrm{sch}}\right)^C / \left(\mathrm{P}_2^{\mathrm{sch}}\right)^A = \lambda^* . \tag{3}$$

The goal of this paper is to provide an efficient way of obtaining operating point **C** (MLP). The difficulty regarding point **C** is that the load flow Jacobian matrix is singular at **C**.

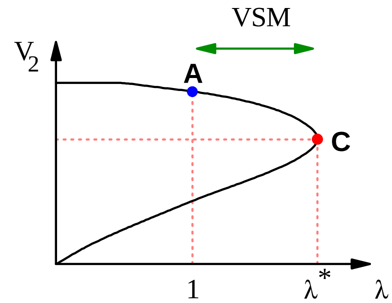

Fig. 3. Typical PV curve for the example two-bus system.

C. Load flow with step size optimization (LFSSO)

LFSSO was first developed for solving the load flow equations of ill-conditioned power systems. For those, the conventional load flow methods exhibit poorer performance, or simply diverge, although the system indeed operates in a stable equilibrium point. This idea was first presented in [12], where the voltages were represented in rectangular coordinates. In [13], an approach based on the representation of voltages in polar coordinates was proposed and in [14] the authors have demonstrated its comparative advantages, including situations where limits on reactive power generation are taken into account. At the *r*th iteration, the state variable vector $\mathbf{x}^{(r+1)} = [\Delta\theta \;\; \Delta\mathbf{V}]^{(r+1)}$ is calculated as

$$\mathbf{x}^{(r+1)} = \mathbf{x}^{(r)} + \mu^{(r)}\,\Delta\mathbf{x}^{(r)} ,$$
$$\Delta\mathbf{x}^{(r)} = -[\nabla_{\mathbf{x}}\mathbf{g}]^{-1}\Big|_{\mathbf{x}=\mathbf{x}^{(r)}} \mathbf{g}(\mathbf{x}^{(r)}, \rho) , \tag{4}$$

where $\mu^{(r)}$ is the optimal multiplier, $\nabla_{\mathbf{x}}\mathbf{g}$ is the Jacobian matrix. Multiplier μ is computed to minimize a quadratic function based on the power mismatches as

$$\min \quad F(\mu) = \tfrac{1}{2}\|\mathbf{g}\|_2^2 = \tfrac{1}{2}\sum_i g_i^2 , \tag{5}$$

where **g** is expanded in Taylor series, considering up to the second-order term, as

$$\mathbf{g}(\mu) = \mathbf{g}(\mathrm{x}^{(r)}, \lambda) + \mu\nabla_{\mathbf{x}}\mathrm{g}(\mathrm{x}^{(r)})^t\,\Delta\mathbf{x}^{(r)} + \mu^2 T(\mathbf{x}^{(r)}) . \tag{6}$$

Also, $T(\mathbf{x})$ corresponds to the second order terms of **g**, as

$$T(\mathbf{x}) = \frac{1}{2}\left(\sum_i \Delta x_i \frac{\partial}{\partial x_i}\right)^2 \mathbf{g}(\mathbf{x}) . \tag{7}$$

Substituting (7) in (6) and applying the local minimum condition $\partial F/\partial\mu = 0$, a cubic equation is obtained and solved for μ.

For well-conditioned systems, μ assumes values close to one and does not affect the iterative process in a significant way. In the case of ill-conditioned systems, μ assumes values such that the iterative process is smoothed out and the solution is obtained, whereas the conventional Newton method would have failed. Recently in [15], the authors recommended the implementation of [12] to get the fastest, most robust performance, regardless of system solvability or size.

For the infeasible cases (either due to an excessive loading or to a contingency), μ assumes very low values (theoretically $\mu \to 0$). Overbye [16] showed that LFSSO leads to a point on the feasibility boundary Σ rather than to simply diverge. With this information (points on boundary Σ), further applications of the LFSSO (as to calculate the MLP and security margins for voltage stability) can be proposed.

Fig. 4 shows the parameter space for the two-bus example system, considering that the voltage at the slack bus is 1.0pu and the transmission line impedance is $(0.02 + j0.5)$pu. Point A is the base case ($\lambda = 1, P_2 + jQ_2 = 0.5 + j0.1$pu). The dashed line corresponds to the load increasing direction for constant power factor. The MLP is also shown and corresponds to $\lambda^* = 1.586\,(P_2 + jQ_2 = 0.7930 + j0.1586\text{pu})$. Point B ($\lambda = 2, P_2 + jQ_2 = 1 + j0.2$pu) is an infeasible point. LFSSO provides point B' $(P_2 + jQ_2 = 0.8776 + j0.0808\text{pu})$ onto the feasibility boundary. Point C ($\lambda = 3, P_2 + jQ_2 = 1.5 + j0.3$pu) is also an infeasible point, and LFSSO provides point C' $(P_2 + jQ_2 = 1.0445 - j0.0915\text{pu})$ onto the feasibility boundary. Note that the point provided by LFSSO gets farther from MLP as λ increases.

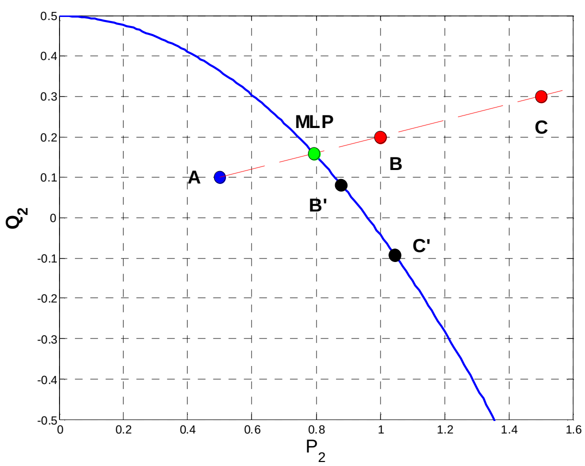

Fig. 4. Parameter space for the example two-bus system.

III. Proposed Method

A. Motivation

From Fig. 4 and (3) the maximum loading factor can be computed from points A and MLP by

$$\lambda^* = \frac{P_2^{\text{MLP}}}{P_2^{\text{A}}} = \frac{Q_2^{\text{MLP}}}{Q_2^{\text{A}}} = 1.586\,. \tag{8}$$

However, the same is not valid for infeasible points B and C, since B' and C' are not on the constant power factor dashed line. The same calculation done in (8) can be done for points B (and B') and C (and C'), and the results can be taken as approximations for λ^*. For point B we have

$$\left(\lambda^*\right)_{\text{estimated}} = \frac{P_2^{\text{B'}}}{P_2^{\text{A}}} = 1.755 \text{ and } \left(\lambda^*\right)_{\text{estimated}} = \frac{Q_2^{\text{B'}}}{Q_2^{\text{A}}} = 0.808\,. \tag{9}$$

These approximated values can be used in an appropriate way to define an iterative procedure for computing the actual MLP. This procedure should start with an estimate of λ^*. The idea is to start with an infeasible operating point. Any infeasible point can be defined, but infeasible points closer to Σ result is smaller computational effort for determining the actual MLP. For realistic systems, this is not a hard task, since operators usually know in advance that, for instance, "the system's VSM right now is not larger than 20%". This conclusion is based on the system's operation history and the operator's experience. In spite of that, the proposed procedure is robust enough to perform well for larger initial estimates.

B. Proposed procedure

The proposed procedure for computing the MLP of an *n*-bus system is detailed below.

(1) Set iteration count $j = 0$. Set an initial estimate for $\left(\lambda^*\right)^j$.

(2) Run LFSSO for the specified loading condition.

(3) Compute estimates for $\left(\lambda^*\right)^{j+1}$ based on the results provided by LFSSO, as

$$\begin{cases} \left(\lambda^*\right)_i^P = \dfrac{P_i^{\text{cal}}}{P_i^{\text{bc}}} \text{ for PQ and PV buses, and} \\ \left(\lambda^*\right)_i^Q = \dfrac{Q_i^{\text{cal}}}{Q_i^{\text{bc}}} \text{ for PQ buses.} \end{cases} \tag{10}$$

where P_i^{cal} is the calculated power at bus *i* provided by LFSSO, and P_i^{bc} is the respective base case value.

(4) The new estimate for $\left(\lambda^*\right)^{j+1}$ is

$$\left(\lambda^*\right)^{j+1} = \alpha \cdot \text{median}\left[\left(\lambda^*\right)_i^P, \left(\lambda^*\right)_i^Q\right], \tag{11}$$

where α is a factor that can be used to speed up the updating process.

(5) Run LFSSO for $\left(\lambda^*\right)^{j+1}$. In case the new operating point is still infeasible, set $j = j + 1$ and go back to step **(3)**. Otherwise, continue.

(6) At this point, $\left(\lambda^*\right)^{j+1}$ corresponds to a feasible point, and $\left(\lambda^*\right)^j$ corresponds to an infeasible point. The MLP can be determined by binary search using these points as initial estimates. Note that LFSSO must be run for each new estimate. The process is interrupted after the difference between two consecutive values is smaller than a predefined threshold. In this paper, the threshold was set to 1%.

IV. Simulation Results

Some simulation results will be shown for small test to large realistic transmission and distribution systems. Factor α was set to 0.9, except where mentioned otherwise.

Table I and Fig. 5 show the results for the IEEE 14-bus test system [17]. The maximum loading factor approaches the solution very rapidly. It is important to point out that the computational effort associated to the process is basically due to running load flows, since the additional calculations take negligible time. The results also show that the initial estimate for λ affects the number of iterations. However, as mentioned before, defining a good initial estimate is not a hard task for experienced operators. In this case, an initial estimate of $\left(\lambda^*\right)^0 = 3.0$ is too large, but the proposed method is able to go towards the correct solution very rapidly.

TABLE I
SIMULATION RESULTS FOR THE IEEE 14-BUS TEST SYSTEM
MAXIMUM LOADING - $\lambda^* = 1.7581$

Iteration	$\left(\lambda^*\right)^j$	$\left(\lambda^*\right)^j$
0	2.0000	3.0000
1	1.6168	1.9208
2	1.8068	1.6029
3	1.7126	1.7618
4	1.7605	1.6824
5	1.7366	1.7221
6	1.7485	1.7420
7		1.7519

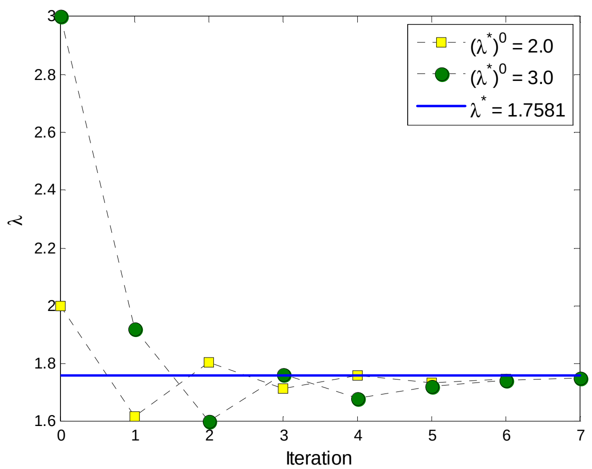

Fig. 5. Simulation results for the IEEE 14-bus test system.

Table II and Fig. 6 show the results for a 33-bus distribution system [18]. The proposed method also performed well in this case.

TABLE II
SIMULATION RESULTS FOR THE 33-BUS DISTRIBUTION SYSTEM
MAXIMUM LOADING - $\lambda^* = 3.7077$

Iteration	$\left(\lambda^*\right)^j$	$\left(\lambda^*\right)^j$
0	4.0000	5.0000
1	3.5183	3.7686
2	3.7592	3.3911
3	3.6387	3.5798
4	3.6989	3.6742
5	3.7291	3.7214
6		3.6978

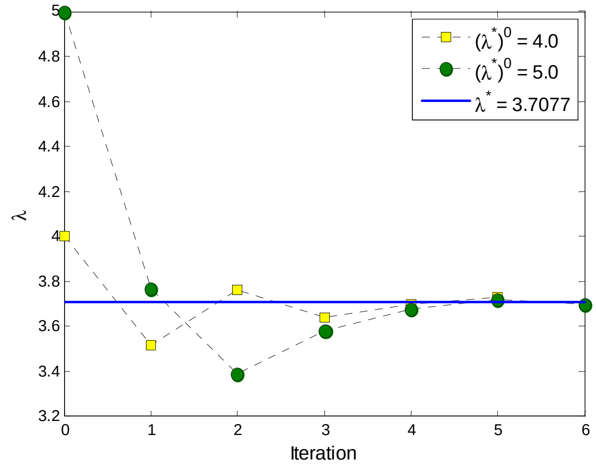

Fig. 6. Simulation results for the 33-bus distribution system.

For many years planning and operation of distribution systems (DSs) were done with little or no analysis at all [19]. As a result, DSs were typically overdesigned. A remarkable development in DSs models and analysis techniques has been observed recently. A direct consequence is the possibility of operating DSs close to their maximum capacities, that is, the ever-increasing demand can be supplied through a better utilization of the existing equipment, postponing investments. Power systems restructuring led this possibility to become a necessity [20]. Of course, operating DSs close to their maximum capacities implies in instability risk increase, including voltage stability [21]. Currently, most DSs operate with a comfortable voltage stability margin. However, this situation will change with the demand increase and equipment stress. Voltage instability in DSs has already been observed in industrial areas under critical loading conditions [21,22]. It is well known that in general distribution system have comfortable voltage stability margins (large maximum loading points). The need for efficient methods for computing voltage stability margins for distribution systems are then clear, and the proposed method showed to be appropriate for that.

Table III and Fig. 7 show the results for a realistic 904-bus system corresponding to part of the Southwestern USA. The proposed method also performed well in this case.

TABLE III
SIMULATION RESULTS FOR THE 904-BUS SYSTEM
MAXIMUM LOADING - $\lambda^* = 1.0443$

Iteration	$\left(\lambda^*\right)^j$	$\left(\lambda^*\right)^j$
0	1.1000	1.3000
1	0.9900	1.1438
2	1.0450	1.0284
3	1.0175	1.0861
4	1.0312	1.0572
5	1.0381	1.0428
6		1.0500

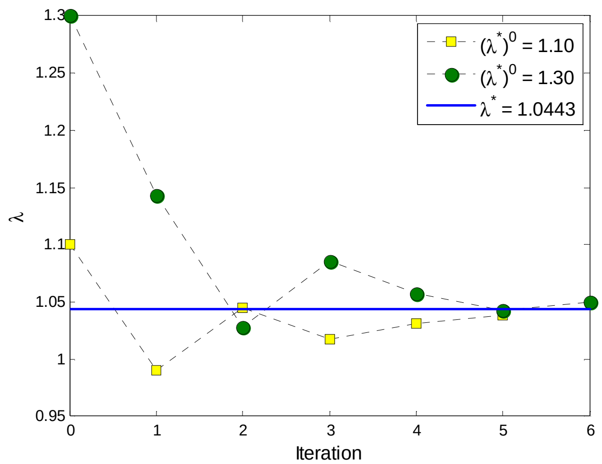

Fig. 7. Simulation results for the 904-bus system.

Table IV and Fig. 8 show the results for a realistic 1030-bus system corresponding to part of the Brazilian interconnected system. The system is critically loaded, presenting a very small voltage stability margin. The proposed method repeated the good performance.

Table V and Fig. 9 show the results for a realistic 1030-bus system considering $\alpha = 0.95$. The proposed method repeated the good performance. In this case the number of iterations was smaller than the previous case, however, this is not a general rule, and cannot be considered valid for all systems.

TABLE IV
SIMULATION RESULTS FOR THE 1030-BUS SYSTEM
MAXIMUM LOADING - $\lambda^* = 1.0036$

Iteration	$(\lambda^*)^j$	$(\lambda^*)^j$	$(\lambda^*)^j$
0	1.0500	1.1000	1.3000
1	0.9133	0.9230	0.9531
2	0.9817	1.0115	1.1266
3	1.0158	0.9673	1.0399
4	0.9987	0.9894	0.9965
5	1.0078	1.0004	1.0182
6		1.0060	1.0073
7			1.0019

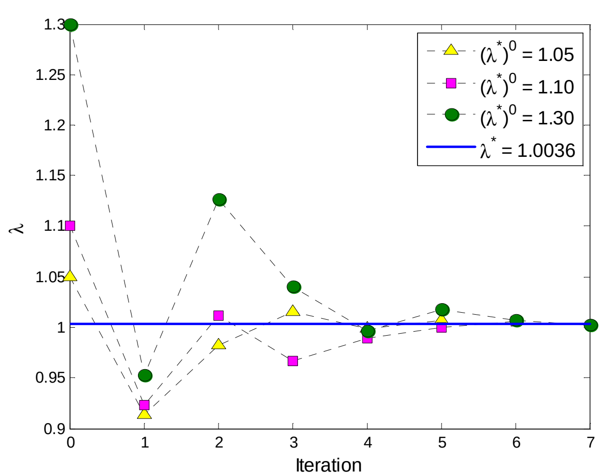

Fig. 8. Simulation results for the 1030-bus system.

TABLE V
SIMULATION RESULTS FOR THE 1030-BUS SYSTEM (α=0.95)
MAXIMUM LOADING - $\lambda^* = 1.0036$

Iteration	$(\lambda^*)^j$	$(\lambda^*)^j$	$(\lambda^*)^j$
0	1.0500	1.1000	1.3000
1	0.9640	0.9743	1.0061
2	1.0070	1.0371	0.9554
3	0.9855	1.0057	0.9808
4	0.9963	0.9900	0.9934
5	1.0016	0.9979	0.9998

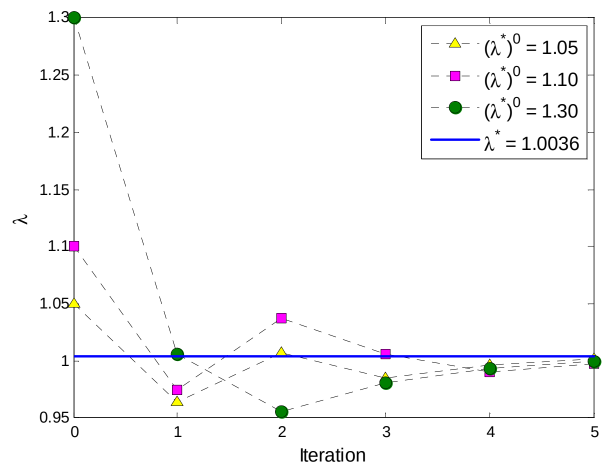

Fig. 9. Simulation results for the 1030-bus system (α=0.95).

Finally, Tables VI and VII show the number of load flow iterations during the calculation process.

TABLE VI
PERFORMANCE OF THE PROPOSED METHOD FOR THE IEEE 14-BUS TEST SYSTEM
MAXIMUM LOADING - $\lambda^* = 1.7581$

Iteration	$(\lambda^*)^0 = 2.0$	$(\lambda^*)^0 = 3.0$
0	4	5
1	4	5
2	5	4
3	3	4
4	3	1
5	3	2
6	5	4
7		5

TABLE VII
PERFORMANCE OF THE PROPOSED METHOD FOR THE 1030-BUS SYSTEM
MAXIMUM LOADING - $\lambda^* = 1.0036$

Iteration	$(\lambda^*)^0 = 1.05$	$(\lambda^*)^0 = 1.1$	$(\lambda^*)^0 = 1.3$
0	13	12	12
1	4	4	4
2	4	6	16
3	4	3	5
4	4	3	4
5	3	3	5
6		3	4
7			3

It is worth pointing out that only one load flow is run in each iteration of the proposed method, and each load flow requires a certain number of iterations, as shown. Note that the number of iterations is small, except some iterations for the realistic system, when λ is much larger than λ^*. It is also important to note that LFFSO provides a solution for feasible cases, or a point on the feasibility boundary for infeasible cases, with the same number of iterations, which makes the process very efficient. The initial point of each iteration is set as the final point of the previous one.

V. Conclusion

The method proposed in this paper showed to be simple, though robust and efficient. The ever-increasing need of efficient voltage stability tools for operation analysis and studies makes the proposed method appropriate for such situations. A good performance was obtained for both transmission and distribution systems.

VI. References

[1] Coordinating group for interconnected operation (GCOI), “Report SCO/SCEL - 01/97 - Analysis of the disturbances on April 24 and 25, 1997 in the interconnected South/Southeast/West system,” 1997 (in Portuguese).

[2] Working Group of the Energy Crisis Administration Chamber (GCE), “Final report,” 2002 (in Portuguese).

[3] U.S.-Canada Power System Outage Task Force, “Final report on the August 14, 2003 blackout in the United States and Canada: causes and recommendations,” 2004. Available: http://www.nerc.com/~filez/blackout.html.

[4] K. Morison, L. Wang and P. Kundur, “Power system security assessment,” *IEEE Power & Energy*, vol.2, 2004.

[5] Brazilian National System Operator (ONS), “Network procedures: guidelines and criteria for electrical studies, submodule 23.3,” (in Portuguese). Available: http://www.ons.org.br/ons/procedimentos/index.html.

[6] WECC Reactive Power Reserve Work Group, “Voltage stability criteria, undervoltage load shedding strategy, and reactive power reserve monitoring methodology, final report,” 1998. Available: http://www.wecc.biz/main.html.

[7] V. Ajjarapu, C. Christy, “The continuation power flow: a tool for steady state voltage stability analysis,” *IEEE Trans. Power Syst.*, vol.7, 1992.

[8] I. Dobson, L. Lu, Y. Hu, “A direct method for computing a closest saddle node bifurcation in the load power parameter space of an electric power system,” *IEEE Intern. Symp. on Circ. and Syst.*, 1991.

[9] N. Flatabø, R. Ognedal, T. Carlsen, “Voltage stability condition in a power transmission system calculated by sensitivity methods,” *IEEE Trans. Power Syst.*, vol.5, 1990.

[10] L.A.Ll. Zarate, C.A. Castro, J.M. Ramos, E.R. Ramos, “Fast Computation of Voltage Stability Security Margins Using Nonlinear Programming Techniques,” *IEEE Trans. Power Syst.*, vol.21, 2006.

[11] P.-A. Lof, G. Andersson, D.J. Hill, “Voltage stability indices for stressed power systems,” *IEEE Trans. Power Syst.*, vol.8, 1993.

[12] S. Iwamoto, Y. Tamura, “A load flow calculation method for ill-conditioned power systems,” *IEEE Trans. Power App. Syst.*, vol. PAS-100, 1981.

[13] C.A. Castro, L.M.C. Braz, “A new approach to the polar Newton power flow using step optimization,” *in Proc. 29th North Amer. Power Symp.*, Laramie, WY, USA, 1997.

[14] L.M. Braz, C.A. Castro, C.A.F. Murari, “A critical evaluation of the size optimization based load flow methods,” *IEEE Trans. Power Syst.*, vol. 15, 2000.

[15] J.E. Tate, T.J. Overbye, “A comparison of the optimal multiplier in polar and rectangular coordinates,” *IEEE Trans. Power Syst*., vol.20, 2005.

[16] T.J. Overbye, “A power flow measure for unsolvable cases,” *IEEE Trans. Power Syst.*, vol. 9, 1994.

[17] Power systems test case archive. Available: http://www.ee.washington.edu/research/pstca/.

[18] M.E. Baran, F.F. Wu, Network reconfiguration in distribution systems for loss redustion and load balancing, *IEEE Trans. Power Delivery*, vol.4, 1989.

[19] W.H. Kersting, “Distribution system modeling and analysis,” CRC Press, 2007.

[20] J. Northcote-Green, R. Wilson, “Control and Automation of Electrical Power Distribution Systems,” Taylor & Francis, 2007.

[21] G. Brownell, H. Clark, “Analysis and Solutions for Bulk System Voltage Instability,” IEEE Computer App. in Power, 1989.

[22] M. Chakravorty, D. Das, “Voltage stability analysis of radial distribution networks,” Electrical Power and Energy Systems, vol.4, 2000.

VII. Biographies

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