Paper accepted for presentation at 2009 IEEE Bucharest Power Tech Conference, June 28th - July 2nd, Bucharest, Romania

Substation grounding system resistance calculations using a FEM approach

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Abstract-- **Grounding systems are very important for the safe construction of a substation or a simple building. This paper examines various methodologies for the calculation of different grounding systems, including single rods, rodbeds and grids. In addition, the use of a Finite Element Method (FEM) formulation in the analysis of grounding systems is presented. Both 2D and 3D models are created in order to get an accurate field calculation. Test cases include both uniform and two layer earth structures. The results obtained by the FEM calculations are compared to those obtained by analytical methods, providing useful information about the most suitable methodology for the calculation of the grounding resistance.**

***Index Terms*-- grounding systems, finite element method, grounding resistance, soil resistivity, two layer stratified earth**

I. Introduction

THE selection of a suitable grounding system is an important aspect in the construction of a high voltage substation. The aim of the grounding system is to drive the ground fault current efficiently to the earth, and to protect the people within and in the surroundings of the substation. To ensure these two aspects, proper upper limits for the step and the touch voltages have been set by international standards [1-2]. The knowledge of the grounding system resistance is essential for the calculation of these voltages in cases of faults.

The grounding systems commonly used, consist of single rods, rodbeds or arrays of rods, grounding grids and combinations of the previous types. The grounding resistance of a system can be calculated by various methods. Depending on the type of the grounding system and the soil structure either uniform or multilayer, different methods have been proposed in the literature. These methods however exhibit significant differences in their results, especially when used in multilayer soils. As a result, the selection of the proper calculating method for each case is a difficult aspect.

In this paper a number of methodologies have been examined and are implemented for three different cases of grounding systems:

- In the first case a single, vertical rod of varying length is examined.
- The second case concerns a rodbed of vertical rods with varying rod lengths, number of rods and cover area.
- The third grounding system examined includes grounding grids of different configuration and sizes

All the above grounding systems are examined for both homogeneous and two layer earth structures.

Several methodologies have been suggested for the calculation of the resistance of a grounding system. Among them are the methods proposed by Schwarz [3], Sullivan [4], Nahman et. al [5, 6, 7, 8] and Salama et al. [9, 10], as well as that of the IEEE Standard [1, 2]. Each of the above methods has been used in the calculation of the resistance of a certain grounding system either in uniform or in two layer earth structure. Moreover, analytical formulas based on classic electromagnetic theory have been also used for the calculation of the rod resistances.

Two different soil configurations are examined in this paper. In the first one, the earth is assumed to be uniform, whereas in the second it is considered to consist of two horizontal layers of different earth resistivities. The results are validated using a Finite Element Method (FEM) formulation, implemented using a commercially available software package. A 2D FEM model is formulated for the case of a single, vertical rod in order to get an accurate field calculation and to achieve a proper comparison with the 3D model which has been used for all the examined cases of rods and grids. The 2D model has been used as reference, as 2D geometries provide better accuracy than the 3D in FEM calculations. However the 2D model can handle only cases with axial symmetry and therefore it can be used in very limited cases. In all other cases the 3D formulation has been used.

II. PROBLEM FORMULATION

A. Homogeneous earth structure

The assumption that the grounding system is buried within homogeneous soil is the simplest one. For this soil model the resistivity of the ground is considered to be constant and independent of the depth. However, since the earth practically consists of layers with different resistivities, this simple model is often inadequate and can lead to erroneous results. For the uniform soil model, the earth resistivity is considered to be 100 Ω·m, a typical value for most common soils.

The most commonly used expression for the calculation of a single rod resistance in uniform soil can be found in the classical electromagnetic theory.

$$R_r = \frac{\rho}{2\pi l} \cdot \ln\frac{4l}{d} \qquad (1)$$

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where:
ρ is the uniform resistivity of the soil (Ω·m)
l is the active length of the rod (m)
d is the diameter of the rod (m)

Another expression which has been proposed by Schwarz [3] and has been used in the IEEE Standard is [1]:

$$R_r = \frac{\rho}{2\pi l}\left(\ln\frac{8l}{d} - 1\right) \qquad (2)$$

A different expression for the calculation of the resistance of a single vertical rod has been proposed by Sullivan in [4].

For cases of more complex grounding systems, consisting of rod beds, grounding grids and combinations of rods and grids in uniform soils, expressions have been suggested by both Schwarz [3] and Sullivan [4]. The former approach has been also implemented in the IEEE Standard [1].

B. Two layer Earth Structure

In the two layer model the earth is assumed to consist of two horizontal layers with different earth resistivities. The first layer, closer to the surface, has a finite thickness, while the second layer is assumed to have infinite depth.

This earth model is more complex than the homogeneous earth model, and leads theoretically to more accurate results. However, as it will be shown, the different proposed methods lead to results that vary substantially, especially in cases where the two earth layers have significantly different resistivity values. As a result, it is in question which of the proposed methods is the most appropriate.

For the determination of the resistivities of the earth layers, from the results of the apparent resistivity measurement on the ground surface, many methods can be found in the literature, concerning both the measurement of the resistivity [13-14], as well as the determination of the parameters of each earth layer [11-12,15-16].

For the calculation of the grounding resistance of the various grounding configurations two major approaches are used. The first one is based on the calculation of an equivalent resistivity for the two-layered soil. Using this equivalent resistivity, the two layer earth is approximated as uniform and the simpler formulas for homogeneous earth can be used.

For the calculation of this equivalent resistivity the IEEE Std [1] proposes the following method. A set of values for the ground resistivity, according to different electrode spacing, is obtained by measurements, e.g. conducted by the Wenner method [13]. The apparent resistivity of the ground is assumed to be the average value of all the measurement values:

$$\rho_\alpha = \frac{\rho_{\alpha_1} + \rho_{\alpha_2} + \ldots + \rho_{\alpha_n}}{n} \qquad (3)$$

where:
$\rho_{\alpha_1}, \rho_{\alpha_2} \ldots \rho_{\alpha_n}$, the measured resistivities by the Wenner method for different electrode spacing (Ω·m)
n, the amount of the measured values.

According to the second major approach, different formulations are proposed, taking into account the different resistivities of the two earth layers, as well as the specific location of the grounding system. In this category belong the different models proposed by Nahman et al. [5-8] and Salama et al. [9-10], applicable in cases of grounding grids, rod beds and combinations of them. All the approaches for the stratified earth can be also used for the case of homogeneous earth, assuming that both earth layers have the same resistivity.

Concerning the two layer earth structure, six different practical soil models have been used in this work. These models are proposed in [11], and they are based on actual ground resistivity measurements [12]. Their characteristics are shown in Table I.

TABLE I
SOIL CHARACTERISTICS FOR THE DIFFERENT CASES EXAMINED

Case	ρ_1 (Ωm)	ρ_2 (Ωm)	h (m)
1	372.729	145.259	2.69
2	246.836	1058.63	2.139
3	57.344	96.714	1.651
4	494.883	93.663	4.37
5	160.776	34.074	1.848
6	125.526	1093.08	2.713

In this table, ρ_1 is the resistivity of the upper soil layer, ρ_2 is the resistivity of the bottom soil layer, and h is the depth of the upper layer.

In order to calculate the equivalent resistivity of the ground for the six cases presented in Table I, using the method proposed by the IEEE Std, experimental measurements of the soil resistivity are also required. For these six cases the measured values of the apparent resistivity on the ground surface are provided in [11] and are shown in Table II. The measurements were obtained using Wenner's method for different electrode spacing, where ρ_m is the apparent resistivity (Ω·m) measured for each electrode spacing [12].

TABLE II
MEASURED GROUND RESISTIVITY VALUES FOR THE CASES OF TABLE I.

Measur.#	Case	1	2	3	4	5	6
1	spacing	2,5	1	0,5	2,5	1	1
	ρ_m	320	255	58,71	451,6	156,4	136
2	spacing	5	1,5	1	5	2	2
	ρ_m	245	290	61,79	366,7	113,1	140
3	spacing	7,5	2,5	1,5	7,5	3	4
	ρ_m	182	315	58,1	250,2	95,2	214
4	spacing	10	3	2	10	4	10
	ρ_m	162	376	61	180	65,3	446
5	spacing	12,5	5	2,5	12,5		20
	ρ_m	168	528	73,79	144,2		685
6	spacing	15	10	3	15		40
	ρ_m	152	690	78	120,2		800
7	spacing			4	20		
	ρ_m			79,13	115,5		
8	spacing			5	25		
	ρ_m			78,19	96,5		

III. FINITE ELEMENT METHOD FORMULATION

FEM modeling allows the calculation of complex grounding system geometries, taking into account the exact topology and the electromagnetic characteristics of all involved elements. The soil characteristics can be defined in detail for every point in the region around the grounding system, while nonlinearities may be also considered.

In this study a commercially available FEM software package, namely COMSOL Multiphysics® [17] has been used. Both 2D and 3D models have been created in order to represent accurately the grounding system in all examined cases.

The FEM in the ground is expressed using the Poisson's equation and the electric scalar potential as:

$$-\nabla(\sigma \cdot \nabla V) = 0 \tag{4}$$

This equation is solved using the boundary conditions that the potential on the problem region boundary is zero V=0. Moreover, the surface between ground and air is regarded as electric insulation (n·J = 0) in order to provide isolation between the two respective subdomains.

At first, the input data, namely the electromagnetic properties of the soil and the rod geometry, are defined. Subsequently, the discretization region in which the problem will be solved is determined. This region should be chosen using the following criteria. It must be large enough, so that all the current flow to the earth from the grounding system will be taken into account, while at the same time, it must be small enough to reduce the computer memory requirements and the necessary computational time. The optimal performance resulted after several trials. The discretization area which has been used in the simulations of this paper has been selected to be a cube with a volume of 50 m^3 or 100 m^3. An arbitrary potential V is applied to the grounded rod. In the cases examined here, the problem is solved assuming a unit potential 1 V on the grounding system [18]. For this potential, the total current I that flows from the grounding system is calculated by the FEM. Finally, the resistance of the rod can be calculated from these two values.

To enhance the numerical performance of the model both automatic and manual mesh generation have been utilized. The final mesh has been chosen in all cases to be more dense near the grounding system, and more sparse towards the boundary surface. This mesh was refined whenever necessary. Subsequently the data input is determined concerning the electromagnetic properties of the soil, the grounding system geometry, as well as the discretization area.

Since the 2D FEM model can lead to accurate results, it has been used for the geometries which present axial symmetry. The results obtained by the 2D FEM formulation for these cases have been compared to those obtained by the 3D model for the same geometries. The aim of this comparison was to determine the topologies, which in the case of the 3D model provide sufficiently accurate results, compared with the corresponding 2D. The investigation showed that the 3D discretization areas should be at least 50 m^3 for the case of a single vertical rod. This has been also assumed to be a proper descretization area for the topologies where the 2D model cannot be used, or an equivalent depending on the actual grounding system spacing for grounding grids and rodbeds. The results obtained by the FEM for all the examined cases are regarded as a reference in order to compare the results by all the other analytical methods.

IV. NUMERICAL RESULTS

The simple case of a single, vertical rod is tested first. The diameter of the rod is considered to be 6.3 mm, while its length ranges from 1m to 10m. Besides the resistance calculation, an investigation concerning the value of the grounding resistance in relation to the burial depth of the rod is conducted. The results are validated using both 2D and 3D FEM formulations.

Next, three different rodbeds are examined. These rodbeds consist of 4, 9 and 25 rods and are denoted as R4, R9 and R25 respectively. Moreover, the length of the rods is varying from 1 to 10 m. The diameter of the rods is 6.3 mm. As previously, the influence of the burial depth of the rods on the ground resistance value is investigated, especially for the two layer soil structures.

Finally, square and rectangular grids of different sizes and with different number of meshes are examined. The grids are buried horizontally in the soil. The results by the methods proposed by Nahman and the IEEE Standard are validated with a 3D FEM formulation. The diameter of the grid conductors is considered to be 1.7 mm.

A. Homogeneous earth structure

In the homogeneous earth structure small differences are recorded among the results of the proposed analytical methods and the FEM. An example of the results obtained for the calculation of the resistance regarding a grounding rod in homogeneous earth is shown in Fig 1. Different rod lengths have been examined ranging from 1 m to 10 m using all the above methods and also the FEM formulation in 2D and 3D. The greatest variation is about 7% and it can be observed between the results obtained by (1) and the formula proposed by the IEEE Std [1].

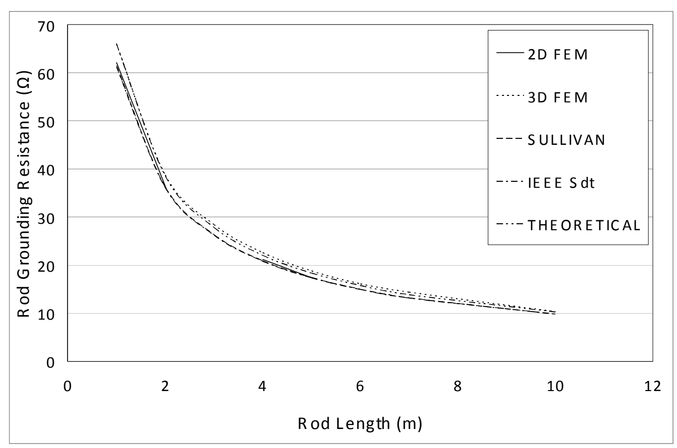

Fig. 1. Rod resistance in the case of homogeneous soil.

The same observations are recorded for a rodbed consisting of rods buried vertically in a uniform soil with ρ=100 Ω·m. The rod length is considered to vary. Fig. 2 shows the comparison among the expression proposed by Schwarz [3], which is implemented in the IEEE Std. [1], and by the FEM formulation. In the figure, rodbeds which consist of 4, 9 and 25 rods are examined.

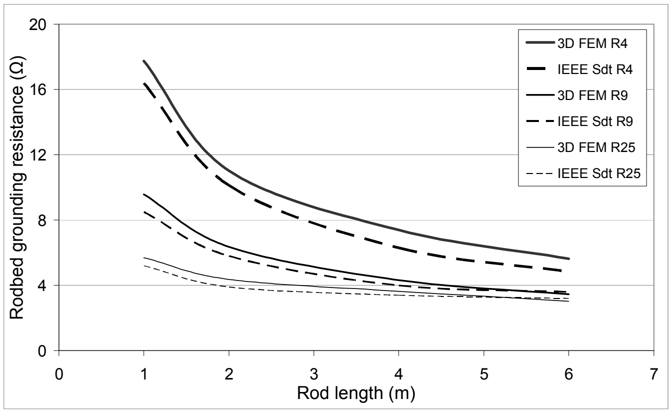

Fig. 2. Rodbed resistance in the case of homogeneous earth.

The recorded differences between the two examined methods are not significant in any case. Apart from this, it is obvious that the differences are greater as the number of rods decreases, while they tend to minimize for extended grounding systems and for larger rod lengths.

Different types of grids have been also examined in the uniform soil. The results concerning both square (S) and rectangle (R) grids with variable number of meshes are shown in Fig. 3. The area covered is $100m^2$ for the square grids and $200m^2$ for the rectangle grids.

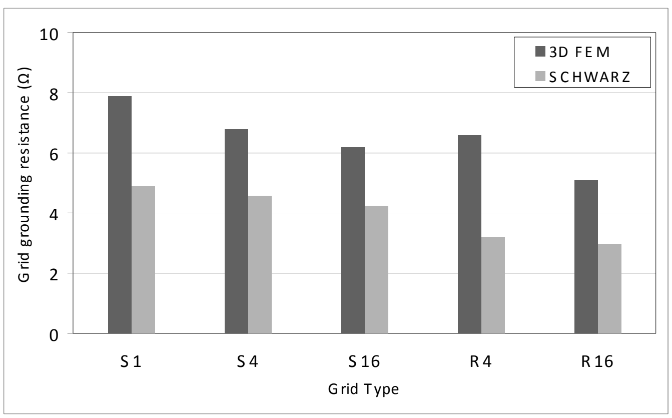

Fig. 3. Grid resistance in the case of homogeneous earth.

In this case, the difference between the results of the FEM formulation and the method proposed by Schwarz are quite significant, even for the extended grounding systems.

B. Two layer earth model

For the two layer earth model, the six different cases of Table I have been examined. As a general rule, from all the different test cases examined for every grounding system arrangement in a two layer earth, a common behavior among earth structures with $\rho_1 > \rho_2$ and those with $\rho_1 < \rho_2$ is detected. Moreover, the recorded differences between the results obtained by all the analytical methods and by the FEM formulation are larger when the resistivity of the first soil layer is greater than the resistivity of the second soil layer.

At first, the case of a single vertical rod of varying length, buried in the six different soil cases is examined. In Fig. 4, the results for case #1 of the Table I are shown.

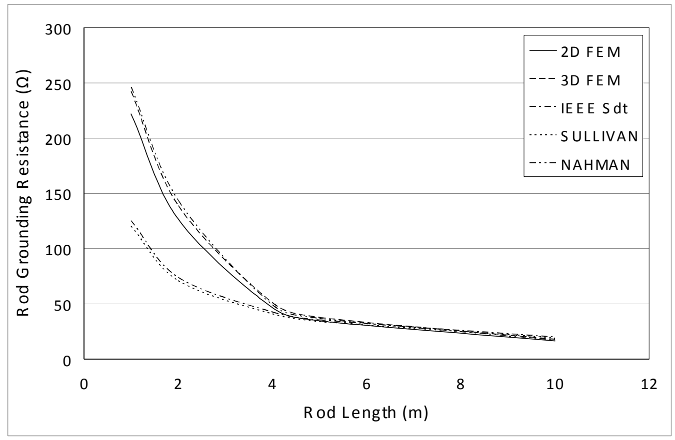

Fig. 4. Grounding resistance of a rod for Case #1 of Table I

There are significant differences between the results of the five methods, especially for small rod lengths. More specifically, the results obtained by the FEM approach are in good agreement to those obtained by the approach by Nahman et al [5]. However, it is important that there is not significant difference detected between the 2D and 3D FEM formulations. In the worst case, the variation of their results is less than 8%. The differences in the results of all five methods are significantly smaller for rod lengths greater than 4 m.

For this earth model, three distinguished areas may be observed in the curves of Fig. 4, respectively to the rod relative position within the two soil layers. When the rod is buried only in the first layer, the resulting curve gradient is the same with the corresponding one of the uniform soil case. This gradient changes when the rod length reaches or marginally exceeds the boundary between the two soil layers, or when the largest part of the rod is buried in the second soil layer.

For case #2 of Table I, the results are presented in Fig. 5.

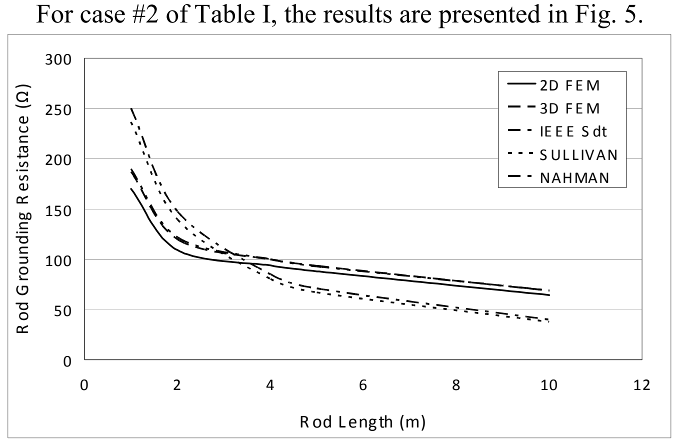

Fig. 5. Grounding resistance of a rod for Case #2 of Table I

For this soil structure case, the variation between the results of the five methods is smaller than in the previous case. Despite this fact, it is important to observe that the results for greater rod lengths do not converge in this case. Similarly to the previous case, the results obtained by FEM are in very good agreement to those obtained by the Nahman et al. method [5]. Moreover, the grounding rod resistance values follow the same pattern as in the previous case.

Finally, even more significant differences between the methods can be observed in case #5 of Table I.

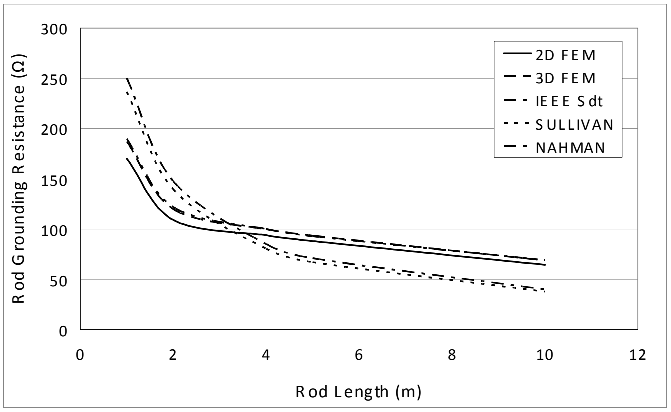

Fig. 6. Grounding resistance of a rod for Case #5 of Table I

For this case, the results acquired by the FEM approach present again almost negligible variations as compared to those obtained by the Nahman and al. method. However, the results of the other two methods do not converge, neither to the results of the FEM, nor between them. Moreover, a steep decrease in the values of the rod resistance for rod lengths lower than 4 m is evident. The influence of the second soil layer, when the rod penetrates into it, is once again obvious.

The behavior of a rodbed buried in the six soil cases of Table I is examined next. A comparison among three different methods for the calculation of the ground resistance of a rodbed buried in two layer earth structure is presented. The rods vary both in length and in number.

In Fig. 7 results are presented regarding a rodbed which consists of 4 rods (R4), and is buried in soil corresponding to case #3 of Table I.

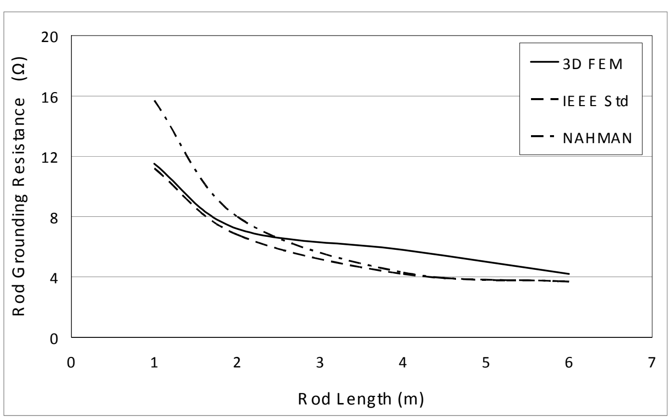

Fig. 7. Grounding resistance of a rodbed (R4) for Case #3 of Table I

For this case, the variation between the results is not significant, especially for rod lengths between 2 and 4 m, which correspond to the most common cases. This happens because the resistivity values of the two soil layers do not exhibit great differences. As a result, this case approximates the corresponding case of the homogeneous earth.

In Fig. 4, results are presented concerning the ground resistance of the same rodbed configuration, but for a soil with characteristics corresponding to case #4 of Table I.

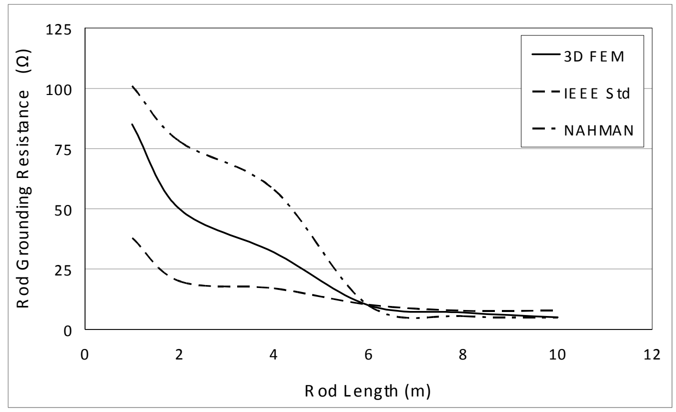

Fig. 8. Grounding resistance of a rodbed (R4) for Case #4 of Table I

For this soil case, significantly greater variations among the results are detected. Especially for small rod lengths, the differences are high. The results obtained by FEM are between the results of the other two methods. The method proposed by Nahman is once again closer to the results obtained by the FEM approach, which are considered as the reference for the comparison.

Significant differences among the results have been also recorded for the cases where the earth resistivity of the first layer is much smaller than that of the second layer.

Finally, the same grid types that were examined in the uniform soil case are examined for the two layer earth structures as well. It should be mentioned that with the FEM software used, grids with sizes greater than $200m^2$ could not be solved due to the computer memory constraints. For larger grid configurations the mesh created in the discretization area was relatively sparse and therefore the accuracy of the corresponding results is not certain.

Two indicative soil cases are presented here. In Fig. 9, the comparison is shown between the results of the FEM formulation and the method proposed by Nahman, considering case #1 of Table I.

The divergence between the Nahman method and the FEM model is obvious. Especially for grids with small number of meshes the differences are significant.

The same observations may be conducted for the other five soil structures that were investigated. Another indicative example is shown in Fig. 10 where the grids are buried in a soil structure with characteristics corresponding to the case #4 of Table I.

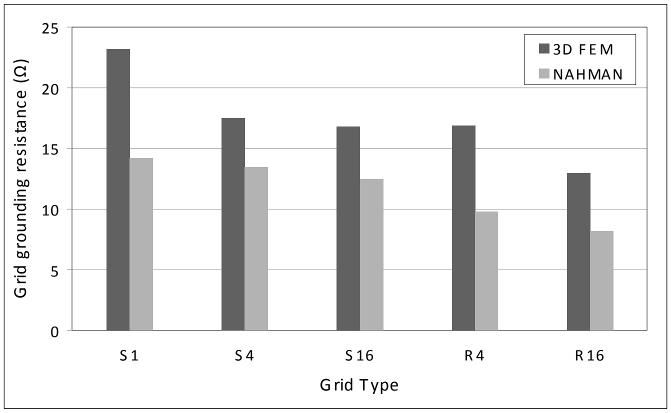

Fig. 9. Grounding resistance of grids for Case #1 of Table I

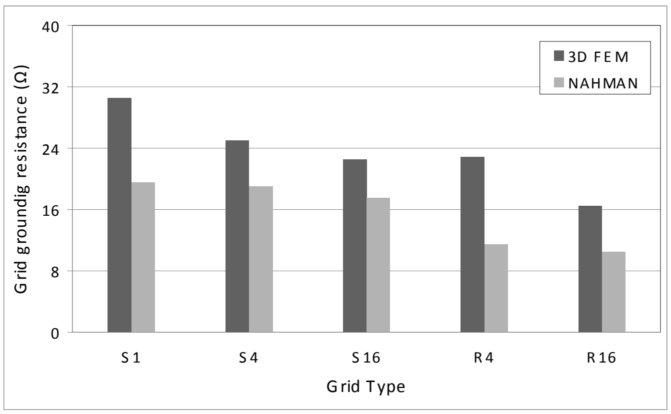

Fig. 10. Grounding resistance of grids for Case #4 of Table I

V. CONCLUSIONS

Different methods for the calculation of the grounding resistance of different grounding systems in homogeneous and two layer soil structures have been examined. The obtained results are compared with the corresponding by a 3D FEM formulation, which are regarded as reference. The differences among the results are negligible for homogeneous soil structure, except the cases of the grids which present significant differences. In the cases where the earth is assumed to consist of two layers, the variations between the results are significant, especially for small rod lengths.

The analytical models examined, use different approaches in the calculation of the resistance of a grounding system in multilayer soils. Some of them are based on the assumption of an equivalent resistivity of a fictitious homogeneous ground. Others consider also the relative position of the system within the soil. The FEM results show that the methods which take into consideration the position of the grounding system in the earth, lead to more accurate results.

Analytically, the Nahman – Salamon method leads to results which are in the best accordance with the corresponding by the FEM. It is also interesting that the method of Schwarz, using the expression for the equivalent resistivity proposed by the IEEE Guide for Safety in AC Substation grounding leads to results which diverge significantly to those by the FEM. This is more obvious in soil cases where the resistivity of the upper soil layer varies greatly from the corresponding one of the bottom soil layer.

For the grounding systems which consist of grounding grids, the different methods show even greater variations in their results.

It must be noted that for the grounding grid configurations which lead to the desired low values of the grounding resistance, all methods seem to converge and this is a very important conclusion. However, the calculation of the resistance in complex grounding system is based on the calculation of the individual resistances of each grounding system element. Therefore the need to search for better approximations to be implemented in the IEEE Std, especially for the case of multilayered earth, is justified.

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