

# Application of MA with RMSLS to Probabilistic Distribution Network Expansion Planning

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**Abstract**—This paper proposes a meta-heuristic method for probabilistic distribution network expansion planning (DNEP). It has been studied for a long time, but recently system planners are faced with uncertainty under competitive power systems. A more flexible method is required to deal with the complicated distribution systems. This paper considers the uncertainty of the nodal specified values and multi-objective optimization. In this paper, a new Memetic Algorithm (MA) that consists of Genetic algorithm (GA) and local search (LS) is proposed to deal with multi-objective optimization. The uncertainty of the nodal specified values is considered in the Monte-Carlo Simulation (MCS). As a multi-objective solver, the  $\epsilon$ -constraint method is employed to solve a multi-objective problem while Random Multi Start Local Search (RMSLS) is used to evaluate local solutions efficiently. The proposed method is successfully applied to a sample system.

**Index Terms**—Distribution network expansion planning, Memetic Algorithm (MA), Uncertainty, Monte-Carlo Simulation (MCS)

## I. INTRODUCTION

THIS paper presents distribution network expansion planning (DNEP) in consideration of load uncertainty. The problem of DNEP may be expressed as a combinatorial optimization problem with multi-objectives. In recent years, distribution networks increase the complexity due to the deregulated and competitive power market as well as the emergence of distributed generation. Therefore, system planners need to rationalize both economic cost and customer requirements under new environment appropriately.

In this paper, Monte Carlo Simulation (MCS) is used to consider the uncertainty of loads in DNEP. In practice, there are correlations between the nodal specific values and distributed wind power generators. To realize the correlation, this paper makes use of the moment matching method (MMM) that adjusts each moment of the random number. The combination method of MMM adjusts the random number from the first-order to the second-order moments. This paper presents a method for considering the correlation with the combination method efficiently. DNEP results in a multi-objective combinatorial optimization problem. It may be solved under the constraints such as the upper and lower

bounds of nodal voltage, the limitation of capacity on the feeders and substations and the radial network conditions. This paper makes use of the  $\epsilon$ -constraint method as a multi-objective optimization solver. The solution is calculated by selecting the  $n^{\text{th}}$  objective function and setting the rest of objective functions as the upper bound constraints. So far, DNEP has been studied for a long time. Simulated Annealing (SA), Genetic Algorithm (GA) and Tabu Search (TS) have been used for solving a combinatorial optimization problem in DNEP. Recently, Memetic Algorithm (MA) is noteworthy for the effectiveness in a way that it combines GA with Local Search (LS) to obtain better solutions efficiently. It has a couple of strategies, *i.e.*, *GA then LS* and *GA with LS*. The former is an algorithm that after finishing the GA search process, LS carries out for enhancing the solution. The latter is an algorithm that LS improves the solution each time GA evaluates the solutions. In this paper, *GA then LS* strategy is used due to computational efficiency. MA carries out local search to obtain an accurate solution after global search is used. In other words, an efficient local search is necessary to evaluate more accurate solutions in MA. As LS, this paper employs Random Multi Start Local Search (RMSLS) that evaluate solutions with multi-point search. However, it has a drawback that some of the solution candidates are infeasible. To overcome the problem, the paper applies the greedy algorithm to the process of generating feasible initial solutions. The initial solutions of RMSLS are determined by GA solutions. The proposed method is successfully applied to a sample system.

## II. PROBLEM FORMULATION

This section describes the problem formulation for DNEP. It is one of complicated problems that handle combinatorial and multi-objective optimization problems. The allocation of feeders, distribution substations and distributed generators are regarded as a combinatorial optimizations problem. The objective functions consist of the installation cost of the feeders, distribution substations and distributed generators, the network active power loss and the average of voltage deviations. As a result, the mathematical formulation may be written as follows:

$$f_1 = \sum_{j=1}^{n_f} c_{fj} x_{fj} + \sum_{j=1}^{n_s} c_{sj} x_{sj} + \sum_{k=1}^{n_d} c_{dk} x_{dk} \quad (1)$$

$$f_2 = P_{\text{loss}} \quad (2)$$

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$$f_3 = \frac{1}{n} \sum_{i=1}^n (V_i - V_{base})^2 \quad (3)$$

Constraints:

$$y = k(z) \quad (4)$$

$$V_i^m \leq V_i \leq V_i^M \quad (5)$$

$$|P_{ij}| \leq T_{ij} \quad (6)$$

$$P_{si} \leq P_{si}^M \quad (7)$$

where

$f_1, f_2, f_3$ : objective functions

$n_f$ : number of feeder candidates

$c_{fi}$ : installation cost of feeder  $i$

$x_{fi}$ : variable denoting whether feeder  $i$  is installed or not,

$n_s$ : number of substation candidates

$c_s$ : installation cost of substation  $j$

$x_{sj}$ : variable denoting whether substation  $j$  is installed or not

$n_d$ : number of distributed generator candidates

$c_{dk}$ : installation cost of distributed generator  $k$

$x_{dk}$ : variable denoting whether distributed generator  $k$  is installed or not

$P_{loss}$ : network active power loss

$V_{base}$ : reference voltage

$V_i$ : voltage of nodal voltage  $i$

$y$ : specified value of power flow equation

$k$ : power flow equation

$z$ : power flow solution

$P_{si}$ : power capacity of substation  $i$

$P_{si}^M$ : upper bound of  $P_{si}$

$|P_{ij}|$ : absolute value of active power between nodes  $i$  and  $j$

$V_i$ : voltage magnitude of node  $i$

$V_i^m (V_i^M)$ : lower(upper) bound of  $V_i$

$T_{ij}$ : upper bound of  $|P_{ij}|$

Equation (1) shows the installation cost of new feeders and substations, where the first terms means the installation cost of new feeders, the second implies the installation cost of new substations, and the third implies the installation cost of new distributed generator. Equation (2) gives the network loss and (3) means the average of the sum nodal of magnitude voltage deviations. Equation (4) means the power flow equation for distribution networks. Equations (5) and (6) imply the upper and lower bounds of nodal voltage, and the limitation of line flow on feeders, respectively. Equation (7) shows the upper bound for power capacity of substations.

### III. MOMENT MATCHING METHOD (MMM)

This section describes MMM for giving random numbers with the correlation between the nodal specified values in MCS. This paper assumes that there exist the correlations between the nodal specified values if the period of DNEP is comparatively short, say, 5 years. According to information on the correlations, this paper aims at selecting the frequent network patterns under uncertain distribution network conditions. Specifically, as uncertain factors, this paper considers the uncertainty of load growth, distributed wind generator, etc. In this paper, MMM is used for generating the random numbers with the correlations. Going into detail,

there are several versions of MMM. This section describes three versions, i.e., the primary sampling, the quadratic sampling and the combination methods as follows:

#### A. Primary Sampling

The primary sampling is a technique that adjusts the random number vectors to fit the first-order moment to that of the input vector. The basic idea is to shift the mean of the normal distribution into the new random number vectors.

$$\mathbf{y} = \mathbf{x} + \boldsymbol{\mu} \quad (8)$$

where

$\mathbf{y}$ : new adjusted random numbers

$\mathbf{x}$ : random numbers of normal distribution

$\boldsymbol{\mu}$ : mean of input variable

#### B. Quadratic Sampling

The quadratic sampling creates the new normal random number vectors  $\mathbf{y} = (y_1, y_2, \dots, y_n)^t$  that adjusts the random number to fit the second-order moment of the created sample random number vector  $\mathbf{x} = (x_1, x_2, \dots, x_n)^t$  to that of the original one.

$$\mathbf{y} = \mathbf{C} \tilde{\mathbf{C}}^{-1} \boldsymbol{\mu} \quad (9)$$

where

$\mathbf{C}$ : lower triangular matrix of covariance matrix of learning data

$\tilde{\mathbf{C}}$ : lower triangular matrix of covariance matrix of  $\mathbf{x}$

#### C. Combination Method

The quadratic and the primary sampling adjust only one moment of created sample random number vectors. To improve a drawback of the primary and the quadratic sampling, two moments are adjusted by combining the quadratic with the primary sampling. As a result, the combination method may be written as follows:

$$\mathbf{y} = \mathbf{C} \tilde{\mathbf{C}}^{-1} (\mathbf{x} - \mathbf{m}) + \boldsymbol{\mu} \quad (10)$$

where

$\mathbf{m}$ : mean of multidimensional normal random numbers.

The combination method adjusts the mean value and the covariance of random numbers to preserve the stochastic characteristics of original ones. This paper utilizes the combination method to improve the performance of random numbers with the correlations.

### IV. PROPOSED METHOD

This section proposes a new MA-based method for solving DNEP. MA carries out LS to improve the solution quality after GA is used. In other words, LS plays a key role to evaluate more accurate solutions. This paper uses RMSLS that is different from any other LS algorithms in a way that multi-point search is used. It is expected that RMSLS evaluates better solutions efficiently. Also, the  $\epsilon$ -constraint method is employed to solve a multi-objective problem.

#### A. MA

MA was developed to improve the performance of GA

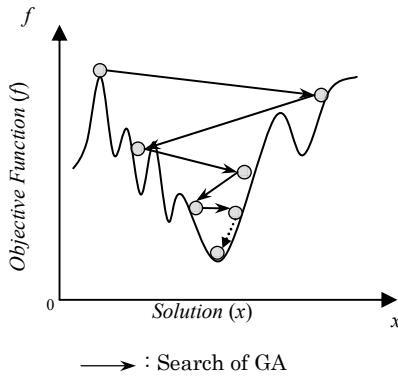


Fig. 1. Concept of MA.

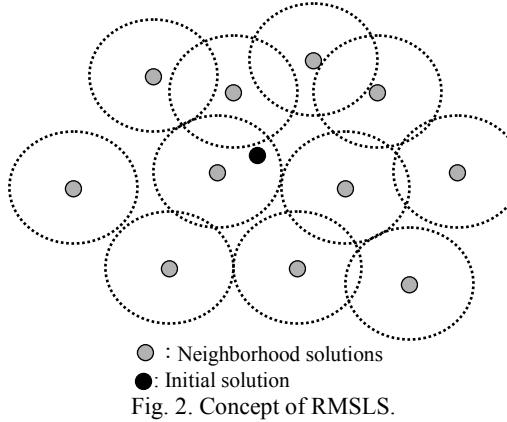


Fig. 2. Concept of RMSLS.

that often gets stuck in a local minimum. MA is based on the concept of the meme that evolves by natural selection through multiplication, mutation, competition, *etc.* If GA is integrated with LS, the hybrid meta-heuristic algorithm is referred to as MA. It is well known that MA has better performance in real-world applications. It has a couple of strategies, *i.e.*, *GA then LS* and *GA with LS*. The former calculates the solutions in a way that LS is used after GA obtains the final solution. On the other hand, the latter evaluates the solutions by repeating the process that after GA finds solutions, LS is used to improve the solutions. *GA then LS* is better in different multi-objective optimization problems although, generally speaking, it is not easy to judge which strategy is better. Thus, this paper employs *GA then LS*.

### B. RMSLS

RMSLS is one of LS algorithms that evaluate a local solution. However, it is different from any others in a way that it makes use of multi-point search to evaluate better solutions. Fig. 2 shows the concept of RMSLS, where it can be seen that there are eleven solution candidates for initial solutions. To estimate more accuracy solutions, RMSLS carries out LS for each initial neighborhood solution after generating initial neighborhood solutions randomly. Also, this paper combines RMSLS with the greedy algorithm to obtain the better solutions efficiently. In other words, RMSLS need generate initial neighborhood solutions with the pre-information of GA randomly.

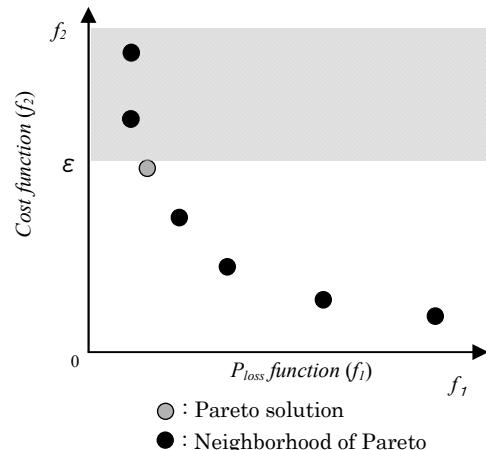


Fig. 3. Concept of  $\epsilon$ -constraint method.

### C. $\epsilon$ -Constraint Method

The  $\epsilon$ -constraint method is used to solve the multi-objective optimization problem of DNEP. The  $\epsilon$ -constraint method was proposed by Haimes, *et al.*. The method aims at evaluating the optimal solution with the scalarized method of the multi-objective functions. The solution is computed by selecting the  $n^{\text{th}}$  objective function while the rest of objective functions are set as the upper bound as the constraints. Fig. 3 shows an example in which cost function  $f_1$  is optimized while cost function  $f_2$  is set as a constant. As a result, the mathematical formulation may be written as follows:

Objective function:

$$f_n(x) \rightarrow \min \quad (11)$$

Constraint:

$$f_m(x) \leq \epsilon \quad (12)$$

where

$f_n$ :  $n^{\text{th}}$  objective function ( $n=1,2,\dots,i$ )

$i$ : number of objective functions

$f_m$ : objective functions excluding  $n^{\text{th}}$  objective function.

$\epsilon$ : upper bound for  $f_m$

$x$ : variable solutions to be optimized

It may be noted that following equation holds for the  $\epsilon$ -constraint method.

$$X_k(\epsilon_k) = \{x | f_k \leq \epsilon_k\} \quad (13)$$

$$\epsilon_k \in E_k = \{\epsilon_k | X_k(\epsilon_k) \neq \varnothing\} \quad (14)$$

## V. SIMULATION

### A. Simulation Conditions

- 1) The proposed method is applied to the 32-node distribution system (see Fig. 5). It is assumed that the system has 2 distribution substations (Nodes 100 and 200), 7 distributed generators (Nodes 33-39) and 83 feeders. The total numbers of bits are 92 so that the number of combinations results in  $1.69 \times 10^{66}$ . Substations and distributed generators require the creation of the neighborhood solutions with the Hamming distance of one and two, respectively

Table 1. Parameters of each method.

Methods	Parameters	
GA	No. of populations	300
	No. of generations	1800
	Crossover rate	0.8
	Mutation rate	0.02
LS	No. of neighborhood solutions	200
RMSLS	No. of neighborhood solution from initial solutions	50
	No. of neighborhood solutions	70

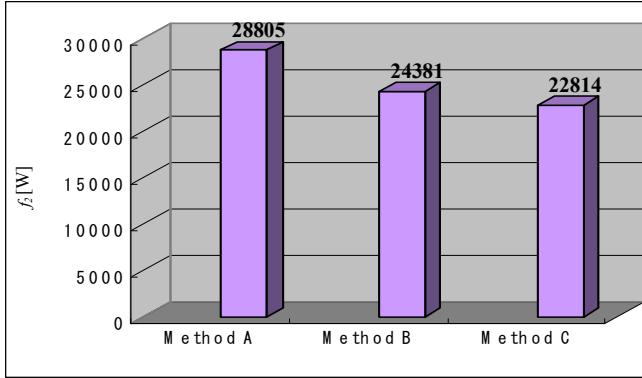


Fig. 6. Average cost function  $f_2$  of each method.

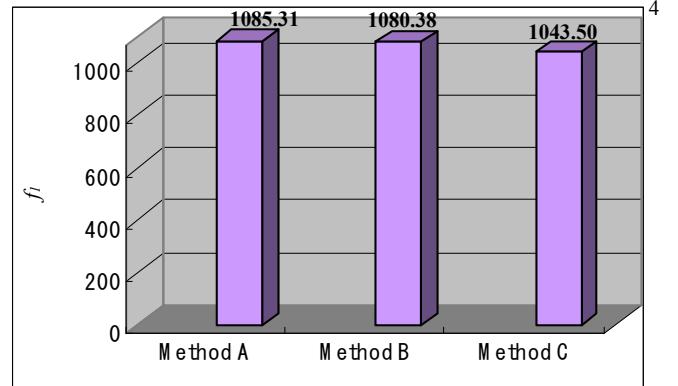


Fig. 5. Average cost function  $f_1$  of each method.

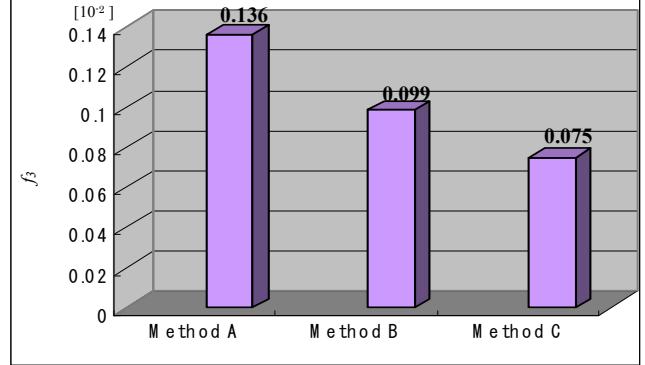


Fig. 7. Average cost function  $f_3$  of each method.

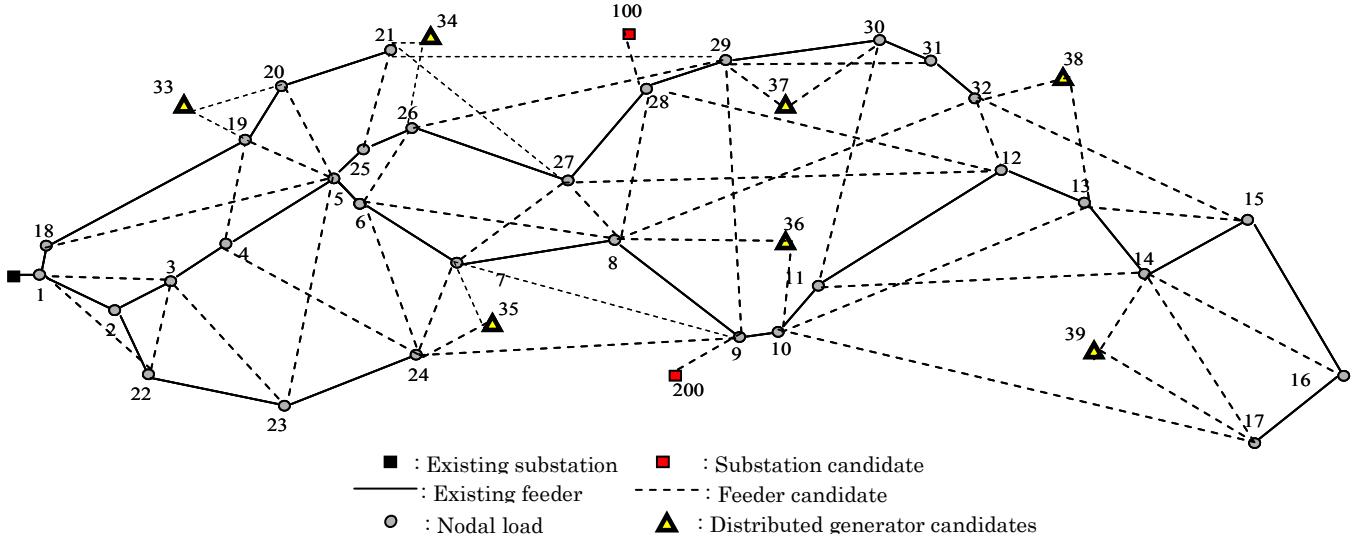


Fig. 4. 32-node distribution system.

- 2) Monte-Carlo simulation (MCS) is carried out to consider the uncertainty of load growth at each node as well as the output power of distributed wind generators (Nodes 34, 37 and 39). The output of other distributed generators is constant. As LS, this paper makes use of variability LS. The combination method of the moment matching method is used to simulate the uncertainty of

load patterns in the future. The random numbers that vary from 80% to 160% of the original data are used to create load patterns.

- 3) The  $\epsilon$ -constraint method sets up the upper bound of cost functions  $f_1$  and  $f_3$ , to be 1100 and  $0.25 \times 10^{-1}$ , respectively.

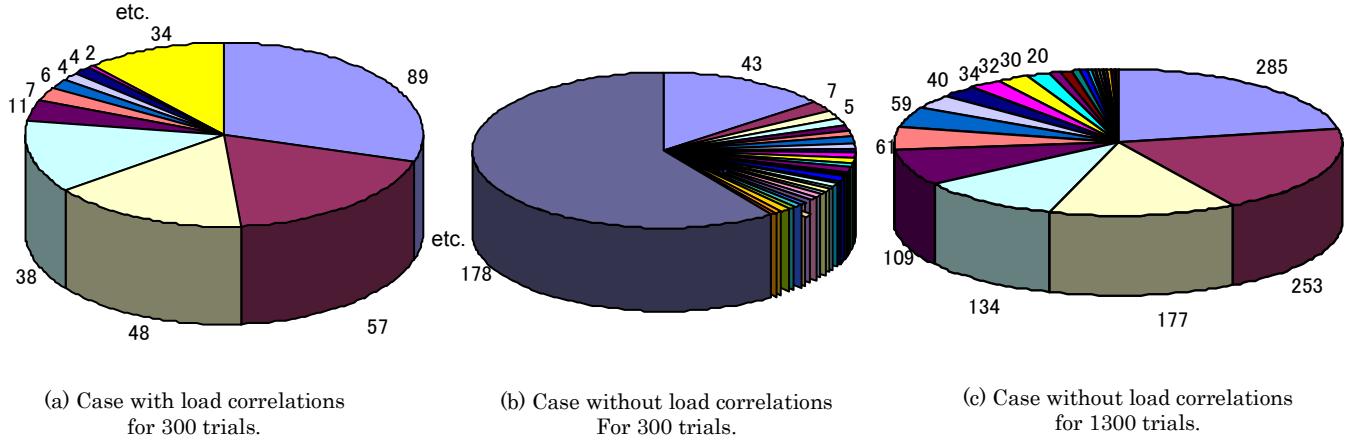


Fig. 8. Frequency of optimal distribution networks

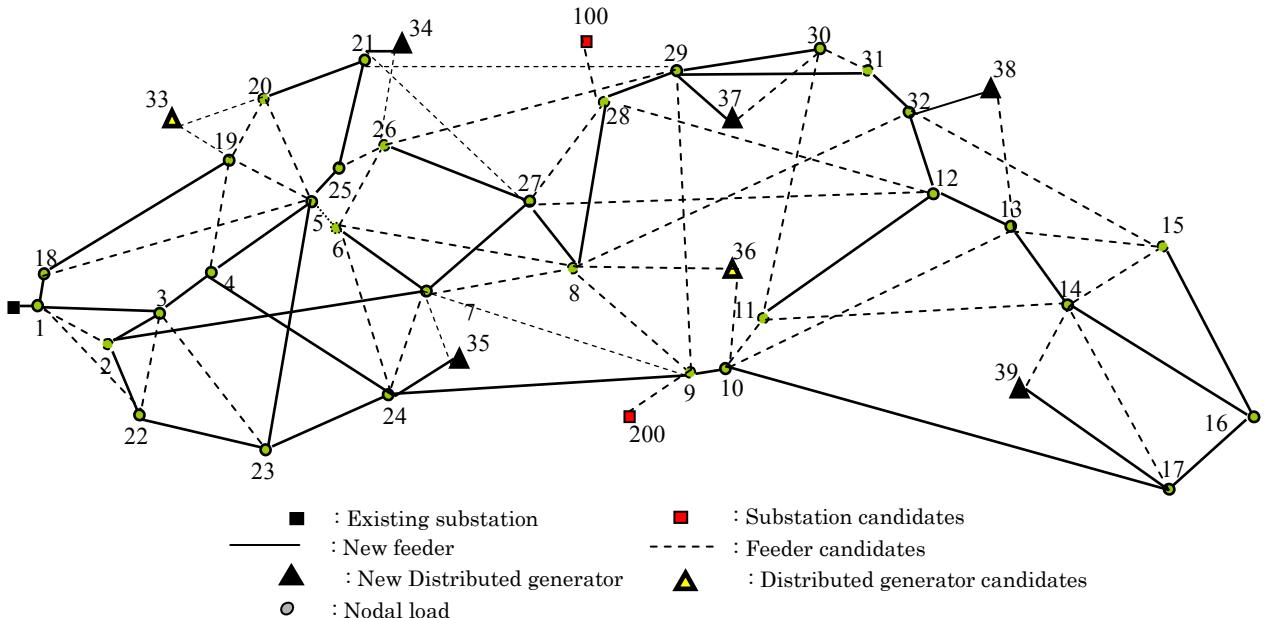


Fig. 9. Most frequent network configuration in 32-node distribution system.

- 4) The proposed method is compared with simple GA and GA using LS in terms of solution accuracy and computational time. The initial solution of RMSLS and LS are based on the solutions obtained through GA. For convenience, the following methods are defined;
  - a) Method A: GA
  - b) Method B: GA with LS
  - c) Method C: GA with RMSLS (proposed method)
- 5) Table 1 shows parameters of each method used in the simulation. GA used the population size of 1800 and the tournament selection strategy. The mutation rate of 0.02 and the crossover rate of 0.8 were used as the genetic operators. Regarding RMSLS, the number of initial neighborhood solutions obtained from GA is 50 while the number of neighborhood solutions from each initial neighborhood solutions is 70.
- 6) RMSLS randomly creates the neighborhood solutions with the Hamming distance of 2, 4 and 6. Similarly, variable LS randomly creates neighborhoods with the

Hamming distance of 2 and 4. The computations were performed on the Dell PC DIMENSION 8300 (Pentium(R) 4 CPU 2.80GHz, Memory 512MB).

### B. Simulation Results

Figs. 5-7 give the average cost functions  $f_1$ ,  $f_2$  and  $f_3$  in consideration of the correlations. Fig. 5 shows that Methods B and C reduced 0.45% and 3.85% of the cost function for Method A, respectively. In the same way, Fig. 6 indicates that Methods B and C reduce 15.36%, 20.79% of the cost function for Method A, respectively. Fig. 7 shows that Methods B and C reduced 27.21% and 44.85% of the cost function for Method A, respectively. Thus, it can be seen that the proposed method gives better solutions. Compared with Method B, Method C reduced 3.41%, 6.34% and 23.7% of cost functions,  $f_1$ ,  $f_2$  and  $f_3$ , respectively. That is caused by the difference between LS algorithms. Method C has advantage to combine GA with RMSLS of multi-point search. Thus, the solutions of RMSLS are more diverse than

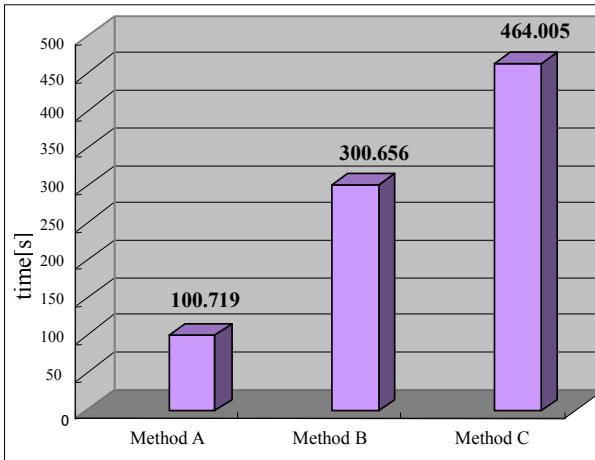


Fig. 10. Average computational time of each method.

these of other LS algorithms in searching solutions. Furthermore, RMSLS made use of variable local search (variable LS). That improves local minimum through the solution diversity. RMSLS has better performance by into variable LS.

Figs. 8 (a) and (b) give the frequency of optimal network configuration solutions for 300 trials with and without the correlations respectively. Compared with Figs. 8 (a), Fig. 8 (b) efficiently captures the feature of the system planning solution for the same trials. Fig. 8 (c) gives the frequency of optimal solutions for 1300 trials without the correlations. MCS without the correlations requires more trials since the results in Fig. 8(a) is closer to those in Fig. 8(c). Namely, the proposed method evaluates more realistic solutions with less trials in MCS. Fig. 9 gives the most frequent network configuration in optimal solutions in Figs. 8 (a)-(c). It can be observed that distributed generators are positively employed in DENP. Fig. 10 shows that Methods A, B and C took 100.719[s], 300.656[s] and 464.005[s] respectively. Method C is slower than Methods A and B. That is because Method C carries out LS for each initial neighborhood solution after generating initial neighborhood solutions randomly. However, as a planning method, the computational time of Method C is sufficiently acceptable.

## VI. CONCLUSION

This paper has proposed the new efficient method for distribution network expansion planning with MA that consists of GA and RMSLS. Also, this paper took into consideration the uncertainty for nodal specific values and output of distributed wind generators. Monte-Carlo Simulation (MCS) was used for examining the most frequent network configuration with and without the correlations between the nodal specified values and the output of the wind power generators. Specifically, this paper made use of the moment matching method to carry out MCS with the correlations. The simulation results has shown that the moment matching method plays an important role to save computational time in a way that the number of trials is

reduced in MCS. The proposed method allows system planner to select planning solution efficiently.

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## VII. BIOGRAPHIES

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