

# Nash Genetic Algorithm Based Optimal Design of Hysteresis Inverters for Active Power Filtering Applications

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**Abstract-** In this paper optimal design of a Current Regulated-Voltage Source Inverter (VSI) to be used for active filtering applications using game theory based multi-objective optimization methods is approached. The Inverter hysteresis band and the dc bus voltage are considered as design variables, while the average switching frequency and the tracking error are considered as two distinct objective functions to be minimized. For the first time, the Nash Genetic Algorithm (Nash-GA), a co-evolutionary optimization approach based on the Nash Equilibrium concept is introduced and used to reach the optimal values for the dc bus voltage as well as the hysteresis band. Both parameters affect the loss and tracking performance of the Inverter. The simulation results obtained in MATLAB/SIMULINK environment prove the superiority of the novel approach presented.

Index Terms: Active Filter, Harmonics, Game theory, Optimization, Nash Genetic Algorithm

## I. INTRODUCTION

Today power utility networks suffer from large harmonic pollution caused by nonlinear loads. Active Power Filters (APF) are used as modern equipments to detect and then compensate the unwanted current harmonics of such nonlinear loads to avoid their adverse impact on the power system [1]-[10]. Moreover, some advanced active filtering systems called Unified Power Quality Conditioner (UPQC) shown in Fig. 1 consist of one series and one parallel compensator [1]. That enables the system to improve the load voltage besides compensation the source current. Such harmonics and/or non-active currents, when detected, should be compensated by a high performance Inverter injecting the undesired harmonics in opposite phase to cancel out the unwanted source line components. The Inverter, which is sometimes controlled by a simple hysteresis controller, should synthesize non-sinusoidal currents, which require high switching frequency and its associated loss. Besides being simple, hysteresis controller provides higher tracking ability for the Inverter [4]. On the other hand, cost and loss of active filtering systems still matter, preventing such

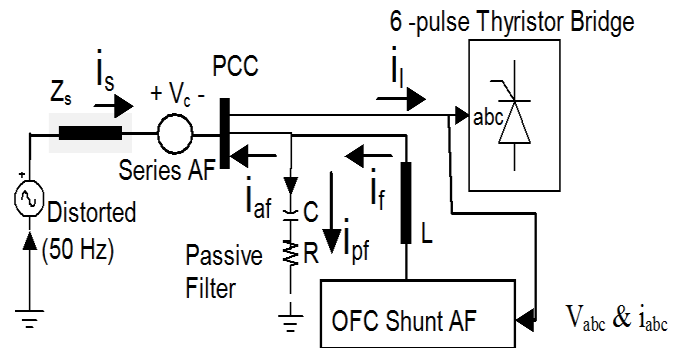


Fig. 1. The UPQC conditioner under study  
( $L_s=2\mu\text{H}$ ,  $C=10\mu\text{F}$ ,  $R=50\Omega$ ,  $L=4\text{mH}$ )

systems from widespread use. Hence, as a mature field of research, active filter design needs optimization approaches to improve the performance and minimize the cost and loss [2],[3],[8],[9]. In this problem we are concerned with at least two different goals must be reached as follow:

1) The Inverter must successfully track its reference produced by the control algorithm of the APF [5]-[7].

2) The Inverter's loss should be minimized to increase the system efficiency and to protect the solid state devices.

This switching loss directly depends on its average switching frequency and the dc bus voltage as well. In this regard, the lower hysteresis band the lower tracking error but higher switching loss. As seen, for an optimal design, we are really concerned with a multi objective design scenario. In this paper we are going to tackle the problem by designing optimal values of the hysteresis band  $H_{\text{sys}}$  as well as dc bus voltage  $V_{\text{dc}}$  via a multi-objective optimization approach for the first time. Among the large family of multi-objective optimization approaches, Game Theory based approaches using evolutionary techniques have found much interest in the recent decades [11]-[16]. In this paper the Nash Genetic algorithm [16], which is based on the Nash Equilibrium (NE) concept is used. In simple words, NE refers to a situation in a game where no player alone (here each objective function) can change its strategy (design variables) improving its situation. To solve the problem, two separate generations are considered. For the first the fitness function is the average switching frequency  $f_s$  working on  $H_{\text{sys}}$ , while the other is the tracking error  $E_{\text{tr}}$ , which try to reach the best  $V_{\text{dc}}$ . The two sets of

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generations share information between each other. The Inverter system used in this study is a set of three single-phase half-bridge Inverters. The results show that the optimization approach can successfully improve the system tracking performance at a minimum cost in terms of switching loss. The organization of the paper is as follows. In section II, the active filtering system under study is introduced. Section III presents a short introduction on multi objective optimization and Nash Genetic Algorithm. Simulation results and discussions are given in section IV, and Section V presents concluding remarks.

## II. SYSTEM UNDER STUDY

Fig.1 shows the UPQC system under study. The series compensator removes all the harmonics and imbalance components of the source voltage, whereas the parallel part is in charge of compensating the current harmonics and power factor improvement [2]. The load is also a 6 pulse front-end thyristor bridge. Fig.2 depicts the source as well as the load voltage waveforms and as seen, the load voltage is almost sinusoidal and balance.

### A. Compensation Strategy

There are numerous compensation strategies such as p-q [17], unity power factor (UPF), Perfect Harmonic Compensation (PHC), and Optimal and Flexible Control (OFC) [2],[3]. Because, the source supply is distorted, in this study, the OFC system is used for controlling the parallel compensator. The compensation strategy here is to maximize power factor subject to some constraint on THD as well as individual harmonics of the source current complying with IEEE 519 standards [10]:

$$\text{Min Power Factor} \quad (1)$$

Subject To : Constraints on Harmonics ( IEEE-519)

It is worth mentioning that under distorted voltage conditions, maximizing power factor as an important power quality requirement does not necessarily mean compensation of all the current harmonics. Hence, based on the compensation strategy, the source current after compensation may still contain harmonics [2].

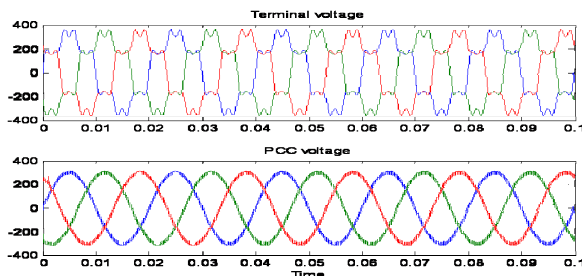


Fig. 2. Up: Source Voltages Down: Load Voltages after series compensation.

## B. Hysteresis Controlled Inverters

Three single-phase full bridge Inverters have been used to generate the compensating currents calculated by the OFC system based on compensation strategy (1). Fig. 3 shows the structure of the hysteresis controlled Inverter system for each phase where  $I_{cr}$  and  $I_f$  and the reference and output currents of the Inverter.

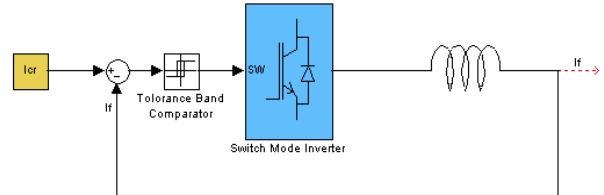


Fig.3 Block diagram of the Hysteresis Controlled Inverter

## III. Multi-Objective Optimization

Generally, a multi-objective optimization (MOP) problem can be represented as:

Minimize:

$$g = f(x) = (f_1(x), \dots, f_i(x), \dots, f_k(x)) \quad (2)$$

Subjected to:

$$x = (x_1, x_2, \dots, x_n) \in X \quad \& \quad y = (y_1, y_2, \dots, y_k) \in Y$$

Where  $f(x)$  is vector of the objective functions to be minimized and  $x$  is the vector of design variables.

### A. Pareto Based Approaches

**Definition:** The vector  $a$  in the search space dominates vector  $b$  if:

$$\forall_i \in \{1, 2, \dots, k\}: f_i(a) \geq f_i(b)$$

$$\exists_j \in \{1, 2, \dots, k\}: f_j(a) > f_j(b) \quad (3)$$

If at least one vector dominates  $b$ , then  $b$  is considered dominated, otherwise it is called non-dominated. Each non-dominated solution is regarded optimal in the sense of Pareto or called Pareto optimal. The set of all non-dominated solutions is called Pareto Optimal Set (POS) and the set of the corresponding values of the objective functions is called Pareto Optimal Front (POF) or simply Pareto Front.

One way of handling a multi-objective or vector objective problems is to combine the goals of the optimization problem and constructing a scalar function

and then using a common scalar optimization approach to solve the problem. The major dilemma of this methodology is unavailability of any straightforward method for combining the objectives or goals of the problem. Game theory concepts are applicable for a multi-objective optimization problem successfully in its own original form needless to any major modifications or combining the objective functions. But of course it requires an evolutionary method to reach globally optimum results [11]-[16]. Some advanced evolutionary method based on Pareto optimality concept are as [15]:

- Niche Pareto Genetic Algorithm (NPGA)
- Hajela's and Lin's Genetic Algorithm (HLGA)
- Vector Evaluated Genetic Algorithm (VEGA)
- Non-dominated Sorting Genetic Algorithm (NSGA)
- Strength Pareto Evolutionary Algorithm (SPEA)

NSGA II [14] and SPEA [15] are the most successful approaches which are based on Pareto optimality concept. The result will be a set of optimal points not a single point. The designer must select one of them which look most suitable for the problem.

### B. Nash Genetic Algorithm

Pareto based approaches are essentially used for cooperative games where the players cooperate to solve a common multi objective optimization. Sometimes, the game is not cooperative and each player wants to optimize its own objective. Nash equilibrium is the solution of a non-cooperative game for MOPs. This concept was introduced by Nash in 1952. According to Nash, each participant in a game has their own strategy set and objective function. Through the game, each player searches for the optimal strategy for its own objective function while the strategies of the other players are fixed. Within this framework, evolutionary gaming run until no player can further improve its objective function, the system is then considered to have reached a state of equilibrium named Nash Equilibrium (NE). Nash Genetic Algorithm (Nash GA) is a co-evolutionary approach for multi objective optimization based on Nash Equilibrium concept [16].

Fig. 4 shows block diagram of the algorithm [16]. The idea of Nash GA is to combine genetic algorithms and Nash concept to find the Nash Equilibrium as a solution to the problem.

Let  $s = (X, Y)$  denotes the potential solution for a dual objective optimization problem for functions  $f_1(X, Y)$  and  $f_2(X, Y)$  and two populations are generated for the players. The optimization task of player 1 is carried out by population1, whereas that of player 2 is performed by population2. Then  $X$  denotes the subset

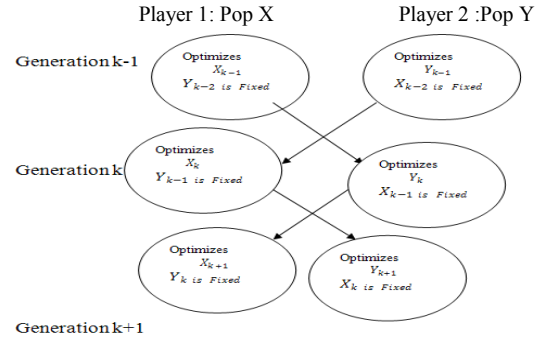


Fig. 4. Nash GA optimization Flowchart

of variables handled by player 1 and is optimized along objective function 1. Similarly,  $Y$  denotes the subset of variables handled by player 2 and optimized based on objective function 2. Hence, this way, player 1 optimizes  $f_1$  with respect to the first objective function through regeneration of population1 for  $X$ , while  $Y$  is fixed. Similarly, player 2 optimizes  $f_2$  with respect to the second objective function by regeneration of population2 for  $Y$ , while  $X$  is fixed. At each generation  $k$ , the best value found by the other player at generation  $k-1$  is used as fixed parameters. In other words, let  $X_{k-1}$  be the best value found by player 1 at generation  $k-1$  and  $Y_{k-1}$  the best value found by player 2 at generation  $k-1$ . At generation  $k$ , player 1 optimizes  $X_k$  while using  $Y_{k-1}$  in order to evaluate  $f_1$  (in this case,  $s = (X_k, Y_{k-1})$ ).

At the same time, player 2 optimizes  $Y_k$  while using  $X_{k-1}$  in order to evaluate  $f_2$  (in this case,  $s = (X_{k-1}, Y_k)$ ). After the optimization process, player 1 communicates the best value  $X_k$  to player 2 who will use it at generation  $k-1$ . Similarly, player 2 sends the best value  $Y_k$  to player 1 who will use it at generation  $k-1$ . Nash equilibrium is reached when none of the players can further improve their objective functions.

## IV. NASH GA BASED ACTIVE FILTERING

Ref. [16] shows that while Nash GA reaches only a single point for each optimization evaluation, it can be more robust and efficient compared to Pareto based approaches. In this paper considering the heavy calculation operation needed to simulate the whole active filter system with high switching frequency, a more efficient approach such as Nash GA can be much more attractive, even though it yields a single local solution for each evaluation.

The compensation strategy used in this paper is adopted from [2] and given by (1) while a Unified Power Quality Conditioner (UPQC) topology shown in Fig. 1 is used. The UPQC contains a series as well as a parallel compensator. As already mentioned, the series part compensates the load voltage and the series part is in charge of removing the source currents harmonics and any other undesired

components. The control strategy of the series part is set to remove all the harmonics as well as the imbalance components. The parallel part is controlled such that the overall load power factor is maximized subject to some constraints on the currents harmonics based on IEEE-519 standards [10]. Its currents THD would be around 5% but with highest possible power factor. Also, any other compensation strategy can be used. This subject is of course beyond the main field of interests of this paper.

Here we assume there are two objective functions to be minimized: a) the tracking error and b) the average switching frequency. The objective functions are affected by both the design variables  $V_{dc}$  (dc bus voltage) and  $H_{sys}$  (Hysteresis band).  $H_{sys}$  theoretically limits the error band, but for a real digital implementation, there is always a finite sampling rate. This allows the error to violate the error band posed by the hysteresis band  $H_{sys}$ . This fact makes the situation more difficult, as there will be no so explicit mathematical relations between the design variables and the objective functions. That highlights the need for using evolutionary approaches such as Genetic Algorithms as part of optimization process, because such approaches do not need any explicit mathematical description of the functions to be minimized.

For the system under study, several different objective functions based on some well-known technical interests can be defined, which are functions of the system parameters such as hysteresis band and dc bus voltage. Some useful objective functions to be minimized are:

- average of switching frequency
- switching loss
- overall kVA needed for the Inverter.
- solid state switches stress.
- tracking error of the Inverter
- capacitive dc bus voltage fluctuations

Any couple of above functions can be used to form a multi-objective optimization problem. Here, as an example to show the successful application of the Nash GA, the objective functions are defined as follow where  $f_s$  and  $E_{rr}$  are average frequency and tracking error (average of the absolute error at steady state) respectively:

$$f_s = f_1(V_{dc}, H_{sys}) = \frac{\text{no of turns on in period } T}{T} \quad (4)$$

$$E_{rr} = f_2(V_{dc}, H_{sys}) = \frac{1}{T} \int |e(t)| dt \quad (5)$$

Where  $T$  is the period and  $e(t)$  is the tracking error signal between the reference current and the real output current generated by the Inverter. There are two sequences of generations corresponding to the two objective functions. Each sequence of generations can modify only one variable by means of the well-known GA operators that is selection, mutation, and crossover. For each step of evolution, the

fitness function of each sequence is affected by the other sequence. For example, in the active power filter problem considered in this paper, the first sequence fitness function is the same  $f_s$  introduced by (4) and the corresponding variable is  $H_{sys}$ . But it must be noted that,  $f_s$  is also a function of variable  $V_{dc}$  being modified by the other sequence.

Generally, it is quite difficult to reach an exact Nash equilibrium point in the whole search space. But the approach tries to reach some local point at least.

### Simulation Results and discussions:

The genetic Algorithms used for each objective functions uses well-know selection, mutation, and crossover operators. Each generation has 8 members to avoid a very time consuming optimization scenario. 10 percent of each generation is moved to the next one. As shown in table 1, after **only 5 generations** the program has reached the NE point, but we recognize it when we evaluate the **6<sup>th</sup> generation** where there is no change in the objective functions. This point is considered NE. However, due to some randomness in GA algorithms to ensure diversity needed for global optimum search, if we continue the optimization for extra generations, we may reach better points or a new NE. This new one can be better or worse than the previous one. To show the reader that a few extra generations does not gain any new NE, in this study we have run the program for 30 generations. Also, Fig. 5 shows the objective functions evolution through the first 30 generations and as seen there is no changes in the objective functions. The sampling frequency for the digital control system of the Inverter is set at 10 kHz. The results are quite interesting. The resulting optimal variables at equilibrium point are as  $H_{sys} = 1.0804$  A and  $V_{dc} = 374$  v, while the best values in the initial generation are 0.5 A and 350v respectively. Comparing NE results with the results of the first generation shown in Table 1 (i.e. the rows corresponding to generation No. 5 and 1 respectively) shows that we have reached much lower tracking error at a cost of around 55% increase in the switching frequency.

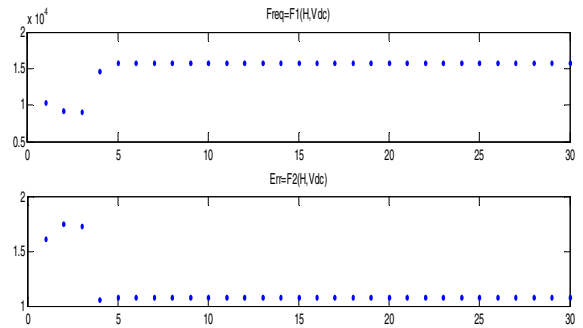


Fig. 5 : Average Frequency and Error through evolutions  
Up: Frequency Down: Tracking Error

Fig. 6 shows the load current, source current after compensation, the Inverter's error signal, and the

compensating current produced by the active filter (Inverter) for the best results generated by the first generation. Fig. 7 depicts the results obtained after 6th generation where the program has reached the Nash Equilibrium. The figures reveal sufficient improvement in the error signal as already predicted in Table 1 numerically. As another point utilizing Nash GA, as can be seen in Fig. 5 as well as Table1, generation 4 gives better results with respect to both the objective functions. It can be due to random behavior of the GA algorithms. This means that NE obtained from the Nash GA is not necessary the best one and may not belong to the exact Pareto Front. As such, one way is to use the NE point as a stop criterion for the program then check all the solutions gained through the whole optimization process. Here, for all the simulations, the passive filter has been removed merely to better illustrate the level of switching ripple in the source current waveforms.

Load characteristics can influence operation of OFC system introducing a transient regime [3]. Once the reference current is calculated in steady state, the Nash GA is used to optimize the Inverter operation.

Table 1: Objective functions and design variables changes through generations for Nash GA algorithm

No	$E_{rr}$	$f_s$ (Hz)	$V_{dc}$ (V)	$H_{sys}$ (A)
1	<b>1.6123</b>	<b>10269</b>	<b>350</b>	<b>0.5</b>
2	1.7485	9183.5	357.88	0.81472
3	1.7248	9038.9	357.88	0.90579
4	<b>1.0574</b>	<b>14586</b>	<b>357.88</b>	<b>0.12699</b>
5 (NE)	<b>1.0804</b>	<b>15746</b>	<b>374.27</b>	<b>0.12699</b>
6	1.0804	15746	374.27	0.12699
7	1.0804	15746	374.27	0.12699
8	1.0804	15746	374.27	0.12699
9	1.0804	15746	374.27	0.12699
10	1.0804	15746	374.27	0.12699
11	1.0804	15746	374.27	0.12699
12	1.0804	15746	374.27	0.12699

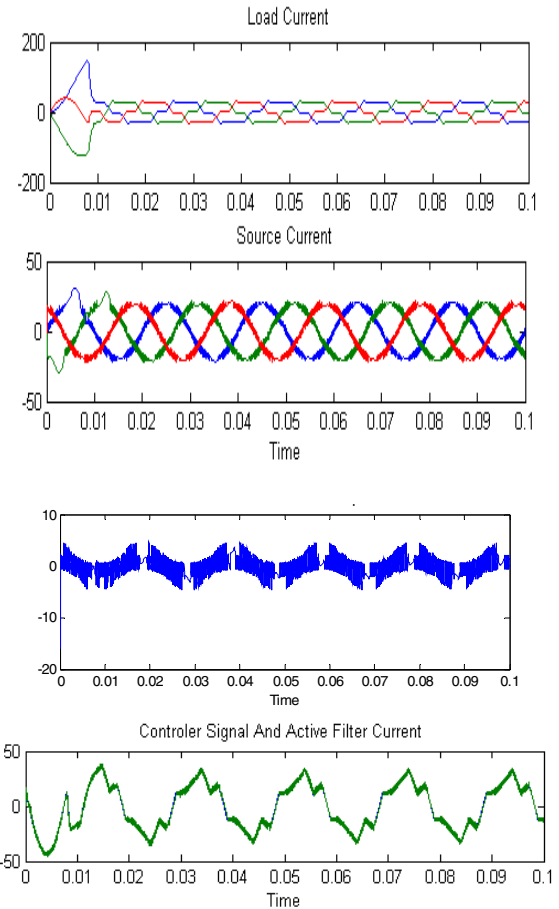


Fig. 6. Results after the first generation.

Up to down: Load Currents, Source Current, Error Current of the Inverter, and The Active Filter Current (Phase a)  
Vert. : Current (A), Horiz.: Time(S)



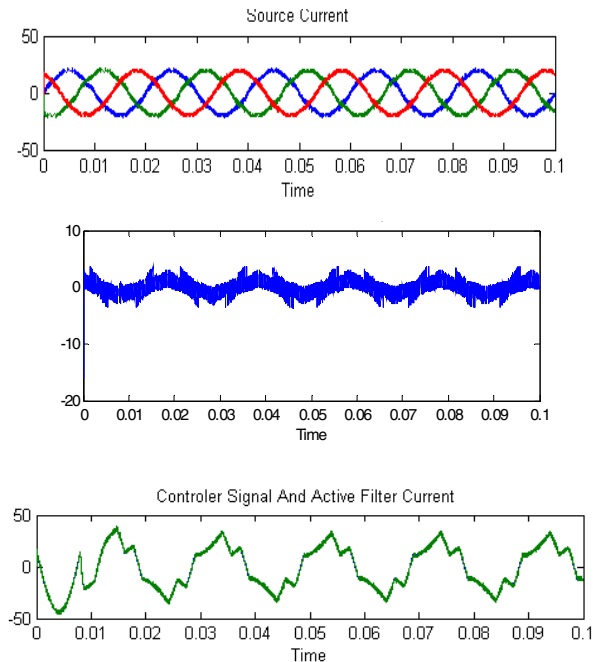


Fig. 7. Nash GA results after 5 generations. Up to down: Source Current, Error Current of the Inverter, and The Active Filter Current (Phase a) Vert. : Current (A), Horiz.: Time(S)

## V. CONCLUSIONS

In this paper, optimal operation of hysteresis controlled Inverters for active filtering applications using Nash GA multi objective optimization method is approached. Without any changes in the structure of the system and only by optimizing the dc bus voltage and hysteresis band of the Inverter we can optimize a set of important system characteristics. In the paper, the Inverter's tracking error as well as average switching frequency are considered as two objective functions to be optimized simultaneously. As addressed, some different sets of objective functions can be taken based on the technical requirements of the system under study. Considering the fact that simulation time for the Inverter system is rather high, a multi objective optimization like Nash GA looks more practical. However, performance of some other multi objective optimization approaches can be verified for the future works.

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