Impact of Load and Generation Price Uncertainties in Spot Prices

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Abstract — **In this paper it is presented a formulation for the DC Optimal Power Flow problem considering load and generation cost uncertainties and the corresponding solution algorithms. The paper also details the algorithms implemented to allow the integration of losses on the results as well the algorithm developed to compute the nodal marginal price in the presence of such uncertainties. Since loads and generation costs are represented by fuzzy numbers, nodal marginal prices are no longer represented by deterministic values, but instead, by membership functions. To illustrate the application of the proposed algorithms, this paper also includes results based on a small 3 bus system and on the IEEE 24 bus/38 branch test system.**

Index Terms— Uncertainties, fuzzy models, DC optimal power flow, multiparametric programming, nodal marginal prices

I. Introduction

THE development of electricity markets and the consequent unbundling process of the vertically integrated companies becomes a current practice in a wide number of countries all over the world. In this context, several authors refer that one of the main goals of a tariff methodology is the identification or development of methods that promote the efficient use of the system and that at the same time guarantee the long-term system reliability. In this sense, several tariff methods were proposed, namely average, incremental, and marginal approaches [1, 2]. Essentially motivated by their intrinsic simplicity and easiness of implementation the average based methods have been the ones traditionally used in the computation of the transmission tariffs. Nevertheless, since this kind of methods can not provide adequate economic signals for the efficient system planning and operation, marginal methods are very attractive given their economic foundation. These methods, however, also display several drawbacks, which are typically related with their inability to adequately remunerate the transmission activity, since their computation usually does not take into account long-term investment cost. This under recovery problem is in fact well known in the literature and it is termed as the Revenue Reconciliation problem [3]. Another drawback of this methodology approach is related with their temporal and spatially volatility, which suggests that their computation should integrate uncertainties on loads and generation cost, especially in a market environment, since they lead to changes in the dispatch policy and thus in marginal prices. Additionally, they should also integrate reliability data regarding system components since this issue also affects the value of nodal marginal prices.

The treatment of uncertainties in power systems has long been addressed. Probabilistic methods were the pioneering methodologies developed in this area. Papers [4] [5] describe the main concepts related with this problem as well as the initially developed algorithms using convolution techniques, the DC model and different linearized versions of the AC power flow problem. Apart from data having probabilistic nature, there are situations in which the uncertainty has not a random nature but it derives, for instance, from the incomplete characterization of the phenomenon under analysis or from insufficient frequency phenomena. In other cases, uncertainty is related with vagueness in the sense that the human language has an intrinsic subjective nature. In this context, since the 80's Fuzzy Set models are under development and application to power systems in order to provide a new framework to model the vague or ill defined nature of some phenomena. This application already occurred or is under way in areas as Fuzzy Power Flow, Fuzzy Optimal Power Flow, risk analysis and reinforcement strategies, generation planning, reliability models, fuzzy reactive power control, fuzzy dispatch and fuzzy clustering of load curves or even in transient or steady state stability analysis.

In this context, reference [6] describes a Fuzzy DC Optimal Power Flow model admitting that, at least, one load is represented by a fuzzy number. As a result, generations, branch flows and power not supplied, PNS, displays fuzzy representations translating data uncertainty. Afterwards, using this Fuzzy OPF, reference [6] also describes how load uncertainties can be reflected in fuzzy distributions for nodal marginal prices. In this paper we are now enlarging this approach in order to consider not only load uncertainties, but also generation cost uncertainties represented by fuzzy numbers. Additionally, the developed solution algorithms use multiparametric linear optimization techniques that allow obtaining more accurate descriptions of the possible behavior of the system under the form of membership functions.

After this Introduction Section, Section II describes the New Fuzzy Optimal Power Flow (NFOPF) model and the algorithm developed to integrate losses and Section III details the

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computation of nodal marginal prices in the presence of load and generation cost uncertainties. Section IV and V present results based on a three bus/three branch system and on the IEEE 24 bus/38 branch test system. Finally Section VI presents the most relevant conclusions.

II. NEW FUZZY OPTIMAL POWER FLOW ALGORITHM

A. General Aspects

Starting from the original concept of the Fuzzy Optimal Power Flow developed by the second author of this paper [6], the NFOPF model [7] is an optimization problem aiming at identifying the most adequate generation strategy, driven by an economic criterion, admitting that, at least, one load or generation cost is represented by a fuzzy number. Similarly to its original model, the NFOPF also uses the DC approach to model the operation conditions of the network. However, instead of running a number of parametric studies, the new developed model uses multiparametric linear programming techniques. This in fact represents an important improvement, since it allows obtaining more accurate membership functions in the sense they actually represent the widest possible behavior of each output variable. Figure 1 presents the corresponding flowchart of the algorithm.

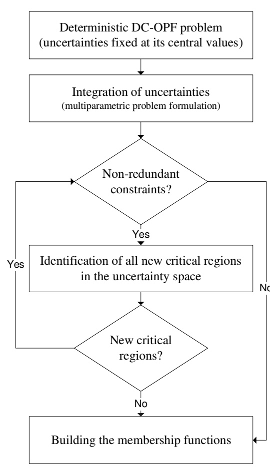

Fig. 1. New Fuzzy DC Optimal Power Flow algorithm [7]

B. Initial Deterministic Study

As it can be seen from Figure 1, the NFOPF algorithm starts with the execution of a deterministic DC-OPF (1-5) considering the central values of the fuzzy numbers that represent loads and generation costs, P_{Lk}^{ctr} and c_k^{ctr} .

$$min\ f = \sum c_k^{ctr}.Pg_k + G.\sum PNS_k \quad (1)$$

subj. $\sum Pg_k + \sum PNS_k = \sum P_{Lk}^{ctr}$ (2)

$$\mathrm{Pg}_k^{\min} \le \mathrm{Pg}_k \le \mathrm{Pg}_k^{\max} \quad (3)$$

$$PNS_k \le P_{Lk}^{ctr} \quad (4)$$

$$P_b^{min} \le \sum a_{bk}.(\ Pg_k + PNS_k - P_{Lk}^{ctr}\ ) \le P_b^{max} \quad (5)$$

In this model Pg_k is the generation in bus k having cost c_k and PNS_k is the power not supplied in bus k, $\mathrm{Pg}_k^{\min}, \mathrm{Pg}_k^{\max}$, $P_b^{\min}$ and $P_b^{\max}$ are the generation and branch flow limits and a_{bk} is the DC sensitivity coefficient of the flow in branch b regarding the injected power in bus k.

Once a feasible and optimal solution of this problem is identified, they are integrated parameters representing each fuzzy load or generation cost leading to a multiparametric optimization problem.

C. Integration of Uncertainties

Following the algorithm presented in Figure 1, after identifying a solution for the deterministic problem (1-5), the algorithm tries to find other optimal and feasible basis in regions of the uncertainty space based on the optimality (6) or feasibility (7) conditions. These regions are called critical regions and this process is conducted by pivoting over the initial basis as well as over all the new ones identified during the search process.

$$\mathrm{C^T}(\Phi_k) - \mathrm{C_0^T}.\mathrm{B_\rho^{-1}}.\mathrm{A} = (\mathrm{c} + \mathrm{c}'(\Phi_k)) - \mathrm{C_0^T}.\mathrm{B_\rho^{-1}}\mathrm{A} \ge 0 \quad (6)$$

$$\mathrm{B_\rho^{-1}}.\mathrm{b}(\Delta_k) = \mathrm{B_\rho^{-1}}(\mathrm{b} + \mathrm{b}'(\Delta_k)) \ge 0 \quad (7)$$

In these expressions, B be is an optimal and feasible basis, ρ the index for the corresponding set of basic variables, A represents the columns of the non-basic variables in the Simplex tableau, C_0 is the cost vector of the basic variables, C^T is the cost vector of the non-basic variables, Φ_k is the vector of the parameters that model generation cost uncertainties and Δ_k is the vector of the parameters modeling load uncertainties.

As a final comment, the ultimate objective to attain when solving a multiparametric optimization problem is to find all possible optimal solutions, their corresponding optimal values and critical regions. These regions can be defined as a closed nonempty polyhedron and can be represented mathematically by a set of linear inequalities in Δ or Φ. Mathematically, this set of constraints can be expressed as the equivalent set of non-redundant constraints, which in turn can be identified through a non-redundant test for linear inequalities, like the one proposed by Gal [8].

D. Consideration of Load Uncertainties

The integration of the parameters that model load uncertainties in the optimal and feasible solution of the deterministic problem (1-5) leads to the multiparametric optimization problem (8-12).

$$min\ f = \sum c_k^{ctr}.Pg_k + G.\sum PNS_k \quad (8)$$

subj. $\sum Pg_k + \sum PNS_k = \sum P_{Lk}^{ctr} + \sum \Delta_k$ (9)

$$\mathrm{Pg}_k^{\min} \le \mathrm{Pg}_k \le \mathrm{Pg}_k^{\max} \quad (10)$$

$$PNS_k \le P_{Lk}^{ctr} + \Delta_k \quad (11)$$

$$P_b^{min} \le \sum a_{bk}.(\ Pg_k + PNS_k - (\ P_{Lk}^{ctr} + \Delta_k\ )) \le P_b^{max} \quad (12)$$

Considering the inequalities associated to the feasibility condition (7), the algorithm identifies a set of non-redundant constraints defining new critical regions. If there are no non-redundant constraints, the algorithm stops. Otherwise, it performs a dual pivoting over the initial optimal and feasible solution to identify new critical regions. This process is repeated until no non-redundant constraints exist or until all identified critical regions correspond to the already known ones. When this is over, all the uncertainty space was covered and we identified all critical regions in which a base B of the problem (8-12) remains feasible and optimal.

At this stage the algorithm proceeds by building the membership functions of the output variables (generations, branch flows and PNS). Given that the problem is linear, each variable in each critical region is represented by a linear expression. In this sense, in order to capture the widest possible behaviour of each variable the algorithm solves several minimizing and maximizing linear optimization problems for several different cut levels and for each function that represent the behaviour of each variable subjected to the non-redundant conditions together with the possible ranges of the input uncertainties regarding the i^{th} cut under analysis. For illustration purposes, if we consider a system having two loads affected by uncertainty, this problem can be formulated by (13-16).

$$\min/\max \text{ f } = v(\Delta_1, \Delta_2) \tag{13}$$

$$\text{subj.} \quad k_{1i}.\Delta_1 + k_{2i}.\Delta_2 \le b_i \tag{14}$$

$$\Delta_1^{\min\ i^{th}-cut} \le \Delta_1 \le \Delta_1^{máx\ i^{th}-cut} \tag{15}$$

$$\Delta_2^{\min\ \mathrm{i^{th}-cut}} \le \Delta_2 \le \Delta_2^{\mathrm{máx\ i^{th}-cut}} \tag{16}$$

In this model $v(\Delta_1,\Delta_2)$ is the linear expression that represents the behaviour of the variable v in terms of the load uncertainty parameters Δ_1 and Δ_2 in the critical region under analysis, k_{1i} and k_{2i} are real numbers, and $\Delta_1^{\min\ i^{th}-cut}$, $\Delta_1^{\max\ i^{th}-cut}$, $\Delta_2^{\min\ i^{th}-cut}$ and $\Delta_2^{\max\ i^{th}-cut}$ are the minimum and maximum values of the load uncertainties in the i^{th} - cut under analysis.

Once all critical regions are analyzed, the final membership function of an output variable is obtained applying the fuzzy union operator to the partial membership functions obtained for that variable. This guarantees that the final result displays the widest possible behavior given the specified uncertainties.

E. Consideration of Generation Cost Uncertainties

The integration of the parameters that model generation cost uncertainties in the optimal and feasible solution of the deterministic problem (1-5) leads to the multiparametric optimization problem (17-21).

$$\min \text{ f} = \sum \mathrm{c_k(\Phi).Pg_k + G.\sum PNS_k} \tag{17}$$

$$\text{subj.} \sum Pg_k + \sum PNS_k = \sum P_{Lk}^{ctr} \tag{18}$$

$$\mathrm{Pg_k^{min} \le Pg_k \le Pg_k^{max}} \tag{19}$$

$$PNS_k \le P_{Lk}^{ctr} \tag{20}$$

$$P_b^{min} \le \sum a_{bk}.(\,Pg_k + PNS_k - P_{Lk}^{ctr}\,) \le P_b^{max} \tag{21}$$

Similarly to what was described in Section II.D, considering the inequalities associated to the optimality condition (6) the algorithm tries to identify new optimal and feasible basis in the uncertainty space and the corresponding critical regions defined by the parameters that model generation cost uncertainties. Once again, if there are no more critical regions the algorithm stops. Otherwise, it performs a primal pivoting over the initial identified optimal and feasible basis. This process is repeated until non-redundant constraint exists or until all identified basis correspond to the already known ones.

When this process is completed the algorithm builds the membership functions of the output variables. This process is conducted taking into account that in this case each variable is constant inside each critical region. As a consequence, to build the membership functions the algorithm just has to solve the linear inequality system defined by the parameters that model generation cost uncertainties in each critical region to check if, at least, one point of a given cut level belongs to the region under analysis. For a given variable, once all partial membership functions are obtained, they are aggregated using the fuzzy union operator to obtain the final membership function of the output variable under analysis.

F. Integration of Active Losses

Active losses in branch b are given by (22). As it can be seen, it depends on the voltage magnitude and phase on the extreme branch nodes i and j and also on the conductance g_{ij}. If we consider that voltage magnitudes are 1.0 pu, we can then obtain the simplified expression (23).

$$Loss_{ij} = g_{ij}.(V_i^2 + V_j^2 - 2.V_i.V_j.\cos\theta_{ij}) \tag{22}$$

$$Loss_{ij} = 2.g_{ij}.(1 - \cos\theta_{ij}) \tag{23}$$

Among several other available techniques described in literature, this algorithm adopts the approach described in [6] to consider an estimate of active losses. The corresponding iterative process evolves as follows:

1. Perform the deterministic DC OPF study (1-5);
2. Compute the voltage phases according to the DC model;
3. Compute an estimate of the active power losses in each branch of the system;
4. Add half of the estimated active power losses for each branch to the loads in the corresponding extreme buses;
5. Perform a new deterministic DC OPF study to update the generation strategy;
6. Compute the nodal voltage phases according with the DC model;
7. Finish if the difference between every voltage phases in two successive iterations is smaller than a specified value. Otherwise return to step 3.

In case we are trying to integrate the effect of the active losses on the results of the algorithms detailed in sections II.D and II.E then, for each extreme point in each identified critical region we must run the algorithm just detailed. As we will show in Sections IV and V, in general, this procedure introduces small deviations regarding the initial results.

III. MARGINAL PRICE COMPUTATION

A. Deterministic Evaluation

The short term nodal marginal price in node k is defined as the impact on the cost function of a short term operation problem regarding to a variation of the load in node k. Given this definition in deterministic terms and for the problem (1-5), short term marginal prices can be computed using (24).

$$\rho_k = \gamma + \sigma_k - \sum_{all\ \ branches} \eta_b . \frac{\partial P_b}{\partial P_{Lk}} + \gamma . \frac{\partial Losses}{\partial P_{Lk}} \tag{24}$$

In this expression:

- γ represents the dual variable of the generation/load balance equation (2);
- the second term represents the contribution from constrains (4) that are eventually on their limits. σ_k is the dual variable of the constraint in node *k*;
- the third term represents the contribution from each branch flow constraint that is on its limit. In this expression, η_b represents the dual variable of the corresponding constraint (5) and the derivative of the flow in branch *b*, P_b, regarding the load in bus *k*, P_{Lk}, is the symmetric of the corresponding sensibility coefficient;
- the fourth term represents the impact on the cost function from varying branch losses in the whole network due a variation of the load in bus *k*.

B. Nodal Marginal Prices Membership Functions

The algorithm presented in Section II could also be used to compute the nodal marginal price membership functions when they are modelled load or generation cost uncertainties. In this context, the computation of the nodal marginal price membership functions comprises three distinct stages. In the first place, the algorithm determines the maximum and minimum possible values of all variables in each cut level. Once this initial stage is completed, the algorithm can evolve to include the impact of the active losses for each identified operating point in each cut level or, this impact can be neglected. Finally, the algorithm determines the nodal marginal prices using (24). When all partial membership functions are known for a given node, the final membership function is obtained applying the fuzzy union operator.

Since nodal marginal prices are described by the dual variables of the original problems their membership functions will be described by ordered pairs of price/membership values, in case of load uncertainties modeling, and by linear segments in case of generation cost uncertainty modeling.

IV. CASE STUDY USING A THREE BUS/THREE BRANCH SYSTEM

A. Data

To illustrate the application of the algorithms described in Section II and III we used the small system presented in Figure 2. In this example, we considered that generator 1 has a capacity of 3 MW and that generator 2 as a capacity of 7 MW. The capacity of branches was set at 5 MW. All branches have the same impedance of 0.05+j1.0 pu, (base of 10 MVA, 10 kV). The generation cost of generator 1 is 10 €/MWh and of generator 2 is 20 €/MWh. The central value of the load on bus 2 is 2.0 MW and on bus 3 is 3.5 MW.

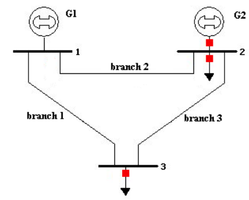

Fig. 2. Three bus/three branch system

B. Considering Load Uncertainties

In order to model load uncertainties, load in nodes 2 and 3 were defined by the trapezoidal fuzzy numbers (25) and (26).

$$P_{L2} = (0.0;1.5;2.5;4.0) \text{ MW} \tag{25}$$

$$P_{L3} = (2.0;3.0;4.0;5.0) \text{ MW} \tag{26}$$

Figure 3 presents the membership functions of the generators at buses 1 and 2 and of the marginal price in bus 3 considering and not considering the effect of active losses.

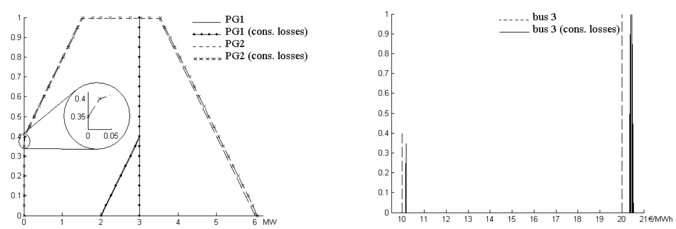

Fig. 3. Membership functions of generator 1 and 2 (at the left) and of the nodal marginal prices at bus 3 (at the right), both considering and not considering the effect of transmission losses.

As we can see from Figure 3 (at left), when the system load is at its minimum (2 MW) only generator 1 is in service since it is the less expensive one. When this generator achieves its maximum capacity of 3 MW, then generator 2 starts to produce. As expected, in this system the effect of transmission losses is neglectable given the system dimension. Without surprise the membership function of nodal marginal price in bus 3 when they are not considered the transmission losses presents a value of 10 €/MWh with a membership value of 0.4 and a value of 20 €/MWh with a membership value of 1.0. In this context it is also important to mention that all buses present the same nodal marginal price membership functions since they are not considered losses and there are no branch congestions or PNS. As expected, this Figure also indicates that a load increase in bus 3 implies an increase of losses and, as a consequence, nodal marginal price in this bus also increases. It is also interesting the fact that the membership degree of the 10.17 €/MWh is now smaller then the one identified for the 10 €/MWh when the losses effect was neglected. This situation results from the fact that when active losses are considered and for uncertainties larger than 0,35, the marginal generator becomes generator 2.

C. *Considering Generation Cost Uncertainties*

In order to model generation cost uncertainties, the generation costs of generators at buses 1 and 2 were defined by the trapezoidal fuzzy numbers (27) and (28).

$$C_{PG1} = (5.0;7.5;12.5;15.0) \text{ €/MWh} \tag{27}$$

$$C_{PG2} = (7.5;15.0;25.0;32.5) \text{ €/MWh} \tag{28}$$

Figure 4 presents the membership functions of generators 1 and 2 and of the nodal marginal price on bus 3.

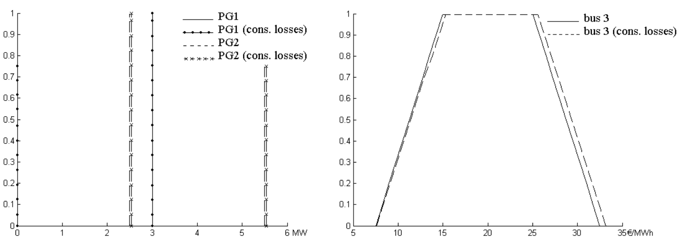

Fig. 4. Membership functions of generators 1 and 2 (at the left) and of the nodal marginal price on bus 3 (at the right), both considering and not considering the effect of active losses

As it can be seen from Figure 4 (at the left), given the specified generation cost uncertainties they were identified two possible generation strategies. In the first one, the two generations are in service and, in the second one generator 2 is at 5.5 MW supplying the entire load, that is 2 MW at bus 2 and 3,5 MW at bus 3. Since generator 2 is always the marginal generator, it defines the system marginal price as it can be seen from the membership function at the right side of Figure 4. This Figure also indicates that the nodal marginal price on bus 3 increases when considering active losses. This in fact shows that a load increase on this node implies an increase of the system active losses.

V. CASE STUDY USING A 24 BUS/38 BRANCH SYSTEM

A. *Data*

The algorithms described in Section II and III were used to build the generation and marginal price membership functions considering a case study based on the IEEE 24 bus/38 branch test system. The original data for this system is given in [9]. Regarding the data in this reference, the load was increased to 4308.05 MW. Table II presents the central values of the loads and Table III the installed system capacity and the central values of the corresponding generation costs.

TABLE II
Load central values

Bus	Load (MW)	Bus	Load (MW)	Bus	Load (MW)
1	220.48	9	385.82	17	0.00
2	270.80	10	216.49	18	226.76
3	3.94	11	40.00	19	265.53
4	32.67	12	10.00	20	103.92
5	105.94	13	162.45	21	100.00
6	187.65	14	262.88	22	100.00
7	218.77	15	650.36	23	0.00
8	398.09	16	225.50	24	120.00

The total installed capacity is 5536 MW according to the data in Table III. Branch data can be obtained from [9] considering that the transformers have a capacity of 400 MW, the capacity of the branches 1 to 6 and 8 to 13 was set at 175 MW and the capacity of the remaining branches was set at 500 MW.

TABLE III
Installed system capacity

Bus/ Gen	Capacity (MW)	Cost (€/MW.h)	Bus/ gen	Capacity (MW)	Cost (€/MW.h)
1/1	40.0	30.0	18/1	310.0	38.0
1/2	40.0	32.0	19/1	800.0	87.0
1/3	152.0	40.0	21/1	700.0	80.0
1/4	152.0	43.0	22/1	100.0	15.0
2/1	40.0	36.0	22/2	100.0	17.0
2/2	40.0	38.0	22/3	100.0	19.0
2/3	152.0	41.0	22/4	100.0	15.0
2/4	152.0	42.0	22/5	100.0	17.0
7/1	150.0	45.0	22/6	100.0	25.0
7/2	200.0	43.0	23/1	200.0	50.0
13/1	250.0	61.0	23/2	50.0	49.0
13/2	394.0	62.0	23/3	310.0	47.0
13/3	394.0	67.0	--	--	--
16/1	310.0	55.0	--	--	--

B. *Considering Load Uncertainties*

In this case we considered trapezoidal fuzzy numbers to model loads. These numbers have at the 0.0 level the uncertainty ranges from +/-10 per cent and at the 1.0 level from +/- 5 per cent of its central value. Figure 5 presents the membership functions of generators 13/3, 19/1 and 21/1 considering and not considering the effect of active losses.

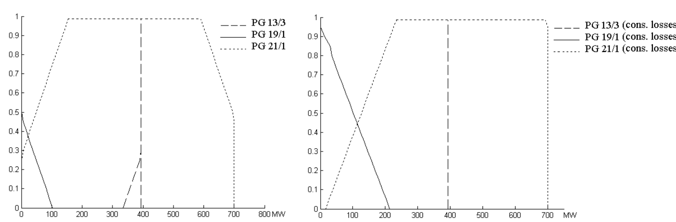

Fig. 5. Membership functions of generators 13/3, 19/1 and 21/1 not considering the transmission losses effect (at the left) and considering this effect (at the right)

As it can be seen from Figure 5 in presence of active losses, generators exhibit greater generation values for the some level of load uncertainty, which in fact corresponds to the losses compensation. An important consequence of this is the fact that when transmission losses are considered generator 13/3 is never the marginal generator. This situation justifies the absence of the 67 €/MWh value in the membership function of the nodal marginal price in bus 10 when they are considered the active losses effect as it can be seen in Figure 6.

When the effect of active losses is considered, the nodal marginal prices increase or decrease depending on the impact of load variations in the active losses. For instance, when generator 21/1 is the marginal one, an increase of the load in node 10 implies an increase of active losses and so the marginal price in node 10 increases.

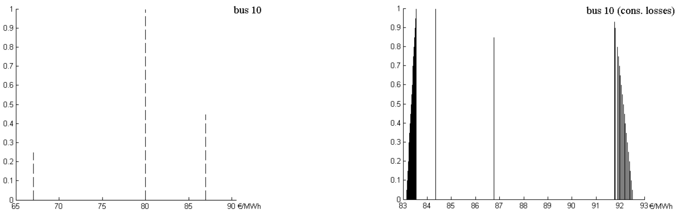

Fig. 6. Membership functions of the nodal marginal price at bus 10 not considering the effect of transmission losses (at the left) and considering this effect (at the right).

C. Considering Generation Cost Uncertainties

In this case we considered that the generation costs of the generators 1/1, 2/1, 2/4, 13/3, 19/1, 21/1 and 23/2 are represented by the trapezoidal fuzzy numbers (29-35). As a consequence, Figures 7 and 8 present the membership functions of generators 13/3, 19/1 and 21/1 and of the nodal marginal price on bus 10 considering the generation cost uncertainties just mentioned.

$$C_{PG1/1} = (26.0;27.5;32.5;34.0) \text{ €/MWh} \tag{29}$$

$$C_{PG2/1} = (33.0;34.5;37.5;39.0) \text{ €/MWh} \tag{30}$$

$$C_{PG2/4} = (37.0;39.5;44.5;47.0) \text{ €/MWh} \tag{31}$$

$$C_{PG13/3} = (58.0;61.0;73.0;76.0) \text{ €/MWh} \tag{32}$$

$$C_{PG19/1} = (74.0;82.0;92.0;100.0) \text{ €/MWh} \tag{33}$$

$$C_{PG21/1} = (71.0;74.0;86.0;89.0) \text{ €/MWh} \tag{34}$$

$$C_{PG23/2} = (77.0;78.5;81.5;83.0) \text{ €/MWh} \tag{35}$$

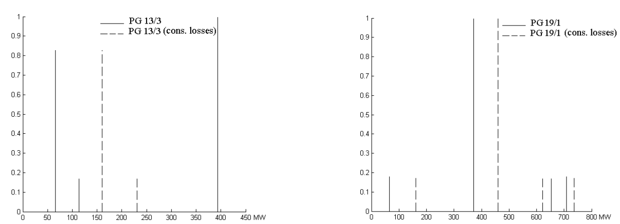

Fig. 7. Membership functions of the generators 13/3 and 19/1 when they are considered the transmission losses effect and when they are not considered.

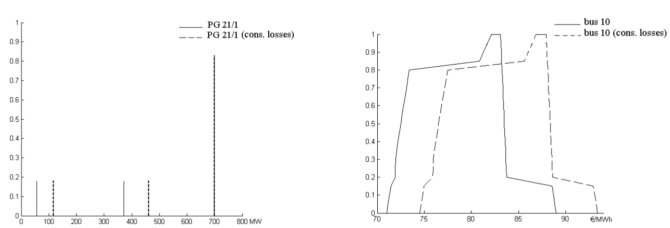

Fig. 8. Membership functions of the generator 21/1 and of the nodal marginal price on bus 10 (considering and not considering the transmission losses effect).

From Figures 7 and 8 we can recognize once again that for some levels of uncertainty generators exhibit larger generation values in order to be able to compensate transmission losses. In Figure 8 it is once again visible that a load increase in bus 10 has an increasing impact on the global system active losses. As a result, the nodal marginal price in this bus when considering transmission losses is larger than the values that were obtained not taking into account this impact.

VI. Conclusion

In this paper we present the NFOPF model as an algorithm to evaluate the impact of load and generation cost uncertainties on the nodal marginal prices computation. It was also described the algorithms developed to integrate an estimate of the active transmission losses on the results. Since this algorithm uses multiparametric programming techniques it allows attaining more accurate representation of the variable membership functions. Given the current volatility and unpredictability of several input parameters, this model could play an important role in analyzing the possible system behavior and its corresponding impact at the agent's making decision process level.

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