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Novel Analytical Approximation Method of Frequency Dependent Thevenin Impedance

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***Abstract*—The Thevenin equivalent representation is widely used in the harmonic distortion studies. For network representation it is necessary to use the driving point impedance in function of the frequency, which can be determined by computer simulation or site measurement.**

The authors are working on the modeling of penetration and elimination of harmonic disturbance originating from the high power railways. In order to improve the accuracy of calculations the double domain simulation method was introduced, which is a combination of frequency and time dependent modeling. The presented novel approximation of the frequency dependent network impedance results in a further improvement regarding the accuracy of the frequency domain simulation.

In this paper the improved accuracy achieved using the proposed analytical approximation method is discussed on the example of a railway substation.

***Index Terms*--Computer simulation, harmonic effect, power quality, traction supply systems, analytical calculation.**

I. Introduction

AFTER the spreading of locomotives supported by DC engines and rectifier units the disturbance originating from railway traction systems has increased. Harmonic filters are used to limit the harmonic currents flowing into the upstream network and to decrease the resonance effect causing current amplification along the 25kV supply line. Reducing the harmonic currents decreases the psophometric current and voltage. [2]

To simulate the harmonic penetration and filtering effects of the power electric traction the combination of frequency and time dependent model should be used. Our novel method, called double domain simulation is improving the accuracy of the frequency domain simulation. To calculate the sophisticated model of the electric locomotive and active harmonic filter as a non-linear load a time dependent model must be used. The traction supply system together with the equivalent supply network impedance could be calculated in the frequency domain. An iteration algorithm is developed converting the variables in every iteration step between the time and frequency domain, this is the so called double domain simulation.

In our paper a new approximation method is discussed for modeling the driving point impedance of the high voltage supply system. This model is the part of the traction system model is developed for studying the Hungarian 25 kV electric traction supply network. In the chapters of the paper the brief summary of the applied model and the double domain simulation is presented, the detailed introduction of the new approximation method will be reported. Finally some calculation results (harmonic currents, and voltages) will be published comparing the new results to the older ones.

II. Modeling the electric traction's supply [3]

The electric railway system is consisting of four main components (Fig. 1.a): [1]

- the locomotives
- the contact line system
- the feeding transformer
- the high voltage supply network

The traction supply system model can be made with these elements as discussed in reference [3]. The locomotive model might be studied in [4]. Because the voltage distortion is caused by the loco itself, an iteration algorithm was developed to convert the variables between the time and frequency domain vice and verse. This is the so called Double Domain Simulation developed by the authors [3]. It's application possibilities are discussed in details in [5].

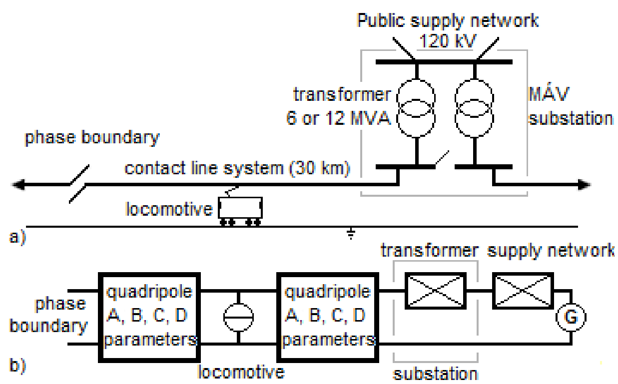

Fig. 1. Simplified circuit for calculation of harmonic effect
a) general traction-current feeding arrangement
b) circuit representation of the traction supply system

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III. Development in the HV system representation

More researches were performed in the topic of calculating the harmonic disturbance caused by high power railways, e.g. [1], [3], [6] and [7]. Taking an overview on the network elements the contact line system, the HV/MV transformer and the locomotives can be calculated with well known parameters. Representing the high voltage supply network the driving point impedance must be measured at the simulated substation (Fig. 2.) The parameters should be inserted to the Thevenin-model of the supply network.

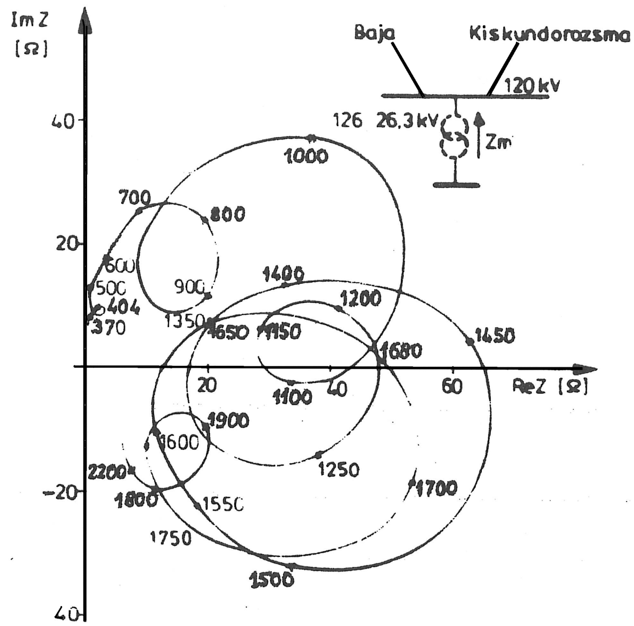

Fig. 2. The high voltage network impedance in function of the frequency (25 kV side)

Because of the difficulty of the measured impedance functions usually the HV network is represented as an infinite network or by an average impedance. In our paper a novel approximation method is discussed, where the impedance curve is separated to its real and imaginary parts as R and X. The independent R and X values are approximated with the combination of polynomial and trigonometrical functions in function of the frequency.

A. Modeling the real part of the impedance (R)

On Fig. 3 the approximation of the R part could be studied. The M function represents the real part of the measured impedance. It is possible to conclude, that the polynomial trend line should be a biquadratic polynomial function, which is the Pol curve. The common equation of this part is:

$$Pol = B_0 + B_1 \cdot x + B_2 \cdot x^2 + B_3 \cdot x^3 + B_4 \cdot x^4 \qquad (1)$$

where x is the variable (now: the frequency) and B_i is the factor.

In this case the maximal difference between the M and Pol functions are 70 %. Reducing the difference, some trigonometrical functions should be used.

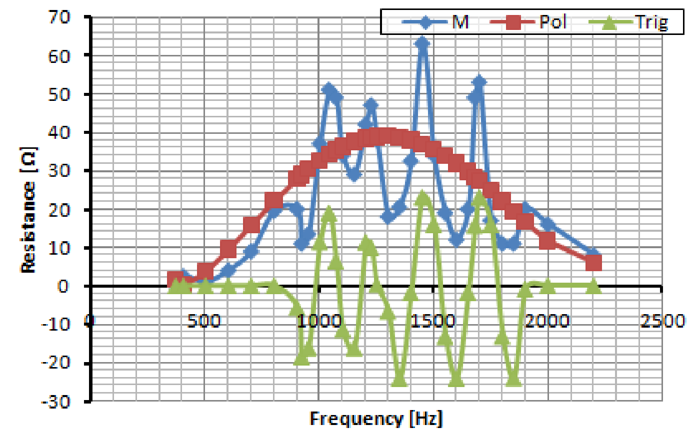

Fig. 3. The polynomial and trigonometrical part of the resistance function
M: measured resistance, Pol: polynomial part, Trig: trigonometrical part

This trigonometrical part could be divided into four parts:

- In two sections of the function (below 800 Hz and over 1900 Hz) compensation is not required.
- Between 900 and 1200 Hz a smaller amplitude sinusoidal compensation is needed.
- Between 1300 and 1800 Hz it is possible to use again a sinusoidal function, but with bigger amplitude and frequency.

Separating the different sections window functions should be used, in this case we use arcus tangent functions. Each frequency period could be conpensated by the following form:

$$Trig_i = C_i \cdot \sin(D_i \cdot x + \varphi_i) \cdot Win \qquad (2)$$

where C_i, D_i and φ_i are the factors, x is the frequency, and Win is the window function:

$$Win = \left[0.5 + \frac{artg(x-W_1)}{\pi}\right] \cdot \left[0.5 - \frac{artg(x-W_1)}{\pi}\right] \qquad (3)$$

where W_1 and W_2 are the lower and upper limits of the window.

After calculating both the polynomial and trigonometrical parts the modeling function could be studied on Fig. 4.

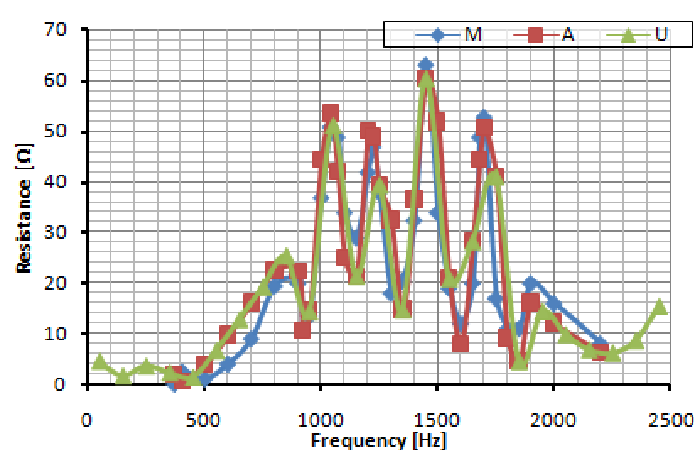

Fig. 4. The approximation of the resistance
M: measured resistance, A: Approximated function, U: Used in Calculations

For the calculations of the double domain simulation the impedance should be known on the odd harmonic orders from 1st to 49th. On Fig 4. the function M shows the representation of the original measured resistance in function of the frequency. Function A is fitted on values we have got measured data. The U curve shows the modeled resistance values which should be used in the further calculations. It means resistance values for each odd harmonic orders from 1st to 49th.

B. Modeling the imaginary part of the impedance(X)

Regarding the ATP-EMTP modeling software it is better to calculate a "virtual inductance" value instead of the X. This value can be easily inserted to the L element of the Thevenin-model. It should be mentioned, that there are some frequencies where the X has negative value. However, in this case the network is capacitive; we represent it with a virtual negative inductance. The resultant negative inductance gives correct X values on the characteristic frequencies, but it is not representing the real frequency function of a capacitive network. The L value is calculated as follows:

$$L = \frac{X}{\omega} = \frac{X}{2\cdot\pi\cdot f} \qquad (4)$$

The polynomial and trigonometrical parts can be studied on Fig. 5. In this case a cubic polynomial function can be used

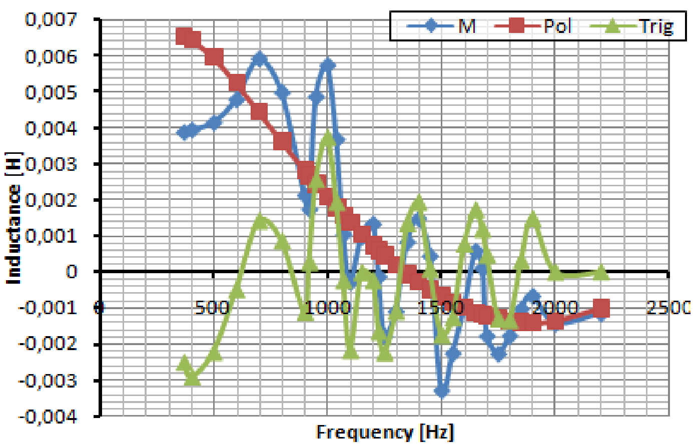

Fig. 5. The polynomial and trigonometrical part of the inductance function
M: measured resistance, Pol: polynomial part, Trig: trigonometrical part

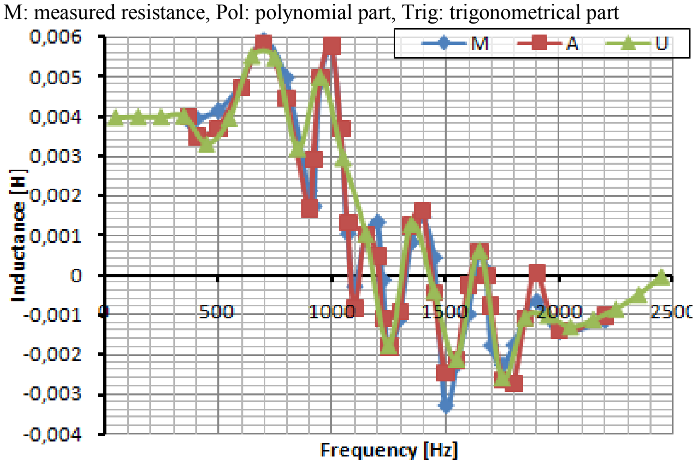

Fig. 6. The approximation of the inductance
M: measured resistance, A: Approximated function, U: Used in Calculations

together with the composition of three trigonometrical functions. The approximation method is near the same than in Chapter III.B, after calculating the Pol and Trig functions (Fig. 5) their sum can be composed which is possible to study on Fig. 6.

IV. Calculation Results

Some characteristic calculations should be taken to determine the accuracy of the novel approximation. In this chapter the comparison of two different calculations are presented. By both cases the double domain simulation is applied. In Chapter IV.A the HV system is represented by frequency independent R and L values, which are constant for every frequency component. In Chapter IV.B the new approximation function is applied. All other parameters are the same in both cases. The calculations are made with V63 locomotives, the traction system length is 30 km, the engine is 10 km far from the substation.

A. Constant R-L representation

In this case the resistance and inductance variables of the HV network impedance were set to an average level. The calculation needs four steps [3], the results can be studied on Fig. 7. and 8.

On the resultant figures a high current distortion could be

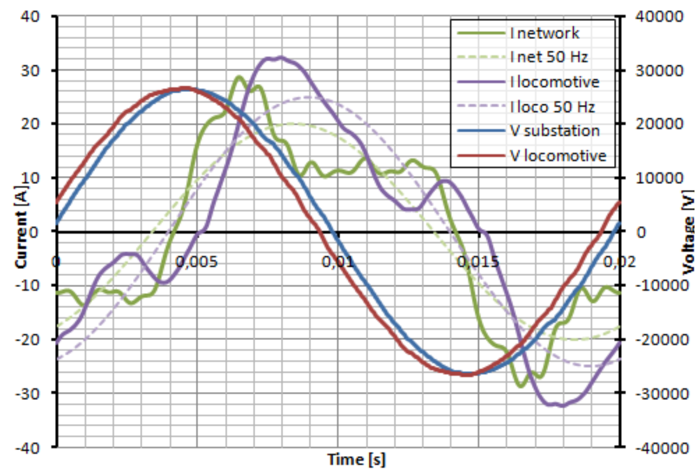

Fig. 7. Time functions with constant R-L representation
green: HV network current, purple: locomotive current, blue: substation voltage, red: locomotive voltage

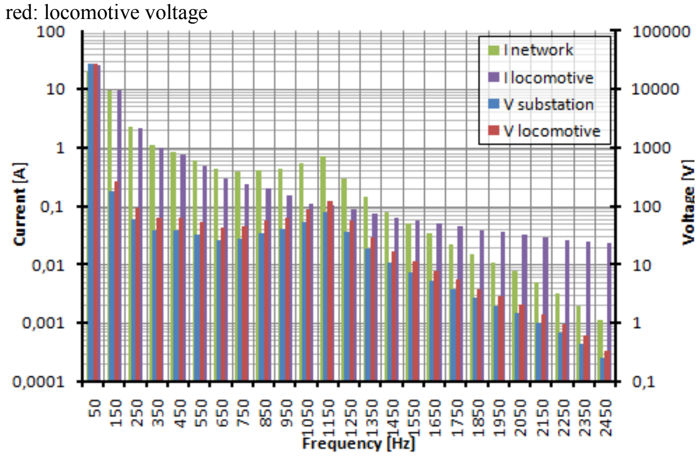

Fig. 8. Spectrums with constant R-L representation
green: HV network current, purple: locomotive current, blue: substation voltage, red: locomotive voltage

noticed. The locomotive consists of DC motors and AC/DC converter (two series controlled half-controlled bridges of thyristors and diodes), this is the origin of the injection of the harmonic currents. The purple columns show the loco current spectrum and the green ones the current by the substation. Comparing these two functions the current amplification can be noticed, which was discussed in details in [3].

B. Novel analytical approximation

Applying our novel approximation as detailed in Chapter III, the calculation needs five steps [3]. The results can be studied on Fig. 9. and 10.

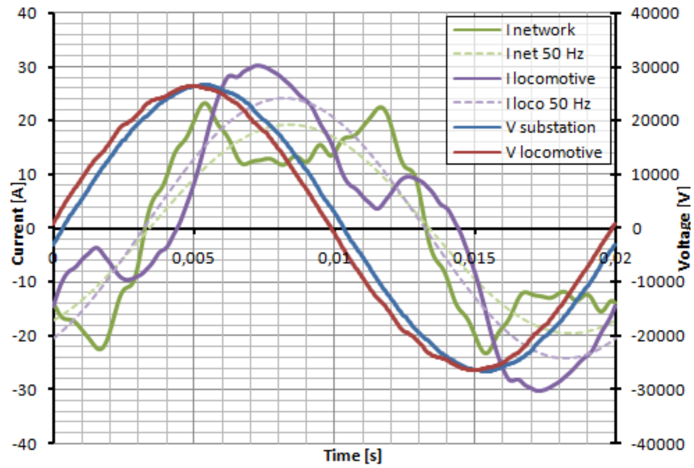

Fig. 9. Time functions with the novel approximation
green: HV network current, purple: locomotive current, blue: substation voltage, red: locomotive voltage

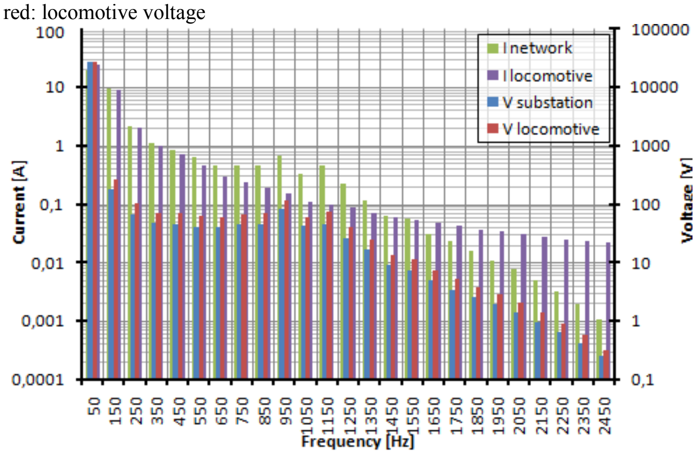

Fig. 10. Spectrums with the novel approximation
green: HV network current, purple: locomotive current, blue: substation voltage, red: locomotive voltage

C. Comparison of the results

Taking into account Fig. 7 and 9, a significant difference can be observed between the network current time functions. However, the locomotive current functions are near the same in both cases (Fig. 11), there is a little difference in the current shapes. It can be concluded, that the change of the network impedance could affect the calculations of the time domain locomotive model, and the resulting network distortion. (It was the main goal why the double domain simulation should be applied replacing the earlier models, where the locomotive is represented by a harmonic current generator and calculated

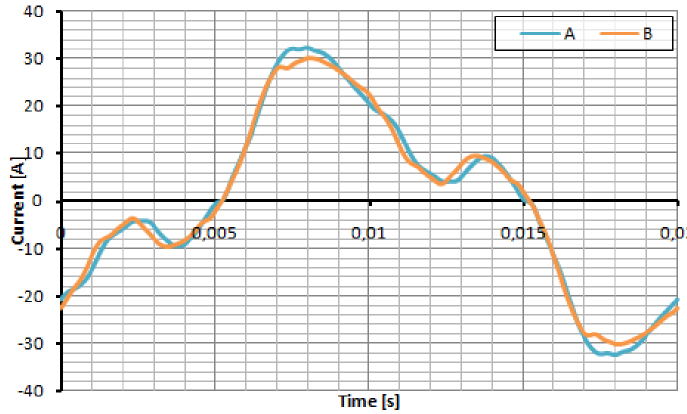

Fig. 11. The locomotive currents (A: in Chapter IV.A, B: in Chapter IV.B)

with a previously measured characteristic current spectrum. [3])

The difference between the network current functions can be studied on Fig. 12. Because of easier comparison, a time shift should be applied for function B. After this adjustment, the functions have their zero points on the fundamental frequency components in the same time points. It was necessary, because the different HV impedances resulted different phase angles, and after this correction the differences can be studied with more accuracy.

There is more difference in the phase angles by other harmonic order current components. That is why there is a near 1 ms difference between the zero points of the functions. The amplitude spectrums can be studied on Fig. 13. It is possible to conclude that the main difference is in the 950…1250 Hz frequency range. These are the same

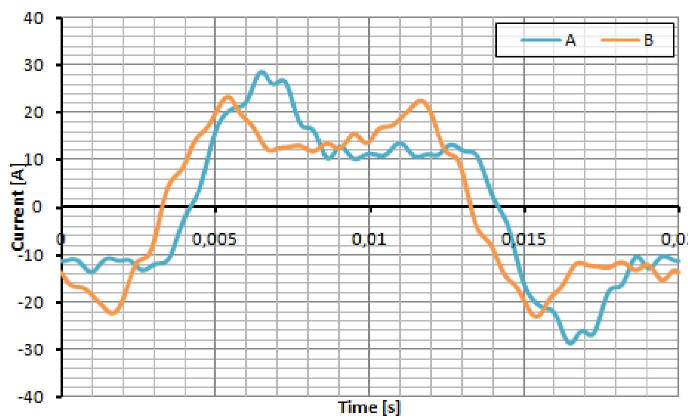

Fig. 12. The network currents (A: in Chapter IV.A, B: in Chapter IV.B)

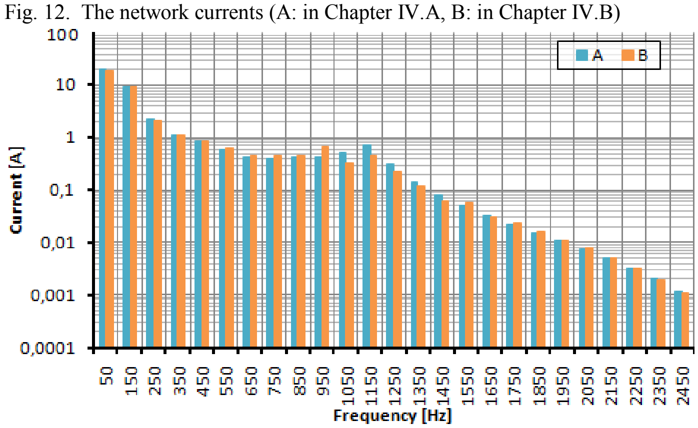

Fig. 13. The network current spectrums

frequencies on which the current amplification has got the highest values [5].

In Table I. some characteristic differences could be studied for the network and locomotive currents. The percentage values are representing the difference between the calculation of Chapter IV.A, where the network R and L values are constant, and the calculation of Chapter IV.B, where the new approximation is applied. It is possible to conclude that however there are big changes on the 950…1250 Hz current components, this will not affect significantly the THD values. There is a minimal change in the RMS values, where the percentages are near the same for the network and locomotive currents.

TABLE I
DIFFERENCES BETWEEN THE CALCULATIONS BY THE NETWORK AND LOCOMOTIVE CURRENTS

	$I_{network}$	$I_{locomotive}$
ΔI_{RMS}	-3,44 %	-2,98 %
ΔTHD_I	0,82 %	0,21 %
ΔI_{50}	-3,76 %	-3,05 %
ΔI_{950}	53,57 %	-1,61 %
ΔI_{1050}	-38,52 %	-0,76 %
ΔI_{1150}	-35,63 %	-5,63 %
ΔI_{1250}	-24,17 %	-0,19 %

V. CONCLUSION

In this paper a polynomial and trigonometrical approximation method was introduced for modeling the frequency dependent HV supply system impedance. The novelty of the application is, that the measured frequency dependent HV supply system impedance is separated to its real and imaginary parts. For both impedance-frequency functions a mathematical approximation was applied. The approximation is based on the composition of polynomial and trigonometrical functions. The new analytical representation shows a significantly better approximation for the HV supply impedance realisation on the characteristic frequencies, than that of a short circuit power representation. With this novel model the network current resonance and the harmonic voltage distortion can be studied with better accuracy.

Further conclusion is that together with the detailed new HV system representation, the double domain simulation method will help to select the most advantageous solution, regarding the composition of reactive power compensation and harmonic filtering.

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