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Optimal Power Flow Solution Using the Penalty/Modified Barrier Method

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***Abstract*—The solution of the optimal power flow problem through the penalty/modified barrier method is described in this paper. This approach features considerable advantages over interior point methods. In this method, the inequality constraints are transformed into equalities by the introduction of slack variables, which are handled by either the modified barrier function or the quadratic penalty function. Then, first order optimality conditions and Newton's method are applied in the solution of the problem. In order to validate the proposed method, electrical power systems of 3, 14, 30, 118, 162 and 300 buses were used as case studies, and the obtained results proved the method's efficiency.**

***Index Terms*—Active power loss minimization, optimal reactive dispatch, nonlinear programming, penalty function, modified barrier function, Lagrangian function, Newton's method.**

I. Introduction

THE optimal power flow (OPF) is a large, non-convex and nonlinear programming problem. It is the ideal tool for an electrical power system analysis in which one can obtain, among others, the spot price for the composition of tariffs; an optimal dispatch of generators, synchronous condensers and static VArs; or even the adjustment of any device for a desired performance of an electrical system. Therefore, its use in energy management systems has become a standard practice. Due to this motivation, it is compelling that the OPF problem keeps being intensely studied and important progresses and formulations have been accomplished since its initial proposal [1], [2].

In the last decade, most of the researches about the OPF problem have been based on variants of interior point methods (IPMs) [3]–[7], and few researches have been based on different approaches [8]–[11].

Although the IPMs are quite established as robust methods for solving the OPF problem, in this paper, we propose the application of the penalty/modified barrier method (PMBM) [12]–[14], which was developed in order to explore and combine the best characteristics of penalty and barrier methods. The presented method is as effective as IPMs in attaining the optimal solution and has some considerable advantages over the IPMs. The ultimate feature of the PMBM approach is based on the merging of the modified barrier function (MBF) [15] and the quadratic penalty function (QPF).

The theory of MBF methods was developed to solve optimization problems with inequality constraints. The MBF methods have several characteristics, such as, being defined and having defined derivatives in the solution, not growing infinitely, and not having the modified barrier parameter driven to zero. Since the MBF may assume different forms, the one that is considered in this work is the modified logarithmic barrier function. The use of such function demands starting points in the relaxed feasible region. On the other hand, the QPF methods allow starting points in the unfeasible region. In addition, both methods also allow solutions on the limits of the inequality constraints.

As a result, the benefits of using such combined methods are chiefly in not demanding a feasible starting point, allowing the optimal trajectory to pass through the feasible and unfeasible regions, and attaining a solution on the frontier of the feasible region.

Regarding this work, we consider that the desired performance of the electrical system is to operate minimizing the active power losses in transmission lines. The choice of such performance is based on the fact that the Brazilian interconnected system is mostly supplied with energy provided by hydroelectric power plants. In addition, its planning and operation follow other directives. Thus, in this work, it will be considered the optimal reactive dispatch (ORD) problem, which is a particular case of OPF where active controls, such as active power injections in the system, are fixed. Due to the increase of the size and complexity of the Brazilian electrical system, the use of robust optimization methods on its operation is compelling. Furthermore, all control variables of the electrical system are assumed to be continuous.

This paper is organized as follows. Firstly, the OPF formulation is presented. Then, the PMBM is discussed. The solution of a generic OPF problem for a 3-bus system by the proposed approach is also demonstrated. The results obtained for other systems are compared with the results provided by the predictor-corrector interior point method (PCIPM). For such comparison, it was considered IEEE electrical systems of 14, 30, 118 and 300 buses and an electrical system of 162 buses.

II. Optimal Power Flow Formulation

The OPF problem can be formulated with the purpose of optimizing a desired performance of an electrical system, subject to some physical and operating constraints. In this work, the desired performance is to determine the point that minimizes the active power losses in transmission lines.

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According to this objective, a generic formulation of such problem is:

$$\begin{array}{ll}\min & P^{\text{ loss}}(\mathbf{V},\boldsymbol{\theta}) \\ \text{s.t.:} & \mathbf{P^G}-\mathbf{P^L}-\boldsymbol{P}(\mathbf{V},\boldsymbol{\theta},\mathbf{t})=\mathbf{0} \\ & \mathbf{Q^G}-\mathbf{Q^L}-\boldsymbol{Q}(\mathbf{V},\boldsymbol{\theta},\mathbf{t})=\mathbf{0} \\ & \mathbf{Q^G_{min}} \leq \boldsymbol{Q^G}(\mathbf{V},\boldsymbol{\theta},\mathbf{t}) \leq \mathbf{Q^G_{max}} \\ & \mathbf{V_{min}} \leq \mathbf{V} \leq \mathbf{V_{max}} \\ & \mathbf{t_{min}} \leq \mathbf{t} \leq \mathbf{t_{max}} \end{array} \tag{1}$$

where:

- $P^{\text{ loss}}(\mathbf{V},\boldsymbol{\theta})$ the is active power loss function;
- NLB is the number of load buses;
- NGB is the number of generation buses;
- $NRCB$ is the number of reactive control buses;
- $\mathbf{P^G} \in \mathbb{R}^{(NLB+NGB-1)}$ and $\mathbf{P^L} \in \mathbb{R}^{(NLB+NGB-1)}$ are the specified active power generation and load vectors at the buses of the system, except for the slack bus;
- $\boldsymbol{P}(\mathbf{V},\boldsymbol{\theta},\mathbf{t}) \in \mathbb{R}^{(NLB+NGB-1)}$ is the calculated active power inflow vector at the buses of the system, except for the slack bus;
- $\mathbf{Q^G} \in \mathbb{R}^{NLB}$ and $\mathbf{Q^L} \in \mathbb{R}^{NLB}$ are the specified reactive power generation and load vectors at the load buses;
- $\boldsymbol{Q}(\mathbf{V},\boldsymbol{\theta},\mathbf{t}) \in \mathbb{R}^{NLB}$ is the calculated reactive power inflow vector at the load buses;
- $\boldsymbol{Q^G}(\mathbf{V},\boldsymbol{\theta},\mathbf{t}) \in \mathbb{R}^{NRCB}$ is the calculated reactive power generation vector at reactive control buses;
- $\mathbf{V}$ is the voltage magnitude vector;
- $\mathbf{t}$ is the transformer tap ratio vector for the in-phase controllable transformers;
- $\mathbf{Q^G_{min}}$, $\mathbf{V_{min}}$ and $\mathbf{t_{min}}$ are the lower limits of the controllable and state variables;
- $\mathbf{Q^G_{max}}$, $\mathbf{V_{max}}$ and $\mathbf{t_{max}}$ are the upper limits of the controllable and state variables.

The tap ratios of in-phase controllable transformers assume discrete values in real power systems. Nevertheless, these variables are considered continuous. Thus, after reaching a solution to the OPF problem, they are adjusted to their closest actual value.

This generic formulation of the OPF problem can be simplified and represented as follows:

$$\begin{array}{ll}\min & f(\mathbf{x}) \\ \text{s.t.:} & \boldsymbol{g}(\mathbf{x})=\mathbf{0} \\ & \boldsymbol{h}(\mathbf{x})\leq\mathbf{0}\end{array} \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the control and state variable vector, representing the vector of the voltage magnitudes ($\mathbf{V}$), the vector of the phase angles ($\boldsymbol{\theta}$), and the vector of the tap ratios for the in-phase controllable transformers ($\mathbf{t}$). The chosen objective function, $f \in \mathbb{R}$, represents the active power losses in the transmission lines. The equality constraints, represented by the vector $\boldsymbol{g} \in \mathbb{R}^m | m < n$, are the load flow equations, and the vector $\boldsymbol{h} \in \mathbb{R}^p$ corresponds to the physical and operating limits of system variables, i.e., limits of voltage magnitudes and tap ratios of in-phase controllable transformers, limits of reactive power injections at reactive control buses and active power injections at generation buses.

III. The Penalty/Modified Barrier Method

All of the following discussion was based on [12]–[14].

When solving (2) by the PMBM, all inequality constraints are transformed into equality constraints by the addition of non-negative slack variables. Therefore:

$$\begin{array}{ll}\min & f(\mathbf{x}) \\ \text{s.t.:} & \boldsymbol{g}(\mathbf{x})=\mathbf{0} \\ & \boldsymbol{h}(\mathbf{x})+\mathbf{s}=\mathbf{0} \\ & \mathbf{s}>\mathbf{0}\end{array} \tag{3}$$

where $\mathbf{s} \in \mathbb{R}^p$ is the vector holding the slack variables.

The non-negativity condition of the slack variables are handled by a continuous and smooth function $\boldsymbol{\phi} \in \mathbb{R}^p$. This function can be either the MBF or the QPF, and it is incorporated into the objective function f through Lagrangian multipliers.

$$\begin{array}{ll}\min & f(\mathbf{x})-\mu.\boldsymbol{\sigma}^T.\boldsymbol{\phi}(\mathbf{s}) \\ \text{s.t.:} & \boldsymbol{g}(\mathbf{x})=\mathbf{0} \\ & \boldsymbol{h}(\mathbf{x})+\mathbf{s}=\mathbf{0}\end{array} \tag{4}$$

where μ is the modified barrier parameter, and $\boldsymbol{\sigma} \in \mathbb{R}^p$ is the Lagrangian multiplier vector associated with the non-negativity condition of the slack variables.

Therefore, the function $\boldsymbol{\phi}$ is defined as:

$$\phi_i(s_i) = \begin{cases} \ln\left(z+\dfrac{s_i}{\mu}\right), \text{ if } s_i \geq -\beta.z.\mu \\ \\ \dfrac{1}{2}.a.s_i^2+b.s_i+c, \text{ if } s_i < -\beta.z.\mu \end{cases} \tag{5}$$

where $i = 1,...,p$, z is the shift parameter used in the relaxation of the feasible region, and β is a parameter associated with the approximation of the point to the boundaries of the feasible region. The coefficients a, b, and c are determined so that the function ϕ_i is continuous and smooth in $s_i = -\beta.z.\mu$:

$$a = -\frac{1}{\left[\mu.z.\left(1-\beta\right)\right]^2} \tag{6}$$

$$b = -\frac{1-2.\beta}{\mu.z.\left(1-\beta\right)^2} \tag{7}$$

$$c = -\frac{\beta.\left(2-3.\beta\right)}{2.\left(1-\beta\right)^2}+\ln\left[z.\left(1-\beta\right)\right] \tag{8}$$

Then, the equality constraints in (4) are incorporated into its objective function through Lagrangian multipliers, resulting in the Lagrangian penalty/modified barrier function (LPMBF):

$$LPMBF = f(\mathbf{x})-\mu.\boldsymbol{\sigma}^T.\boldsymbol{\phi}(\mathbf{s})+\boldsymbol{\lambda}^T.\boldsymbol{g}(\mathbf{x})+\boldsymbol{\pi}^T.\left(\boldsymbol{h}(\mathbf{x})+\mathbf{s}\right) \tag{9}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^m$ and $\boldsymbol{\pi} \in \mathbb{R}^p$ are Lagrangian multiplier vectors.

The first-order necessary conditions are applied to the LPMBF, generating a system of nonlinear equations:

$$\nabla_{\mathbf{d}} LPMBF = 0 \tag{10}$$

where

$$\mathbf{d} = \begin{bmatrix} \mathbf{x} \\ \mathbf{s} \\ \boldsymbol{\lambda} \\ \boldsymbol{\pi} \end{bmatrix} \tag{11}$$

is the vector that holds the unknown primal $(\mathbf{x}, \mathbf{s})$ and dual $(\boldsymbol{\lambda}, \boldsymbol{\pi})$ variables of the problem.

Newton's method is applied in order to solve the system of nonlinear equations (10) to find the search direction vector:

$$\mathbf{H}(\mathbf{d}).\Delta\mathbf{d} = -\nabla_{\mathbf{d}} LPMBF \tag{12}$$

where $\Delta\mathbf{d}$ is the search direction vector, and $\mathbf{H}$ is the Hessian matrix of the LPMBF.

Using the search directions obtained from the solution of (12), the variable vectors $\mathbf{x}$ and $\mathbf{s}$, and the Lagrangian multiplier vectors $\boldsymbol{\lambda}$ and $\boldsymbol{\pi}$ are updated in the *k-th* Newton's iteration as follows:

$$\mathbf{d}^{k+1} = \mathbf{d} + \alpha.\Delta\mathbf{d}^{k} \tag{13}$$

where α is a scalar step size.

The step α is obtained through Armijo's rule, in which the value of α is determined in order that the new point corresponds to a decrease in the LPMBF value.

It is also important to realize that some elements of the Hessian matrix and the gradient of the LPMBF depend on the function $\boldsymbol{\phi}$. Due to this fact, the function $\boldsymbol{\phi}$ must be evaluated throughout Newton's iterative process. This iterative process stops when the following criterion is satisfied:

$$\|\nabla_{\mathbf{d}} LPMBF\| \leq \xi \tag{14}$$

where $\xi > 0$ and a typical value for ξ is $\xi = 1.10^{-3}$.

After determining the solution of (12), the modified barrier parameter in the *K-th* iteration is updated according to:

$$\mu^{K+1} = \frac{\mu^{K}}{\gamma} \tag{15}$$

where $\gamma > 1$ and typical values for γ are $\gamma = 2$ or $\gamma = 10$.

The β parameter is updated according to:

$$\beta^{K+1} = min\left[1 - \frac{1-\beta^{K}}{\gamma}\ ;\ \beta_{max}\right] \tag{16}$$

where $0 < \beta_{max} < 1$. However, according to [14], [16], this parameter can be considered constant throughout the iterative process, with $\beta = 0.9$.

The update of the Lagrangian multipliers associated with the non-negativity condition of the slack variables is, according to [14], as follows:

$$\sigma_i^{K+1} = \begin{cases} \mu^{K}.\sigma_i^{K}.\left(\dfrac{1}{\sigma_i^{K}.z + s_i}\right), \text{ if } s_i \geq \beta^{K}.z.\mu^{K} \\ \\ \mu^{K}.\sigma_i^{k}.\left(a^{K}.s + b^{K}\right), \text{ if } s_i < \beta^{K}.z.\mu^{K} \end{cases} \tag{17}$$

where $i = 1, ..., p$.

Regarding the initialization of the aforementioned Lagrangian multipliers, according to [14], it is as follows:

$$\sigma_i^{0} = \frac{\mu^{0}}{s_i} \tag{18}$$

where the index 0 refers to the initial step of the algorithm of the method. It is important to start the iterative process with a good estimate of these Lagrangian multipliers since, according to [15], the iterative process may converge to a solution without having the modified barrier parameter driven to zero.

The iterative process of the PMBM stops when the Karush-Kuhn-Tucker (KKT) conditions are satisfied.

Algorithm 1 Penalty/Modified Barrier Method

```
Set K = 0, and ξ and γ with their typical values.
Initialize d^K, σ^K and μ^K.
Make the LPMBF.
Make ∇_d LPMBF.
Make H(d).
Set STOP = 0.
while STOP = 0 do
    Set k = 0.
    Evaluate ∇_d LPMBF for d^k = d^K, σ^K and μ^K.
    while ||∇_d LPMBF|| > ξ, do
        Evaluate H for d^k, σ^K and μ^K.
        Solve H(d^k).Δd^k = −∇_{d^k} LPMBF.
        Compute the step length α.
        Update d according to d^{k+1} = d + α.Δd^k.
        Evaluate ∇_d LPMBF for d^{k+1}, σ^K and μ^K.
        Set k = k + 1.
    end while
    Set d^K = d^k.
    Evaluate the KKT optimality conditions.
    if the KKT optimality conditions are satisfied, then
        The iterative process stops.
        The optimal solution is given by d^K, σ^K and μ^K.
        Set STOP = 1.
    else
        Update σ according to (17).
        Update μ according to μ^{K+1} = μ^K / γ.
        Set K = K + 1.
        The iterative process continues.
        Set STOP = 0.
    end if
end while
```

IV. Test Results

In order to verify the efficiency of the proposed method, tests with different electrical systems have been done.

The algorithm of the PMBM was implemented in C++ and Fortran computational languages, using double-precision variables. Most of the work in this algorithm is in the solution of the linear system (12). Furthermore, the usage of classic methods for the factorization of the Hessian matrix is not practicable. In addition, some of the OPF Hessian matrix features must be taken into account in the resolution of (12). For instance, this Hessian matrix is sparse and symmetric, and its structure remains constant throughout the resolution of the problem.

Due to such characteristics, the MA57 subroutine was used to factorize the Hessian matrix and solve (12). The MA57 subroutine is a Fortran code for the direct solution of large sparse linear equation systems, and it is in HSL 2002, formerly known as Harwell Subroutine Library[1].

The studied cases were the minimization of active power losses in the transmission system. The performance of the approach was evaluated in six electrical systems. Some of them were standard IEEE test systems. The tests with the standard IEEE electrical systems were done with the initial conditions from the data available on www.ee.washington.edu/research/pstca.

In the following tests it will be considered the ORD problem.

The main characteristics of the studied systems are summarized in Table I, in which the *Reactive Control Buses* column represents the number reactive control buses, the *Controllable Transformers* column represents the number of controllable in-phase transformers in the system, and the *Control Variables* and *State Variables* columns are respectively the number of control and state variables.

TABLE I
MAIN CHARACTERISTICS OF THE STUDIED SYSTEMS

Number of Buses	Number of Lines	Reactive Control Buses	Controllable Transformers	Control Variables	State Variables
3	2	1	-	2	3
14	17	4	3	30	22
30	41	5	4	63	53
118	186	53	9	244	181
162	280	11	43	366	311
300	409	68	50	649	530

A. Illustrative Example

The PMBM approach for solving the OPF problem will be illustrated for the power system given in Figure 1.

The ORD problem for this system can be formulated as:

[1]The HSL 2002 was developed at the Rutherford Appleton Laboratory, in the Computational Science and Engineering Department of the Science & Technology Facilities Council, in the United Kingdom.

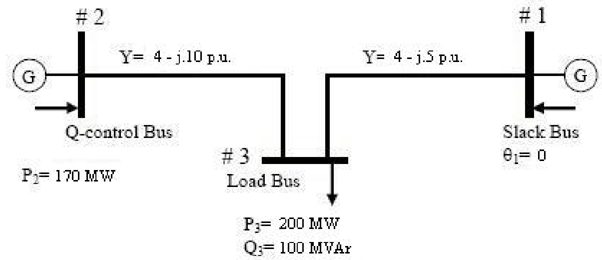

Fig. 1. 3-bus system

$$\begin{array}{ll} \min & f(\mathbf{V},\boldsymbol{\theta}) \\ \text{s.t.:} & \Delta P_2(\mathbf{V},\boldsymbol{\theta}) = 0 \\ & \Delta P_3(\mathbf{V},\boldsymbol{\theta}) = 0 \\ & \Delta Q_3(\mathbf{V},\boldsymbol{\theta}) = 0 \\ & Q_2^{min} \leq Q_2(\mathbf{V},\boldsymbol{\theta}) \leq Q_2^{max} \\ & V_1^{min} \leq V_1 \leq V_1^{max} \\ & V_2^{min} \leq V_2 \leq V_2^{max} \\ & V_3^{min} \leq V_3 \leq V_3^{max} \end{array} \tag{19}$$

where:

$$\begin{aligned} f(\mathbf{V},\boldsymbol{\theta}) \quad = \quad & g_{13}.(V_1^2 + V_3^2 + 2.V_1.V_3.\cos\theta_{13}) + \\ & g_{23}.(V_2^2 + V_3^2 + 2.V_2.V_3.\cos\theta_{23}) \end{aligned}$$

$$\Delta P_2(\mathbf{V},\boldsymbol{\theta}) = P_2^G - P_2^L - V_2.\sum_{i=1}^{3} V_i.\,(g_{2i}.\cos\theta_{2i} + b_{2i}.\sin\theta_{2i})$$

$$\Delta P_3(\mathbf{V},\boldsymbol{\theta}) = P_3^G - P_2^L - V_3.\sum_{i=1}^{3} V_i.\,(g_{3i}.\cos\theta_{3i} + b_{3i}.\sin\theta_{3i})$$

$$\Delta Q_3(\mathbf{V},\boldsymbol{\theta}) = Q_3^G - Q_3^L - V_3.\sum_{i=1}^{3} V_i.\,(g_{3i}.\sin\theta_{3i} - b_{3i}.\cos\theta_{3i})$$

$$Q_2(\mathbf{V},\boldsymbol{\theta}) = V_2.\sum_{i=1}^{3} V_i.\,(g_{2i}.\sin\theta_{2i} - b_{2i}.\cos\theta_{2i})$$

and:
g_{km} is the conductance of the line between buses k and m
b_{km} is the susceptance of the line between buses k and m
θ_{km} is the voltage angle difference between buses k and m

According to the given formulation above, g_{22} and g_{33} are, respectively, the sum of the conductances of the lines connected to buses 2 and 3.

In (19), positive slack variables are introduced to transform the inequality constraints into equality ones.

In Table II, it is shown the starting point for the iterative process. In Table III, the limits of the voltage magnitudes and reactive power generation are presented. Then, the optimization process for the 3-bus system from Figure 1 is displayed in Table IV.

TABLE II
STARTING POINT

Bus (k)	V_k (p.u.)	θ_k (rad)	P_k^G (p.u.)	P_k^L (p.u.)	Q_k^G (p.u.)	Q_k^L (p.u.)
1	1.000	0.0	-	-	-	-
2	1.000	0.0	1.700	0.000	-	-
3	1.000	0.0	0.000	2.000	0.000	1.000

TABLE III
LIMITS OF VOLTAGE MAGNITUDES AND REACTIVE POWER INJECTION

Bus (k)	V_k^{min} (p.u.)	V_k^{max} (p.u.)	Q_k^{min} (p.u.)	Q_k^{max} (p.u.)
1	0.950	1.100	-	-
2	0.950	1.200	-99.99	99.99
3	0.990	1.010	-	-

TABLE IV
OPTIMIZATION PROCESS OF THE 3-BUS SYSTEM

Iteration	P^{loss} (MW)	ΔP (MW)	ΔQ (MVAr)
0	4.1406	117.1151	159.0702
1	12.8973	0.060	0.0159
2	12.7593	0.001	0.0001
3	12.6765	0.001	0.0001

B. Comparison of Methods

The viability of the proposed approach for some electrical systems can be verified through the comparison between the results given by the PMBM and the PCIPM. Therefore, comparative studies of such optimization processes have been performed.

In Table V the summary of the results of a comparative study between the PCIPM and PMBM for the test systems is shown. In this table, the *It.* column means the number of iterations, the *CPU Time* column is the CPU processing time in seconds, and the P^{loss} column is the active power losses for the electrical system.

TABLE V
SUMMARY OF THE OPTIMIZATION PROCESS OF THE STUDIED SYSTEMS

	PCIPM		PMBM		
Systems	It.	CPU Time (s)	It.	CPU Time (s)	P^{loss} (MW)
14-bus	3	0.02	3	0.02	12.68
30-bus	3	0.06	3	0.05	16.65
118-bus	7	0.50	6	0.36	110.00
162-bus	4	1.41	4	0.92	152.07
300-bus	10	2.34	9	1.90	398.00

Analyzing Table V for these five systems and using the PCIPM solutions as a reference, it can be concluded that the PMBM has shown a good performance considering the number of iterations and processing time.

Although the number of iterations and processing time are smaller for the PMBM, this method's effectiveness is in having considerable enhancements over IPMs, such as not demanding a feasible starting point, allowing the optimal trajectory to pass through the feasible and unfeasible regions, and attaining a solution on the frontier of the feasible region. All these characteristics make the PMBM suitable for its application in solving OPF problems.

V. CONCLUSION

This paper has presented a new and efficient algorithm to minimize the active power losses in transmission lines via the Lagrangian penalty/modified barrier method. The main features of the PMBM are: not requiring any feasible starting point; the conditioning of the involved Hessian matrix is greatly enhanced since it is defined in the solution and the modified barrier parameter does not need to be driven to zero to attain the solution; allowing the optimal trajectory to pass through the feasible and unfeasible regions, which is quite advantageous for non-convex problems; having a finite convergence property since the solution can be on the frontier of the feasible region. Therefore, these characteristics make the PMBM appropriate for being applied to the solution of the optimal power flow.

With the intention of demonstrating the validity and effectiveness of the PMBM, tests in IEEE electrical power systems, ranging from 3 to 300 buses, were performed. The results showed the good performance and the robustness of the proposed approach.

As future work, we will be studying a realistic 2256-bus power system corresponding to the Brazilian South-Southeastern interconnected system. In order to ensure the quality of supply of energy, safety and economy in several operating conditions, optimal control strategies are, therefore, necessary. Since the Brazilian interconnected system is mostly supplied with energy provided by hydroelectric power plants and its planning and operation follow other directives, it will be considered the optimal reactive dispatch problem.

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BIOGRAPHIES

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