Ill-conditioned Optimal Power Flow Solutions and Performance of Non-Linear Programming Solvers

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***Abstract*-- In this paper, the performance of the most popular non-linear programming solvers (e.g. IPOPT, KNITRO, LOQO, MINOS and SNOPT) is evaluated for applications in power systems and treatment of ill-conditioned systems related to Optimal Power Flow (OPF) solutions. Regarding the applications, the maximum loading point problem modeled as an optimization problem is used as an OPF example. Also, it is used to describe the ill-conditioned OPF solutions, the sensitivity matrices obtained in the OPF process, and conditions related to linear independence (LI) loss singularity analysis. Simulations using a simple two-bus and IEEE test systems are carried out to evaluate the performance of solvers for solving LI loss singularity cases, which were obtained upon the original data systems. Some of the Lagrange multipliers corresponding to the constraints tend to infinity when the LI condition is violated, which implies that the OPF solution is ill-conditioned. In this case, the number of iterations increases (convergence problems) significantly for all solvers, and some solvers presented oscillatory process.**

***Index Terms*-- Maximum loading point, voltage stability, load flow analysis, step size optimization.**

I. Introduction

THE application of optimization techniques to power system planning and operation problems has been an area of active research in the recent past. Optimal Power Flow (OPF) is a generic term that describes a broad class of problems in which we seek to optimize a specific objective function while satisfying constraints dictated by operational and physical particulars of the electric network. The general problem of OPF subject to equality and inequality constraints was formulated in 1962 [1]. Because very fast and accurate optimization methods have evolved, it is now possible to solve the OPF efficiently for large practical systems.

A wide variety of optimization techniques has been applied to solving OPF problems. The techniques can be classified as [2]: nonlinear programming (NLP), quadratic programming, Newton-based solution of optimality conditions, linear programming (LP), hybrid versions, and interior-point (IP) methods. Now more than twenty years after Karmarkar's publication [3], IP methods are a well understood area both in theory and practice. The current implementations are sophisticated optimization tools capable to solve very large linear programs. Moreover, the IP methods have proved to be significantly more efficient than the best available simplex implementations for many LP problems [4]. Many applications of IP methods in NLP were implemented with relative success.

Ill-conditioned systems were studied under several power systems analysis scenarios as: load flow [5], state estimation [6], OPF [7], among others. For ill-conditioned systems, a small change in some system parameter produces large changes in the unknowns, thus different problems can occur during the solution process as: non-convergence (oscillatory process) and divergence, mainly because the estimated point is close to singularity condition in sensitivity matrices. In order to explain ill-conditioned OPF solutions, it is necessary to perform sensitivity analysis using nonlinear parametric programming, which can be achieved from the first-order Karush-Kuhn-Tucker (KKT) optimality conditions for a point respect to changes in the model parameters. For explaining and solving ill-conditioned OPF problems few publications have been realized until now. In [7], parametric optimization techniques were used. It allowed the development of methods for the solution of the OPF problem and the differentiation of various critical conditions where most algorithms fail to find the solution.

Analyzing the structure of the optimal solution, the ill-conditioning occurs when (*i*) the solution falls within the infeasible region, and (*ii*) it becomes a singular point. There are different types of singularity in the optimal solution path, as the optimality loss and linear independence loss. The authors are interested in the singularity due to linear independence loss, since optimality loss cases are difficult to obtain for realistic power systems applications. The number of iterations obtained when different solvers are applied to small and large scale systems depends whether the solution is close to singularity points, thus different examples were obtained to study the characteristics of this singularity.

The goal of this work is to describe the ill-conditioned OPF solutions (as singular points), the sensitivity matrices obtained in the OPF process, and conditions related to singular points (as feasibility loss). The maximum loading point problem formulated as an optimization problem is used as an OPF example. Simulations using a simple two-bus and IEEE test systems, are carried out using the most popular non-linear programming solvers to evaluate their performances. Solvers as IPOPT, KNITRO, LOQO, MINOS and SNOPT are used in this paper since some of them are of public domain in an executable or even in a source code form.

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978-1-4244-2235-7/09/$25.00 ©2009 IEEE

II. Singularities in the Optimal Solution Path

A. NLPP standard form and KKT stationary conditions

Nonlinear parametric programming (NLPP) tracks the behavior of the parameterized solution of the corresponding nonlinear program. The NLPP standard form is as follows.

$$\min_{x} f(x, \varepsilon) \tag{P1}$$

s.t.

$$g_k(x, \varepsilon) = 0,\ k\in K,\ K=\{1,\ldots,m\},$$

$$h_i(x, \varepsilon) \le 0,\ i\in I,\ I=\{1,\ldots,l\},$$

where $x\in\Re^n$ is the decision variable vector, $\varepsilon\in\Re^p$ is the parameter vector, g and h are function vectors corresponding to the equality and inequality constraints, respectively. The Lagrangian of (P1) is given as:

$$L(x,\varepsilon,\lambda,\pi) = f(x,\varepsilon) + \sum_{k\in K}\lambda_k g_k(x,\varepsilon) + \sum_{i\in I}\pi_i h_i(x,\varepsilon)\,. \tag{1}$$

A point x^* that is a local minimum for (P1) satisfies the first-order KKT stationary conditions:

$$\nabla_x f(x^*,\varepsilon) + \sum_{k\in K}\lambda_k \nabla_x g_k(x^*,\varepsilon) + \sum_{i\in I_0}\pi_i \nabla_x h_i(x^*,\varepsilon) \tag{2}$$

$$\pi_i h_i(x^*,\varepsilon) = 0\,,\quad i\in I, \tag{3}$$

$$\pi_i > 0,\quad i\in I_0, \tag{4}$$

$$g_k(x^*,\varepsilon) = 0\,,\quad k\in K, \tag{5}$$

$$h_i(x^*,\varepsilon) \le 0\,,\quad i\in I. \tag{6}$$

I_0 is the index set of the active inequalities at point x^* defined as: $I_0 = \{\ i\in I / h_i\,(x^*,\varepsilon) = 0\}$. In addition to the KKT conditions, the gradients of the active constraints at point x^* are assumed to form a set of linearly independent vectors which is an assumption known as the linear independence constraint qualification (LICQ) which guarantees the uniqueness of the Lagrange multipliers at point x^*.

The stationary conditions (2) and (3), combined with the feasibility relation (5), form a set of parameterized nonlinear equations. Since the Lagrange multipliers that correspond to the inactive inequalities π_i ($i\notin I_0$) are equal to zero, then the equation set takes the following form:

$$F(x,\lambda,\pi,\varepsilon) = \begin{bmatrix} \nabla_x^T L(x,\lambda,\pi,\varepsilon) \\ g_k(x,\varepsilon) \\ h_i(x,\varepsilon) \end{bmatrix} = 0\,,\ k\in K,\ i\in I_0. \tag{7}$$

The number of equations is equal to $n + m + r$, where $r\le l$ is the number of the active inequality constraints, including $n + m + r + p$ unknowns. Thus the system has p degrees of freedom, equal to the dimension of the parameter space.

B. Optimal solution path

The set of the KKT points for (P1) for different values of the parameter vector ε is obtained by solving (7) with the addition of the strict complementary condition (4), that requires the Lagrange multipliers which correspond to the active inequality constraints π_i ($i\in I_0$) to be positive. That sequence of KKT points is named "optimal solution path" which involves local minima, local maxima, saddle points or boundary points of the feasible region of (P1).

The continuous deformation of the optimal solution path could be studied through perturbations of one free parameter, thus, the definition of *regular* and *singular* points was necessary for understanding its behavior.

C. Regular and singular points of F

A point $z_0 = (x_0,\ \lambda_0,\ \pi_0)$ which satisfies (7), for some parameter values ε_0, and has a non-singular Jacobian of F respect to z ($\nabla_z F$) is called a *regular* point of F. If $\nabla_z F$ is singular, then the point is called a *singular* point of F.

The entire analysis of the behavior of the optimal solution path as the parameter vector ε varies is based upon the following theorem [8]:

Theorem: Let (z_0, ε_0) be a solution of (7) and assume that f, ~~h, g are~~ twice continuously differentiable in a neighborhood of (x_0, ε_0). Then a necessary and sufficient condition that $\nabla_z F$ is nonsingular is that each of the following three conditions hold:

C1. Strict complementary condition; $\pi_i > 0$, for $i\in I_0$.

C2. $\{\nabla_x g_k\ (x,\varepsilon),\ k\in K \cup \nabla_x h_i\ (x,\varepsilon),\ i\in I_0\}$ is a set of linearly independent vectors; (LICQ).

C3. The reduced Hessian of the Lagrangian $Z^T\nabla^2_x LZ$ is nonsingular at (z_0,ε_0), where Z is a matrix whose columns form a basis for the null space of the gradients of the active constraints.

D. Singularities in the optimal solution path

For simplicity, the Jacobian $\nabla_z F$ and components have the following form:

$$W = \nabla_z F = \begin{bmatrix} H & J^T \\ J & 0 \end{bmatrix}, \tag{8}$$

where $H = \nabla_x^2 L$ and $J = \begin{bmatrix} \nabla_x g_{k\in K} \\ \nabla_x h_{i\in I_0} \end{bmatrix}$.

For the detection and study of the behavior of the optimal solution path around singular points, the analysis is realized for matrix (8). This contains the full Hessian of the Lagrangian function which is evaluated with procedures that require great computational effort.

The type of singularity in the optimal solution path depends upon the violated conditions C1-C3. For example, the linear independence (LI) loss and optimality loss singularities are due to that C2 and C3 are violated, respectively. The authors are interested in the singularity of linear independence loss since optimality loss cases are difficult to obtain for realistic power systems applications.

Definition of LI loss singularity: Let (z_0, ε_0) be a solution of ~~(7) and the rank of J at that poin~~t is equal to $m + r - 1$ while around this point the rank of W is equal to $n + m + r$, which implies that W becomes singular. It can then be shown that the optimal solution path of (P1) exhibits a quadratic turning point [9].

III. Maximum Loading Point Problem and OPF Solution Paths

This section deals with the application of optimization techniques to power system planning and operation problems, involving the well known Optimal Power Flow (OPF) problem. The maximum loading point (MLP) problem is chosen as an OPF example and the main characteristics of LI loss singularity are described using a simple two-bus test system. In Sec. V, IEEE test systems are used to verify similar behavior related to LI loss singularity.

A. Maximum loading point

The operating point is obtained when the load flow equations are solved, thus $\Delta P = P_{sch} - P_{cal} = 0$ and $\Delta Q = Q_{sch} - Q_{cal} = 0$, where $\Delta P \in \Re^{nPQ+nPV}$ and $\Delta Q \in \Re^{nPQ}$ are the mismatches of real and reactive powers, respectively; *nPQ* and *nPV* are the number of *PQ* and *PV* buses, respectively, the subscript *sch* and *cal* belong to scheduled and calculated terms, respectively. This work considered a constant direction of generation and load increase, which is defined as proportional to the base case, so $P_{sch} = \rho P_{sch\text{-}bc}$ and $Q_{sch} = \rho Q_{sch\text{-}bc}$ where $\rho \in \Re$ is the loading factor, *bc* is base case ($\rho_{bc} = 1$). Also, $P_{sch\text{-}bc} = P_{g\text{-}bc} - P_{l\text{-}bc}$ and $Q_{sch\text{-}bc} = Q_{g\text{-}bc} - Q_{l\text{-}bc}$ where *l* and *g* are the terms associated to load power and generation, respectively. This load increase direction is usually adopted by utilities and regulatory agencies for the definition of secure loading margins [10,11].

The loading factor reaches its value maximum $\rho = \rho_{cr}$ (*cr* stands for critical point) on voltage stability boundary Σ; this point is named maximum loading point (MLP). Boundary Σ divides the space in two regions: (*i*) region where there are two solutions for system, or feasible region; and (*ii*) region where there are not solutions, or unfeasible region.

B. MLP problem in OPF form

The basic computation of the MLP can be defined as an OPF problem according to

$$\min\ f = -\rho \tag{9}$$

s.t.

$$g\,(x,\varepsilon) = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = 0\,, \tag{10}$$

$$h\,(x,\varepsilon) = \begin{bmatrix} Q(x,\varepsilon) - Q_{g,\max} \\ Q_{g,\min} - Q(x,\varepsilon) \end{bmatrix} \le 0\,, \tag{11}$$

where $x=(\theta,V,\rho)$ is the decision variable vector; $\theta \in \Re^{nPQ+nPV}$ and $V \in \Re^{nPQ}$ are bus voltage angles and magnitudes, respectively; $\rho \in \Re$ is the loading factor; *nPQ* and *nPV* are the number of *PQ* and *PV* buses, respectively. In (9), the maximization the loading factor is sought. Eq. (10) represents the load flow (LF) equations with a constant generation and load increase direction proportional to loading factor ρ and scheduled powers (active and reactive) in base case. Eq. (11) represents the reactive generation limits; $h \in \Re^{2nPV+2}$ considering the *PV* and *slack* buses; $Q_{g,\max}$ and $Q_{g,\min}$ are upper and lower reactive power generation limits, respectively.

An analysis of the behavior of the optimal solution path is performed for (8) using (9)-(11). Without loss of generality, it is assumed that ε is one-dimensional. The reactive compensation variable vector B_c at some *PQ* buses is chosen as parameter ε.

C. Two-bus test system and OPF solution path for MLP problem

The two-bus test system shown in Fig. 1 has a lossless transmission line 1-2 with a reactance x= -0.25 p.u. (capacitive). Bus 1 (*slack*) has E = 1 p.u.; the lower and upper reactive generation limits of bus 1 are fixed to $Q_{g\text{-}min}$= -100 and $Q_{g\text{-}max}$ = +100 MVAR, respectively. Bus 2 (load) has $P_{l\text{-}bc}$= 50 MW and $Q_{l\text{-}bc}$ = 30 MVAR.

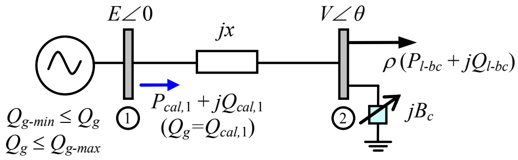

Fig. 1. Two-bus test system with variable reactive compensation.

Subscripts *g* and *l* are associated to generation and load powers, respectively, and *bc* is base case (ρ_{bc} = 1). The reactive compensation at bus 2 is variable, being capacitive when B_c is positive and inductive when B_c is negative.

The LF equations (10) originate a set of operating points for each different value of B_c, and is defined as

$$\begin{aligned} \Delta P_2(\theta,V,\rho) \quad &= P_{sch,2}(\rho) - P_{cal,2}(\theta,V) = 0\,, \\ \Delta Q_2(\theta,V,\rho,B_c) &= Q_{sch,2}(\rho) - Q_{cal,2}(\theta,V,B_c) = 0\,, \end{aligned} \tag{12}$$

where $b = -1/x$, $P_{cal,2}(V,\theta) = -VEbsen\theta$, $P_{sch,2} = -\rho P_{l-bc}$,

$Q_{cal,2}(V,\theta,B_c) = -V^2(b+B_c) + VEb\cos\theta$, $Q_{sch,2} = -\rho Q_{l-bc}$,

also to NLPP problem is defined $x = [\theta\ V\ \rho]^T$ and $\varepsilon = B_c$.

Fig. 2 shows the OPF solution path respect to ρ for MLP problem.

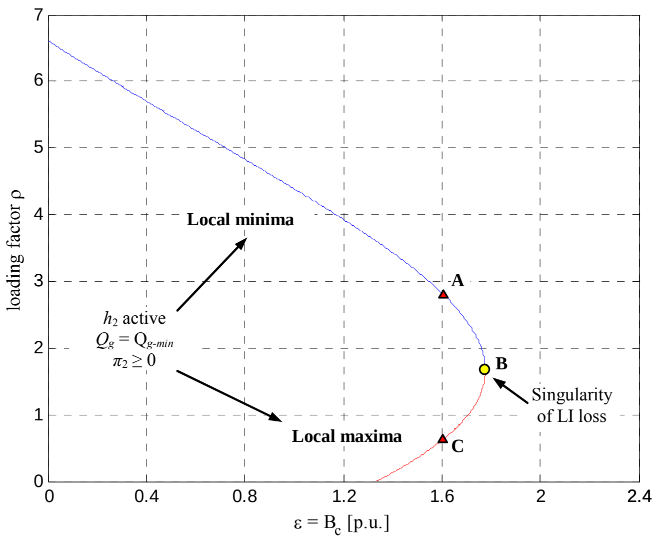

Fig. 2. OPF solution path for MLP problem.

1) *OPF solutions*

It is important to clarify that the number of decision variables of the two-bus system is n = 4 (θ_2, V_2, ρ and Q_{g-1}) and the number of equality constraints is m = 2 (ΔP_2 and ΔQ_2). According to Fig. 2, the OPF solution is obtained when the inequality constraint associated to $Q_{g-1,min}$ is active, thus the number of active inequality constraints is r = 1.

In Fig. 2, the local minima segment represents the OPF solutions and local maxima segment represents the impractical OPF solutions (due to that the problem objective is to minimize ρ). For all points on the two segments, except point B, the rank of J is equal to $m + r$ = 3 and W is nonsingular, thus the system is well-conditioned.

For example, points A and C are calculated for the same ε = 1.6 but only point A minimizes ρ. Also, both are defined when only h_2 (inequality constraint related to $Q_{g\text{-}min}$) is active, thus the Lagrange multiplier that corresponds to h_1 becomes greater than to zero ($\pi_2 \ge 0$).

2) *Singularity of LI loss*

At point B ($\varepsilon = +1.7765$ p.u.) the rank of J at that point is deficient (equal to $2 < m + r$) while around this point is equal to 3, which implies that W becomes singular. It can be shown that the OPF path of the MLP problem exhibits a quadratic turning point close to point B with transition from local minima to local maxima; thus, it is a saddle point of W.

IV. Non-Linear Programming Solvers

Several efficient non-linear programming (NLP) codes have been developed in recent years and most of them are based on interior-point, CPLEX and sequential quadratic programming algorithms. There exist several commercial vendors e.g. AT&T, CPLEX, DASH and IBM as well as numerous research codes, some of them public domain in an executable or even in a source code form. The codes studied and used in this paper are as follows.

A. MINOS (Modular In-core Nonlinear Optimization System)

This package uses a stable implementation of the primal simplex method to solve linear programming problem. For linearly constrained problems, a reduced-gradient method is employed with quasi-Newton approximations to the reduced Hessian. For nonlinear constraints, MINOS solves a sequence of subproblems in which the constraints are linearized and the objective is an augmented Lagrangian, step length control is heuristic but superlinear convergence is often achieved [15]. This software is sold through Stanford University Office of Technology Licensing, coded in Fortran 77. (http://www.sbsi-sol-optimize.com).

B. IPOPT (Interior Point OPTimizer)

This package includes a primal-dual interior-point algorithm with filter line-search method to ensure global convergence. In [12] it is provided a comprehensive description of the algorithm, including the feasibility restoration phase for the filter method, second order corrections, and inertia correction of the KKT matrix. Heuristics are also considered for allowing faster performance. IPOPT is an open source software coded in C++, C, Fortran and MATLAB. (http://www.coin-or.org/Ipopt/).

C. KNITRO (Nonlinear Interior-point Trust Region Optimizer)

This package provides three algorithms for solving NLP problems: a) interior-point direct algorithm, b) interior-point conjugate gradient algorithm, and c) active-set algorithm. It is possible to use every independent form algorithm or to use a crossover procedure implemented internally switching the three algorithms during the solution process. The primary technical reference is [13]. The interior-point direct algorithm applies barrier techniques and directly factorizes the KKT matrix of the nonlinear system. The interior point conjugate gradient algorithm applies barrier techniques using the conjugate gradient method to solve KKT subproblem. The active-set algorithm implements the sequential linear-quadratic programming method. All algorithms have fundamental differences that lead to different behavior on NLP problems. Together, they provide a suite of different ways to tackle difficult problems. This software is sold through Ziena Optimization, available in C++, C, Fortran. (http://www.ziena.com/knitro.html).

D. LOQO

This package is based on an infeasible primal-dual interior-point method and solves both convex and non-convex optimization problems, including smooth constrained optimization problems. To convex problem, LOQO finds a globally optimal solution. Otherwise, it finds a locally optimal solution near to a given starting point. LOQO is coded in Fortran 77 and more information can be found in [14]. It is an open source software but requires a license file before it can be used. (http://www.princeton.edu/~rvdb/).

E. SNOPT (Sparse Nonlinear OPTimizer)

This package uses a sequential quadratic programming algorithm. Search directions are obtained from quadratic programming subproblems that minimize a quadratic model of the Lagrangian function subject to linearized constraints. An augmented Lagrangian merit function is reduced along each search direction to ensure convergence from any starting point. Information about SNOPT can be found in [16]. This software is sold through Stanford University Office of Technology Licensing, coded in Fortran. (http://www.sbsi-sol-optimize.com).

V. Performance of NLP solvers

The latter research codes were used to solve the MLP problem for test systems, as a simple two-bus (see Sec. III) and IEEE test systems. Some changes were made upon the original data systems to obtain singularity cases.

A. Two-bus system

Variable reactive compensation (parameter $\varepsilon = B_c$) was applied at bus 2, as shown in Sec. III. The results, as Lagrange multiplier norm and number of iterations, are shown in Figs. 3 and 4, respectively.

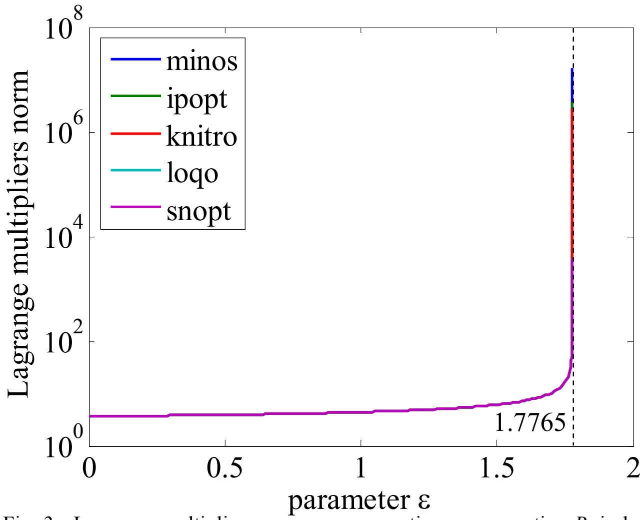

Fig. 3. Lagrange multipliers norm versus reactive compensation B_c in bus 2, two-bus system.

In Fig. 3, when parameter ε is near to +177.65 MVAR the Lagrange multipliers λ_1, λ_2 and π_1 (associated to ΔP_2, ΔQ_2 and $Q_{g-1,max}$, respectively) correspond to the constraints that cause the rank deficiency and tend to infinity when the LI condition (**C2**) is violated, which imply that W becomes singular and the OPF solution is ill-conditioned.

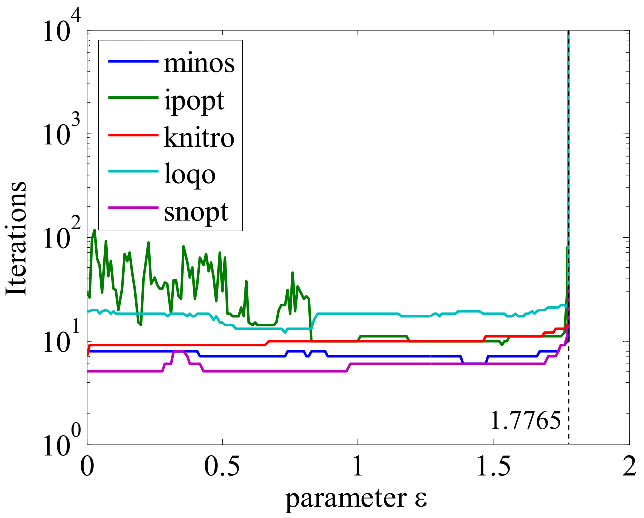

Fig. 4. Number of iterations versus reactive compensation B_c in bus 2, two-bus system.

According to Fig. 4 SNOPT and MINOS solvers presented number of iterations smaller than 10 considering the simulations of well-conditioned OPFs (before LI loss singularity). IPOPT presented large oscillations in well-conditioned OPF cases for parameter ε between 0 and 1. Also, the number of iterations increases (convergence problems) significantly for all solvers near to singularity.

B. IEEE14 system

The variable reactive compensation (parameter $\varepsilon = B_c$) was applied at bus 5, also the minimum reactive power limits of generators at buses 3 and 6 are fixed to -50 and -10 MVAR, respectively. The results are shown in Figs. 5 and 6.

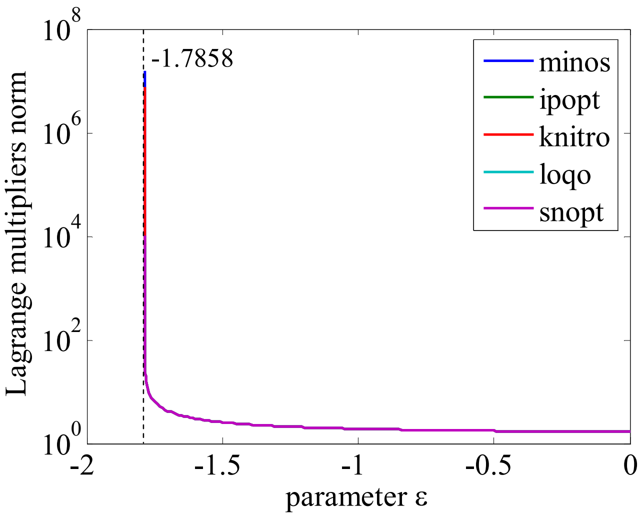

Fig. 5. Lagrange multipliers norm versus reactive compensation B_c in bus 5, IEEE14 system.

In Fig. 5, when parameter ε is near to -178.58 MVAR the Lagrange multipliers associated to the constraints that cause the rank deficiency tend to infinity and the OPF solution is ill-conditioned. According to Fig. 6 near to singularity, the number of iterations increases significantly for all solvers. Considering the simulations of well-conditioned OPFs, SNOPT, MINOS and KNITRO solvers maintained numbers of iterations smaller than 10. The number of iterations near to LI loss singularity of all solvers is shown in Fig. 7.

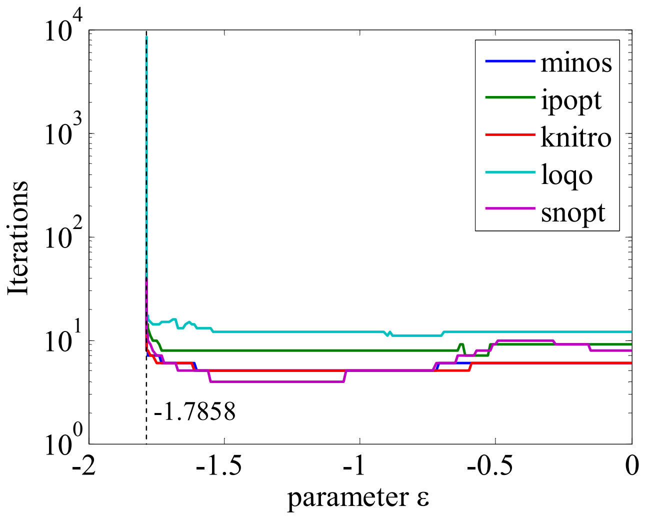

Fig. 6. Number of iterations versus reactive compensation B_c in bus 5, IEEE14 system.

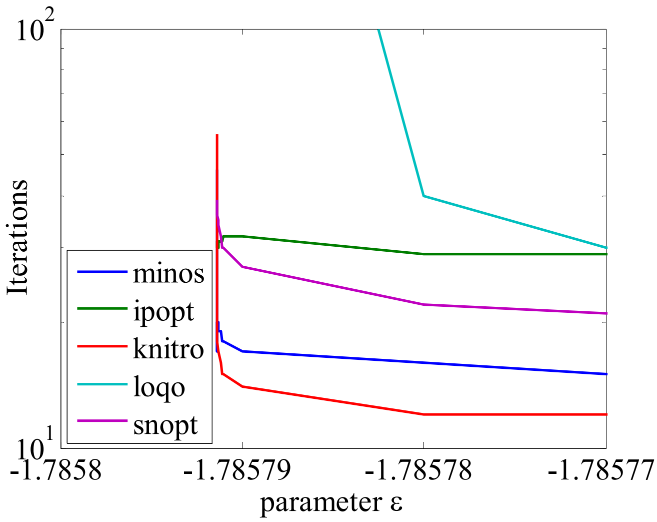

Fig. 7. Number of iterations performance near to LI loss singularity, IEEE14 system.

According to Fig. 7 LOQO solver presented the worst performance for calculating the solution with adequate precision, KNITRO and MINOS solvers presented the best performance. IPOPT solver slightly decreased in iterations near to singularity but it is consequence of interruption criteria.

C. IEEE57 system

The variable reactive compensation (parameter $\varepsilon = B_c$) was applied at bus 15, also the reactive power limits of generators at buses 2, 6, 8, and 9 are fixed to +300/-300, +200/-200, +300/-300 and +300/-300 MVAR, respectively. The results are shown in Figs. 8 and 9.

Analogously to previous cases, when parameter ε is near to -585.82 MVAR the Lagrange multipliers associated to the constraints that cause the rank deficiency tend to infinity and the OPF solution is ill-conditioned (see Fig. 8). According to Fig. 9 near to singularity, the number of iterations increases for all solvers. Considering the simulations of well-conditioned OPFs, SNOPT, MINOS and KNITRO solvers maintained number of iterations smaller than 10. The number of iterations near to LI loss singularity of all solvers is shown in Fig. 10.

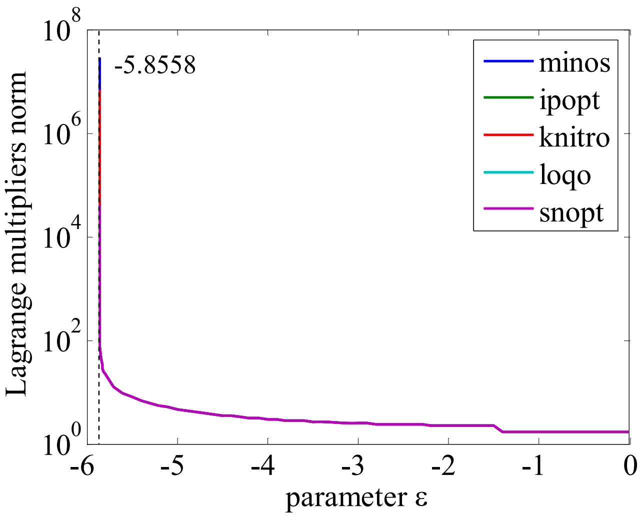

Fig. 8. Lagrange multipliers norm versus reactive compensation B_c in bus 15, IEEE57 system.

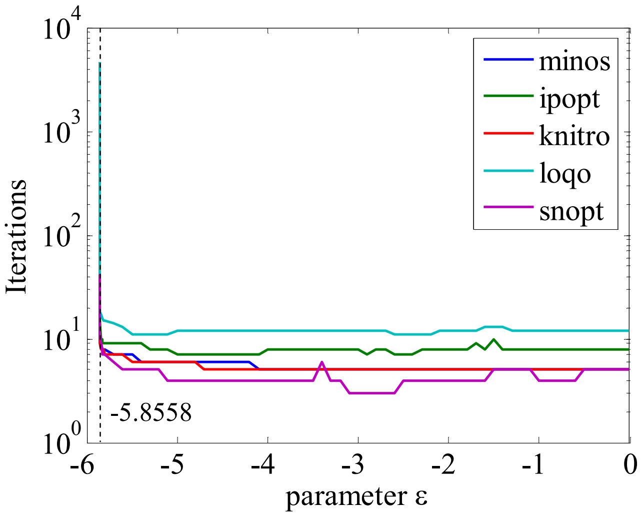

Fig. 9. Number of iterations versus reactive compensation B_c in bus 15, IEEE57 system.

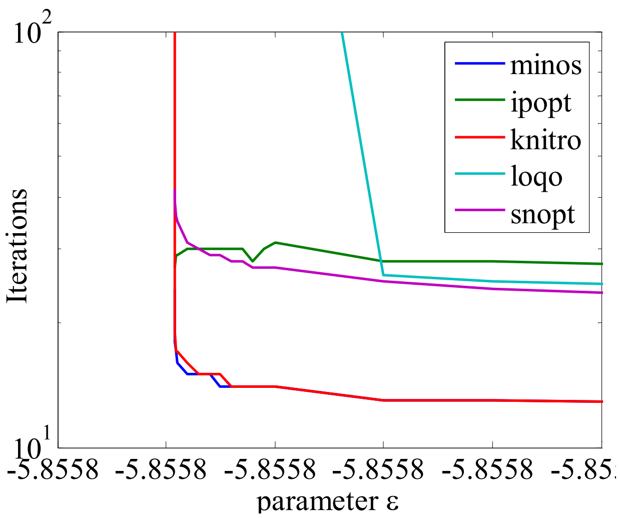

Fig. 10. Number of iterations performance near to LI loss singularity, IEEE57 system.

According to Fig. 10, LOQO solver presented the worst performance again for calculating the solution with adequate precision, KNITRO and MINOS solvers presented the best performance. Additionally, IPOPT solver showed a small number of iterations near to singularity, which allows the understanding of the interruption criteria.

VI. Conclusions

In this work the ill-conditioned OPF solutions (as singular points), the sensitivity matrices obtained in the OPF process, and conditions related to LI loss singularity were described. The two-bus system allowed studying the main characteristics of the LI loss singularity using the optimal solution path and Lagrange multipliers.

The performance of solvers as IPOPT, KNITRO, LOQO, MINOS, and SNOPT were not similar near to LI loss singularity, but all of solvers reproduced the typical characteristics of this singularity as Lagrange multipliers that tend to infinite.

Considering the simulations of well-conditioned OPFs (before of LI loss singularity), SNOPT, MINOS and KNITRO solvers maintained number of iterations smaller than 10. Near to LI loss singularity, LOQO solver presented the worst performance again for calculating the solution with high precision, and KNITRO and MINOS solvers presented the best performance in all simulation cases.

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VIII. Biographies

Manfred F. Bedriñana (S'00) was born in Lima, Peru. He received the B.S. degree in electrical engineering (first-class honors) from National University of Engineering (UNI), Peru, and the M.S. degree in electrical engineering at Federal University of Maranhão (UFMA), Brazil. Currently he is working towards his Ph.D. degree in Electrical Engineering at UNICAMP. His research areas are security assessment of electrical energy systems and electricity markets.

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