

# Technical Challenges in Modernizing Distribution Electric Power Systems with Large Number of Distributed Generators

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**Abstract**--In this paper we consider possible technical challenges which may arise while attempting to modernize today's electric power systems by connecting many distributed generators (DG) of diverse technologies closer to the end-users within the distribution networks. We report that connecting larger (~1 MW) DGs at the network locations, which result in minimal delivery losses, may lead to frequency instabilities not currently experienced in distribution grids supplied from single power generators. The paper further points out that there is a tradeoff between efficiency and the robustness of future energy systems. For example, adding storage can help, but this adds cost. The paper closes with a discussion of designing advanced enhancing robustness methods to ensure both safety and efficiency.

**Index Terms**--Distributed Generation, Future Distributed Energy Systems, AC Optimal Power Flow (OPF), Small Signal Stability, Combustion-Turbine, Hydro Plants, and Robustness.

## I. INTRODUCTION

Recent pressures for more sustainable energy have led to active efforts to deploying smaller-scale power plants close to the end users. These plants are broadly referred to as distributed generation (DG). DG units offer potential advantages, for example, increased efficiency through waste heat recovery, loss power reduction and higher reliability and availability (Zerrifi et al [1]). Some DGs also fall under the category of renewable resources and are cleaner than the traditional large fossil fuel plants. In this paper we are primarily concerned with medium sized generators (~1 Mw) located in the distribution system that supply a significant fraction of the power they generate to the system (as opposed to adjacent local loads).

In order to provide sound support for effectively integrating larger DG systems into the legacy distribution systems, it is essential to: (1) assess current operating and planning practices with respect to their ability to best integrate and utilize these new energy resources; (2) identify potential technical challenges brought about by the DG deployment; and, (3) introduce technically innovative ways for facilitating the best integration of DGs without creating reliability and safety problems. This paper begins to address all three of these issues and illustrates these issues with an example electric power distribution system.

This paper first outlines the future structure of distribution systems with DG, noting how they are different from today's systems. Section II explores optimum location and utilization of DGs in such a system so as to improve efficiency and reduce power losses in future energy systems.

Section III examines small signal stability of future energy systems in which a large portion of electric demand is provided by DGs. It demonstrates that high penetration of larger DGs supplying a significant portion of the power they generate to the distribution system (rather than to their local loads) may destabilize frequency in local distribution networks. This phenomenon has only recently been observed and studied by several authors like Cardell et al [2] and [3], Lopes et al [4], Guttromson [5] and Donnelly et al [6]. These instabilities are partially explained in terms of electromechanical oscillations caused by the presence of small synchronous generators. However, an in-depth precise explanation and effective solutions of this phenomenon have not previously been provided.

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As a result, in Section III we investigate in depth the nature of frequency instability and determine the dependence of instability on the network's and DGs' parameters. Finally, in Section IV several methods for stabilizing frequency in distribution networks are proposed.

## II. TECHNICAL LOCATING DGs BASED ON STATIC UTILITY CRITERIA

Today's electric power systems have hierarchical structures in which electric power is produced by large central power plants and transferred through transmission and distribution networks to end users. However, that structure has had some major drawbacks such as low efficiency in producing and transferring electric power<sup>1</sup>, high cost of expanding transmission and distribution networks, and negative environmental impacts from large coal power plants and from transmission lines. As a result, there are proposals to transform today's electric power systems into systems with many new distributed generators on the distribution side of the electric networks. Fig. 1 shows a schematic of such an evolving future distribution energy systems in which new candidate DGs are located.

For illustrative purpose, shown in black is a point of connection between the transmission and distribution system. This is generally modeled as an ideal power source. Represented in grey are two Combustion-Turbines (C-T) connected to nodes 13 and 14. In the future the same electric power distribution system may be expected to serve a small hydro plant (shown in blue) and/or small wind plant (shown in green).

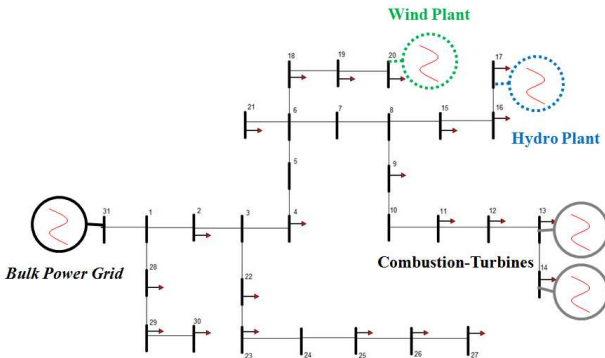


Fig. 1. Schematic of a typical future distributed energy system.

Since viewed strictly from the point of grid performance, one of the main criteria has been to design and operate the distribution systems with as little loss as possible, in this section we focus on minimizing power loss in future energy systems by optimally locating and utilizing DGs. This often requires extensive studies by the local utility to whose distribution system DG units want to connect. Because of this, it is important to establish guidelines for assessing various DGs for their relative impact on system-wide distribution losses.

Here we use AC Optimum Power Flow (AC OPF) algorithm to find optimum locations of candidate DGs in Fig. 1. The objective function of this optimization algorithm is to minimize total loss in the network and the decision variables are both real power and voltages of the candidate DGs. An in depth explanation of AC OPF is out of the scope of this paper, and further expression can be found in [12].

We began with an exhaustive assessment of all candidate locations of two Combustion Turbines (C-T) of real and reactive power capacity  $0.7MW \leq P_{DG} \leq 0.8MW$  and  $-0.4MVar \leq Q_{DG} \leq 0.4MVar$ , respectively, using AC OPF.

The results show that among 900 possible combinations of locating two C-Ts in the 30-bus system with total load demand of 15 MW, only one combination is globally optimum (buses 13 and 14). In addition, at this combination, 50% of total losses can be reduced by providing just 10% of total demand by DGs. In other words, two DGs with average capacity of 0.75 MW can reduce 0.7 MW power losses in the aforementioned distribution system if DGs are optimally located and utilized in the system.

However, in order to optimally utilize DGs, they need to be dispatchable, otherwise losses cannot be minimized in the network when loads are changing. As a result, it turns out that making the most out of the future energy systems is not feasible unless larger DGs are supported by dispatchable control systems. Furthermore, Apt et al [13] and [27] and Cardell et al [3] reflect the fact that technologies

<sup>1</sup> Efficiency of centralized power plant is 30% whereas efficiency of Combine Heat and Power (CHP), one form of DG, is more than 95%.

like wind<sup>2</sup> and solar which have non-dispatchable characteristic may degrade efficiency of future energy systems unless these technologies are equipped to some advanced mechanical or power electronic controllers. Clearly larger DGs that will provide a significant portion of their power to the system should have control systems to make them dispatchable. However, in the next section we show that this alone is not sufficient since control systems like governor-controllers may destabilize the frequency of distribution systems.

### III. TECHNICAL SMALL SIGNAL STUDY OF FUTURE DISTRIBUTION SYSTEMS

In this section we assess small signal stability of distribution systems with high penetration of DGs equipped to G-C systems. Often even the most attractive solutions obtained using static optimization may not be robust with respect to small perturbations. Earlier work has indicated that potential technical problems may be seen in frequency instabilities [2][8].

Here we present the results of a systematic small signal stability study for the system shown in Fig. 1. We stress that it is possible to identify fundamental causes of potential instabilities, based on electrical distance<sup>3</sup> between DGs. We demonstrate that there is a critical electrical distance between DGs, which results in strong coupling between the G-Cs of two DGs and proceeds to frequency instability in the geographically local distribution network. The critical electrical distance that results in frequency instability highly depends on DGs' and distribution networks' parameters such as the inertia of DGs, the gain of G-Cs, then number of DGs in the system and topology of the network.

We calculate critical electric distance using numerical methods. Finding an analytical formula for calculating this number is left to future work.

As noted in Section II, our particular interest is the relation between potentially optimal loss-based

locations of the new DGs and their effect on system stability. Using the small signal dynamic model shown in appendix A, we find that the distribution system with two combustion power plants located at nodes 13 and 14 which result in the optimal loss is indeed small signal unstable. Since small-signal instability is one form of non-robustness, this is a clear indication that substantial addition of DG that supply power to the distribution system may lead to major technical problems. More specifically, small signal instability of DGs in local distribution networks could lead to local blackouts if a modest number of DGs are meeting 10-15% of a distribution systems power needs. Moreover, Guttromson [5] has shown that if penetration of DGs is significantly high (>20%), frequency instability can even produce disruption at the level of transmission networks.

#### A. Frequency Stability of Combustion-Turbines

In this section we analyze the small signal stability of radial distribution networks with operating C-T generators equipped with G-C systems. Dynamic model of the whole inter-connected system which consists of a dynamic model of DGs and distribution systems are presented in appendix A. here we exhibit the results of exhaustive small signal study on the system shown in Fig. 1. Our findings indicate that out of 900 possible combinations of locating two C-Ts, 192 cases are small signal unstable. That is, in 192 cases the system matrix has positive eigenvalues. The participation factor method, which has been fully developed in [17], shows that instabilities can be categorized in three scenarios:

- Case A: DG units close to each other, but far away from the sub-station;
- Case B: one DG unit close to the sub-station and the other far from it; and,
- Case C: both DG units close to the sub-station and close to each other;

In case A, participation factor-based analysis of the system shows that both DGs participate equally in an unstable eigenmode or a positive eigenvalue. The degree of instability is measured by the real part of the positive eigenvalue,  $\sigma_{\max}$ .

Furthermore, our eigenvalue analysis shows that electrical distance between DGs has a significant

<sup>2</sup> Note that currently wind plants are mainly connected to the transmission level; however, by dramatic increase of penetration of these plants in the early future, we project that medium size and small wind plants will be connected to the distribution system to make providing-30%-of-the-whole-electricity-by-wind feasible (RPS).

<sup>3</sup> Electrical distance between two nodes is the impedance (short circuit impedance) between the points of interest; further explanation can be found in [9].

impact on instability. As shown in Fig. 2, increasing the electrical distance between DGs results in a decrease in instability and after a certain point the system becomes stable. Similarly, decreasing electrical distance between DGs leads to a higher unstable eigenmode. Based on our numerical analysis the upper bound for critical electrical distance between DGs is extracted as 0.5 per unit (p.u.). This number can vary by changing the size of DGs, their inertia, gain of G-Cs and topology of the system.

One example of Case A scenario is the system when two combustion plants are placed at the optimal loss locations (buses 13 and 14). Fig. 3 shows how the system of two C-Ts located at optimum locations becomes unstable when small perturbation, equal to 0.1 p.u., occurs at node 15.

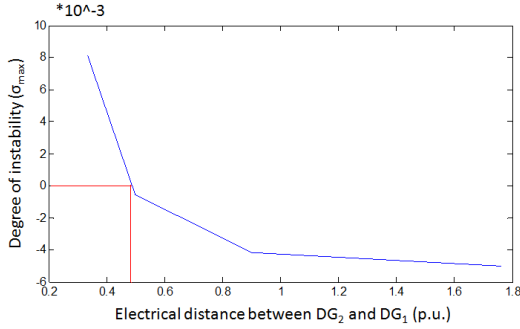


Fig. 2. Illustration of the degree of instability as a function of electrical distance between DGs (Case A).

A possible explanation of this phenomenon is that, due to the short electrical distance between DGs, their G-Cs are strongly coupled. This causes interruption in operation of local G-Cs, which have no communication with each other, or the rest of the system. Therefore, given any perturbation in the system, both generators are acting against each other to compensate power mismatch; however, they don't observe that the next-door generator is also reacting to the perturbation. Thus, they suddenly observe another perturbation in the system because of the next-door DG, so they again try to react to the new perturbation and this cascading phenomenon makes both unstable. This phenomenon has been observed and introduced for the first time in this paper and can be generalized for the case that  $n$  DGs with local G-Cs are placed in the system ( $n > 1$ ).

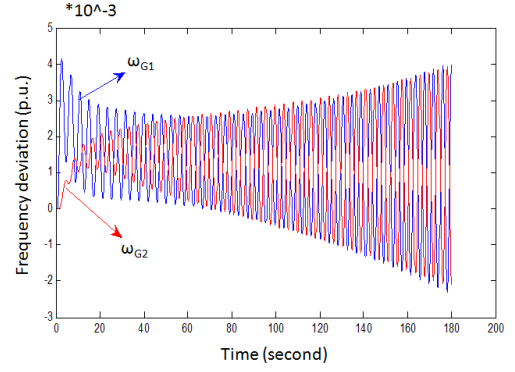


Fig. 3. Illustration of how the frequency deviation of the distribution system with two C-Ts placed at optimum locations diverge to infinite when small perturbation, equal to 0.1 p.u., occurs at bus 15.

Case B has similar qualitative behavior to case A, however, in this condition the DG electrically close to the sub-station is mostly participating in an unstable eigenmode and the second DG does not have a significant impact on instability. Similar to case A, by increasing electrical distance between the first DG and sub-station, instability decreases and eventually the DG turns to stable mode.

The physical explanation of this phenomenon is that the short electrical distance between the DG electrically closer to the sub-station and the sub-station causes strong coupling between them, which eventually leads to frequency instability of the DG. Since the sub-station is modeled as a bulk generator with infinite inertia, only the DG becomes unstable but the sub-station remains stable; moreover, the second DG further from the sub-station remains stable because there is a weak interaction between that DG and the sub-station. Participation factor-based analysis also shows that the second DG has no significant impact on instability.

In a different fashion, Case C has combined qualitative behavior of both case A and B. In this case both DGs are electrically close to each other and close to the sub-station. Hence, instability is exacerbated because of strong coupling between DGs combined with the coupling between DGs and the sub-station, so the degree of instability (magnitude of  $\sigma_{max}$ ) increases. Likewise, increasing electrical distance between DGs and that between the DGs and sub-station decreases the degree of instability.

A participation factor-based analysis for this scenario also shows that both DGs are contributing equally to the unstable eigenmode. However, by

moving one DG further from the sub-station the role of that DG decreases and nature of instability turns to case B. Note that there is no precise border between these three scenarios. The same system can reflect each case by simply changing the location of DGs. These scenarios can be generalized for the n-DG system. Eigenvalues of the system in three examples we have considered here are also provided in appendix B.

These cases have been explored in order to illustrate that it may be possible to create guidelines and new standards, beyond those of IEEE 1547<sup>4</sup>, for placing DGs in a way that helps to assure distribution system robustness.

#### IV. POTENTIAL ROBUSTNESS ENHANCEMENTS METHODS

In this section we propose three major approaches that are designed to assure frequency stability in distributed systems with DGs that export a substantial amount of power they generate to the distribution system (rather than using it locally).

##### A. Optimum locating DGs with respect to robustness

Recalling from section III, it is possible to establish guidelines for assuring the robustness of distributed networks based on the critical electrical distance between DGs. DGs need to be located such that the electrical distance between them is more than the critical point. For instance, in the distribution system shown in Fig. 1, when DGs are located at optimum static locations, frequency is initially unstable; however, by increasing the electrical distance between them, frequency stability is restored. Thus, when DGs are located at buses 11 and 14 or 10 and 14, DGs are stable. Fig. 4 illustrates a schematic of the distribution system in which two C-Ts are located at buses 14 and 10. Fig. 5 also shows how frequency deviation of two C-Ts diverges when they are electrically far from each other (more than 0.5 p.u.). Eigenvalues for this system are provided in appendix B.

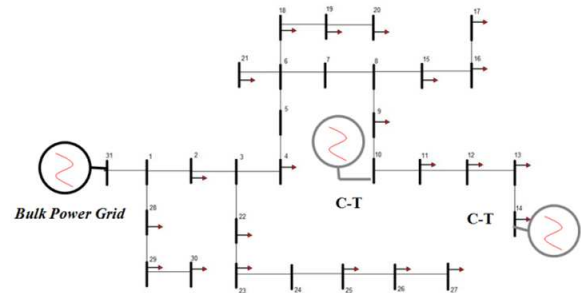


Fig. 4. A schematic of the distribution system in which two C-Ts are located at buses 14 and 10.

In spite of the fact that there is almost no cost associated with re-locating DGs, if they are mainly used to export power rather than serve local loads, this method is not efficient because the most effective use of DGs cannot be made since optimum locations are among unstable cases. Furthermore, as shown in Fig. 5, DGs do not have fast dynamic response and depending on the electrical distance between DGs damping of frequency deviation may last for several minutes. Therefore, we need to design new methods if we are to achieve both efficiency and robustness.

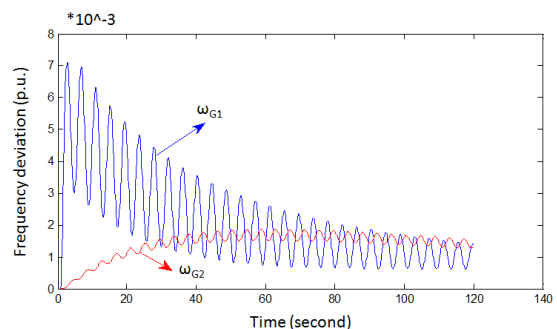


Fig. 5. Illustration of the dynamic response of C-Ts, located at buses 10 and 14, when small perturbation occurs in the system.

##### B. Increasing the inertia of DGs

It has been observed earlier and mentioned in [3] that one of the main causes of frequency instability in distribution networks with large number of DGs is the low inertia of most DG units. Perhaps simply increasing the inertia of DGs can improve robustness of the system. To explore this possibility we re-analyze small signal study of all the cases explored above with the inertia of the DGs increased by a factor of ten. We found that doing this restored stability in all of the unstable cases. For example, Fig. 6 illustrates dynamic responses of the systems with two C-Ts when DGs are located at

<sup>4</sup> IEEE 1547 is the Standard for Interconnecting Distributed Resources with Electric Power Systems approved by the IEEE Standards Board in June 2003. Error! Reference source not found..



the optimum locations and their inertia has been increased tenfold. The system is now small signal stable.

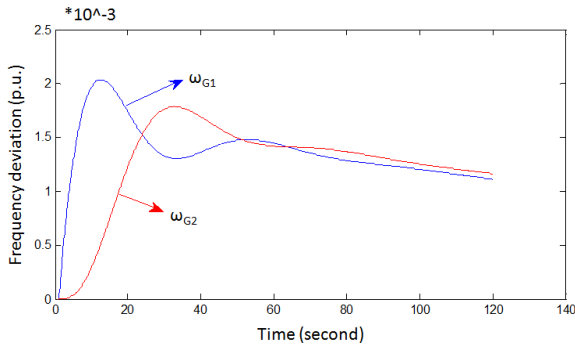


Fig. 6. Illustration of the dynamic response of the system with two C-Ts when DGs are located at the optimum locations and their inertia has been increased tenfold.

Inertia can be increase by using flywheels or with electrical storage devices. The latter approach is very expensive because battery or capacitor is costly. Second, both flywheels and electrical storage devices have sluggish dynamic response, consequently dynamic reaction of DGs evolves very slowly and in many cases they cannot response properly to load deviations. Hence, it turns out that we need to develop a new method if we wish to assure *both* robustness and efficiency.

### C. Designing Advanced Control Systems

In this section we briefly describe the effect of advanced control systems on robustness and efficiency in future energy systems. We begin by concisely explaining controllability in linear systems. A system is stabilizable if it is fully controllable and a linear system can be recognized as controllable if its controllability matrix is full-rank [11].<sup>5</sup> Further explanation can be found in [11].

On this basis it is straightforward to show that all the systems discusses above are fully controllable, and also in all cases each sub-system (DG) individually is locally controllable. Hence, the system is either locally or globally stabilizable.

A control strategy can pursue two general goals: 1) assure stability of the system; 2) improve the efficiency of the system through optimum control design. The two are not mutually exclusive. By

implementing optimum control logics one can both enhance robustness and at the same time improve efficiency. It is worth mentioning that our criteria for efficiency depend on the needs of consumers and utilities. Control design can either seek to minimize the cost of controllers or improve the quality of electricity, by minimizing fluctuations. In this section we just concentrate on qualitative aspects of the advanced control design and its quantitative features are left for future work.

Since in general it is feasible to stabilize future energy systems both locally and globally, the question automatically raises of which control strategy (centralized or decentralized) is more appropriate for future energy systems. Our answer is that choosing between these two strategies depends highly on existing system design, regulation and policy. If the system already has a hierarchical structure and the policy of the utility is to conserve this structure, then a centralized control system is likely to be a better option. On the other hand, if the system has been converted to a deregulated structure and policy is in favor of deregulation, then a proper control strategy for the system would more likely be decentralized control.

## V. CONCLUSIONS AND FUTURE WORK

This paper has demonstrated that, as the utility studies are carried out when considering new DGs that will export significant power to the distribution system, it is necessary to develop a systematic framework for assessing potential technical problems unique to the transformation of electric power systems into future, more distributed, systems.

To that end first we have shown that by providing a small portion of electric demand (~10%) by DGs, a significant portion (>50%) of power losses can be eliminated in distribution networks and consequently in transmission lines, if DGs are optimally located and utilized in the system.

Second our technical finding involves the fact that high penetration of DGs, contributing significant amounts of power to the today's distribution network, is not feasible because of frequency instability, unless new methods for enhancing-robustness are adopted. Further there is

<sup>5</sup> Full-rank matrix is an square matrix which has no zero eigenvalue [11]

often a tradeoff between efficiency and robustness of future energy systems.

Moreover, based on our technical findings we recommend three major methods of enhancing robustness, that is, i) optimum locating of DGs with respect to robustness criteria, ii) enlarging inertia of DGs, and iii) designing advanced control strategies.

Finally, as mentioned earlier the most efficient way of warranting robustness of evolving future energy systems is designing advanced centralized or decentralized control systems. However, choosing between these two control-strategies raises the question of tradeoff between high communication and observation or advanced local control systems. The best solution will depend critically on technical design and regulation policies of the system. A quantitative analysis of the third robustness enhancing method is also left for future work.

## VI. APPENDIX

### A. Dynamic Model

In order to investigate dynamic stability of the system, we need to first create a dynamic model of the whole system. To this end we have developed a linearized state-space model of all segments of the network and have connected them with interconnected system variable. The first dynamic model being presented is the state-space model of a combustion-turbine with G-C.

$$\frac{d}{dt} \begin{bmatrix} \omega_G \\ V_{CE} \\ W_F \\ W_F d \end{bmatrix} = \begin{bmatrix} \frac{-D}{M} & 0 & \frac{c}{M} & 0 \\ -\frac{K_D}{b} & \frac{-1}{b} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & \frac{-\delta}{\alpha} & \frac{-\beta}{\alpha} \end{bmatrix} \begin{bmatrix} \omega_G \\ V_{CE} \\ W_F \\ W_F d \end{bmatrix} + \begin{bmatrix} -1 \\ M \\ 0 \\ 0 \end{bmatrix} P_G + \begin{bmatrix} 0 \\ \frac{K_D}{b} \\ 0 \\ 0 \end{bmatrix} \omega^{ref} \quad (A.1)$$

In this model,  $\omega_G$  is frequency,  $V_{CE}$  is fuel control,  $W_F$  is fuel flow and  $W_F d$  is the derivative of fuel flow.  $M$  and  $D$  are the inertia and damping coefficients respectively.  $a$ ,  $b$  and  $c$  are transfer function coefficients for the fuel system, and  $K_D$  is the governor gain.  $\beta$  and  $\delta$  are algebraic functions of the parameters, defined as  $\beta = b + c\tau_F$  and  $\delta = c + aK_F$ , where  $\tau_F$  is the fuel system time constant, and  $K_F$  is the fuel system feedback gain. Numerical values of combustion turbine parameters are presented in appendix B. These equations are derived and simplified from the equations and models found in [14], [15] and [16] and used for the first time in [3].

Equation (A.1) can be also written in a general form as;

$$\frac{d}{dt} x_{LC} = \mathbf{A}_{LC} x_{LC} + \mathbf{C}_M P_G + \mathbf{B}_{LC} u_{LC} \quad (A.2)$$

where  $x_{LC}$  is the local state vector,  $u_{LC}$  is the control input which controls the variable  $\omega^{ref}$ , the reference frequency for the governor, and the bold variables represent matrices.  $\mathbf{A}_{LC}$  is referred to as the local system matrix, whose elements consist of the linear coefficients of the generator parameters [3].

To complete the system model, the dynamic model of generators should be coupled to each other via the distribution network. Mathematically, the local state space model of each generator is expanded to include the system-coupling variable. This coupling variable is chosen to be active power output, or  $P_G$  [3]. In other words, to connect the generator dynamic models to the distribution system, first the equilibrium point for the full system was determined by the AC OPF study mentioned in Section II. Then the power flow equations were linearized around the equilibrium point by using the Jacobian Matrix. Hence, the differential equation for active power is found as following [3];

$$\frac{d}{dt} P_G = \mathbf{K}_P \omega_G + \mathbf{D}_P \frac{d}{dt} P_L \quad (A.3)$$

where  $\frac{d}{dt} P_L$ , representing load disturbance, is an input variable to the system,  $K_P$  and  $D_P$  are derived from the Jacobian matrix of the distribution network, and  $P_G$  is the system coupling variable. Including  $P_G$  from (A.3) and extending the model of (A.2) to the general case, the full system model takes the form as;

$$\begin{bmatrix} \frac{d}{dt} x_{LC1} \\ \vdots \\ \frac{d}{dt} P_{G1} \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{LC1} & 0 \\ 0 & \ddots \\ \mathbf{K}_P E & \cdots \\ \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_{LC1} \\ \vdots \\ P_{G1} \\ \vdots \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{M1} & 0 \\ 0 & \ddots \\ 0 & \cdots \\ \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_{LC1} \\ \vdots \\ P_{G1} \\ \vdots \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{LC1} & 0 \\ 0 & \ddots \\ 0 & \cdots \\ \vdots & \ddots \end{bmatrix} [u_{LC1} \cdots u_{LCn} | 0 \cdots] + \mathbf{D}_P \frac{d}{dt} P_L \quad (A.4)$$

In the equations for  $\frac{d}{dt}P_{Gi}$  only  $\omega_{Gi}$  of the local space for each generator has a non-zero element, so the matrix  $E$  has block diagonal elements of the form  $[1 \ 0 \ 0 \ \dots]$ . In this form the matrix  $E$  is zero except for elements corresponding to  $\omega_{Gi}$ , where the entry is equal to 1 to provide the coupling between  $P_{Gi}$  and  $\omega_{Gi}$  (via  $K_P$ ), the Jacobian matrix [3].

## B. Parameters

Parameters of Combustion-Turbine generator [3]. Coefficients are in per unit

| Combustion-Turbine parameters |      |          |      |
|-------------------------------|------|----------|------|
| M                             | 11.5 | $\alpha$ | 0.45 |
| D                             | 2.0  | a        | 1.0  |
| c                             | 1.0  | $\tau_F$ | 0.40 |
| $K_D$                         | 25.0 | $K_F$    | 0.0  |
| b                             | 0.05 |          |      |

Eigenvalues of 30 Bus System when DGs are placed in different locations

| DGs at buses 13 and 14 (case A) | DGs at buses 1 and 14 (case B) | DGs at buses 2 and 3 (case C) | DGs at buses 10 and 14 (stable case) |
|---------------------------------|--------------------------------|-------------------------------|--------------------------------------|
| -19.9943                        | -19.9943                       | -19.9943                      | -19.9943                             |
| -19.9943                        | -19.9943                       | -19.9943                      | -19.9943                             |
| 0.0081 +                        | 0.0134 +                       | 0.1091 +                      | -0.0130 +                            |
| 1.5217i                         | 1.5270i                        | 1.6931i                       | 1.5029i                              |
| 0.0081 -                        | 0.0134 -                       | 0.1091 -                      | -0.0130 -                            |
| 1.5217i                         | 1.5270i                        | 1.6931i                       | 1.5029i                              |
| -0.0186 +                       | -0.0182 +                      | -0.0143 +                     | -0.0184 +                            |
| 1.4984i                         | 1.4987i                        | 1.5018i                       | 1.4985i                              |
| -0.0186 -                       | -0.0182 -                      | -0.0143 -                     | -0.0184 -                            |
| 1.4984i                         | 1.4987i                        | 1.5018i                       | 1.4985i                              |
| -1.1387                         | -0.9628                        | -1.1190                       | -1.1378                              |
| -0.9989                         | -1.1366                        | -0.6989 +                     | -1.1130                              |
| -0.1969                         | -0.2437                        | 0.6914i                       | -0.0050                              |
| -0.0037                         | -0.0066                        | -0.6989 -                     | -0.0407                              |
|                                 |                                | 0.6914i                       |                                      |
|                                 |                                | -0.0321                       |                                      |

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