Profit-Based Head-Sensitive Behavior of a Hydro Chain: Mixed-Integer Nonlinear Method

J. P. S. Catalão, *Member, IEEE*, H. M. I. Pousinho, V. M. F. Mendes

***Abstract*—This paper is on the problem of short-term hydro scheduling, particularly concerning a head-sensitive hydro chain. A new mixed-integer nonlinear method is proposed for optimizing power generation efficiency. The proposed method considers not only the nonlinear dependence between power generation, water discharge and head, but also start-up costs for the hydro units and discontinuous operating regions, in order to obtain more realistic and feasible results. Numerical results, based on one of the main Portuguese cascaded hydro systems, illustrate the proficiency of the proposed method. Finally, conclusions are duly drawn.**

***Index Terms*—Hydroelectric power generation, mixed-integer nonlinear method, power generation scheduling.**

I. Nomenclature

The notation used throughout the paper is stated as follows:

I, i Set and index of reservoirs.
K, k Set and index of hours in the time horizon.
λ_k Forecasted energy price in hour *k*.
p_{ik} Power generation of plant *i* in hour *k*.
SU_i Start-up cost of plant *i*.
y_{ik} Binary variable which is equal to 1 if plant *i* is started-up at beginning of hour *k*.
z_{ik} Binary variable which is equal to 1 if plant *i* is shut-down at beginning of hour *k*.
Ψ_i Future value of the water stored in reservoir *i*.
v_{ik} Water storage of reservoir *i* at end of hour *k*.
a_{ik} Inflow to reservoir *i* in hour *k*.
M_i Set of upstream reservoirs to reservoir *i*.
q_{ik} Water discharge by reservoir *i* in hour *k*.
s_{ik} Water spillage by reservoir *i* in hour *k*.
h_{ik} Head of plant *i* in hour *k*.
l_{ik} Water level in reservoir *i* in hour *k*.
η_{ik} Power efficiency of plant *i* in hour *k*.
u_{ik} Binary variable which is equal to 1 if plant *i* is on-line in hour *k*.
$\overline{v}_i, \underline{v}_i$ Water storage limits of reservoir *i*.
$\overline{h}_i, \underline{h}_i$ Head limits of plant *i*.
$\overline{q}_i, \underline{q}_i$ Water discharge limits of plant *i*.
A Constraint matrix.
$\overline{b}, \underline{b}$ Upper and lower bound vectors on constraints.
x Vector of decision variables.
$\overline{x}, \underline{x}$ Upper and lower bound vectors on decision variables.
$\overline{\eta}_i, \underline{\eta}_i$ Power efficiency limits of plant *i*.
$\overline{l}_i, \underline{l}_i$ Water level limits of reservoir *i*.

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II. Introduction

In this paper, the short-term hydro scheduling (STHS) problem of a head-sensitive hydro chain is considered. Hydro plants with only a small storage capacity available are known as run-of-the-river. Due to the reservoirs small storage capacity, the operating efficiency becomes sensitive to the head—head change effect [1]. In a cascaded hydraulic configuration, where hydro plants can be connected in both series and in parallel, the release of an upstream plant contributes to the inflow of the next downstream plant. The cascaded hydraulic configuration coupled with the nonlinear head change effect, augments the problem dimension and the complexity.

In the STHS problem a time horizon of one to seven days is considered, usually discretized into hourly periods. The STHS problem is treated as a deterministic one. Where the problem includes stochastic quantities such as inflows to reservoirs or energy prices, the corresponding forecasts are used [2]. The main goal in the profit-based STHS problem is to maximize the value of total hydroelectric generation throughout the time horizon, while satisfying all hydraulic constraints, aiming the most efficient and profitable use of the water [3].

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [4]-[5]. However, direct application of DP methods for hydro chains is impractical due to the well-known DP curse of dimensionality.

Artificial intelligence techniques have also been applied to the STHS problem [6]-[9]. However, due to the heuristics used in the search process only sub-optimal solutions can be reached.

A natural approach to STHS is to model the system as a network flow model [10], because of the underlying network structure subjacent in hydro chains. The network flow model is often simplified as a linear or piecewise linear one. Linear programming (LP) is a widely used method for STHS [11]. However, LP typically considers that hydroelectric power generation is linearly dependent on water discharge, thus ignoring head-dependency to avoid nonlinearities.

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Mixed-integer linear programming (MILP) is becoming often used for STHS [12]-[15], where integer variables allow modeling of start-up costs, which are mainly caused by the increased maintenance of windings and mechanical equipment and by malfunctions of the control equipment [16].

Hydro scheduling is in nature a nonlinear optimization problem. A nonlinear model has advantages compared with a linear one. A nonlinear model expresses hydroelectric generation characteristics more accurately and the head change effect can be taken into account [17]-[18]. However, the nonlinear model cannot avoid water discharges at forbidden intervals, and ignoring start-up costs may give schedules unacceptable from an operation point of view.

In this paper, a new mixed-integer nonlinear programming (MINLP) method is proposed to solve the STHS problem. The proposed method considers not only head-dependency but also start/stop of units. The proposed method has been applied on a case study based on one of the main Portuguese cascaded hydro systems.

This paper is organized as follows. Section 3 provides the mathematical formulation of the STHS problem. Section 4 presents the proposed MINLP method to solve the STHS problem. Section 5 provides the results from the case study. Finally, concluding remarks are given in Section 6.

III. Problem Formulation

A. Objective Function

In this paper, the objective function to be maximized is expressed as

$$F=\sum_{i=1}^{I}\sum_{k=1}^{K}\left(\lambda_k\, p_{ik}-SU_i\, y_{ik}\right)+\sum_{i=1}^{I}\Psi_i\,(v_{iK}) \qquad (1)$$

In (1), the first term is related to the revenues of each plant i in the hydro chain, whereas the second term represents the start-up costs. The last term expresses the future value of the water stored in the reservoirs in the last period K, as in [19].

B. Hydro Constraints

1) Water Balance: The water balance equation for each reservoir is formulated as

$$v_{ik}=v_{i,k-1}+a_{ik}+\sum_{m\in M_i}(q_{mk}+s_{mk})-q_{ik}-s_{ik} \qquad \forall\, i\in I,\quad \forall\, k\in K \qquad (2)$$

assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period.

2) Head: The head is considered a function of the water storages in the upstream and downstream reservoirs

$$h_{ik}=h_{ik}\,(v_{f(i)k},v_{t(i)k}) \quad \forall\, i\in I,\quad \forall\, k\in K \qquad (3)$$

3) Power Generation: Power generation is considered a function of water discharge and hydro power efficiency

$$p_{ik}=q_{ik}\,\eta_{ik}(h_{ik}) \quad \forall\, i\in I,\quad \forall\, k\in K \qquad (4)$$

Hydro power efficiency is expressed as the output-input ratio, depending on the head. The hydroelectric power generation characteristics can be graphically represented by a family of nonlinear curves, also known as unit performance curves, each curve for a specific value of the head (see Fig. 1).

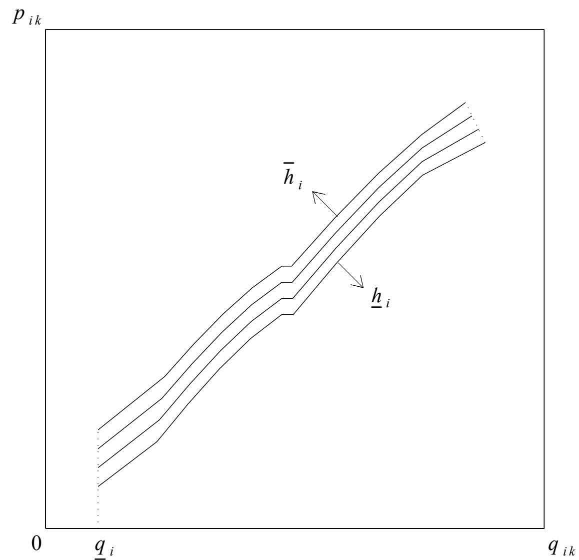

Fig. 1. Unit performance curves.

4) Water Storage: Water storage has lower and upper bounds

$$\underline{v}_i\le v_{ik}\le \bar{v}_i \quad \forall\, i\in I,\quad \forall\, k\in K \qquad (5)$$

5) Water Discharge: Water discharge has lower and upper bounds

$$u_{ik}\,\underline{q}_i\le q_{ik}\le u_{ik}\,\bar{q}_i \quad \forall\, i\in I,\quad \forall\, k\in K \qquad (6)$$

6) Water Spillage: A null lower bound is considered for water spillage

$$s_{ik}\ge 0 \quad \forall\, i\in I,\quad \forall\, k\in K \qquad (7)$$

Water spillage by the reservoirs exits when the water storage exceeds its upper bound. The spillage effects were considered in [20].

7) Logical Status of Commitment: The following constraints

$$y_{ik}-z_{ik}=u_{ik}-u_{i,k-1} \quad \forall\, i\in I,\quad \forall\, k\in K \qquad (8)$$

are necessary to model the start-up and shut-down status of the plants. Although variables z_{ik} may seem superfluous since they only appear in (8), extensive numerical simulations have proven their ability in considerably reducing computation time [12].

The initial water storages and inflows to reservoirs are assumed known. Also, the energy prices are considered as deterministic input data for the STHS problem. Nevertheless, several techniques are available in the literature to forecast these prices [21]-[26].

IV. Mixed-Integer Nonlinear Method

The MINLP problem can be stated as to maximize

$$F(x) \tag{9}$$

subject to

$$\underline{b} \leq A\, x \leq \overline{b} \tag{10}$$

$$\underline{x} \leq x \leq \overline{x} \tag{11}$$

$$x_j \text{ integer} \quad \forall\, j \in J \tag{12}$$

In (9), the function $F(\cdot)$ is a nonlinear function of the vector x of decision variables. Equality constraints are defined by setting the lower bound equal to the upper bound, i.e. $\underline{b} = \overline{b}$. The variables x_j are restricted to be integers. The lower and upper bounds for water discharge imply new inequality constraints that will be rewritten into (10).

As expressed in (3) and (4), water level and hydro power efficiency depend respectively on water storage and head.

The linearization of hydro power efficiency of plants is considered, given by

$$\eta_{ik} = \alpha_i\, h_{ik} + \eta_i^0 \quad \forall\, i \in I, \quad \forall\, k \in K \tag{13}$$

where the parameters α_i and η_i^0 are given by

$$\alpha_i = (\overline{\eta}_i - \underline{\eta}_i) \,/\, (\overline{h}_i - \underline{h}_i) \quad \forall\, i \in I \tag{14}$$

$$\eta_i^0 = \overline{\eta}_i - \alpha_i\, \overline{h}_i \quad \forall\, i \in I \tag{15}$$

Also, a linearization of the water level function is considered, given by

$$l_{ik} = \beta_i\, v_{ik} + l_i^0 \quad \forall\, i \in I, \quad \forall\, k \in K \tag{16}$$

where the parameters β_i and l_i^0 are given by

$$\beta_i = (\overline{l}_i - \underline{l}_i) \,/\, (\overline{v}_i - \underline{v}_i) \quad \forall\, i \in I \tag{17}$$

$$l_i^0 = \overline{l}_i - \beta_i\, \overline{v}_i \quad \forall\, i \in I \tag{18}$$

Substituting (13) into (4), power generation is given by

$$p_{ik} = q_{ik}\, (\alpha_i\, h_{ik} + \eta_i^0) \quad \forall\, i \in I, \quad \forall\, k \in K \tag{19}$$

Therefore, substituting (3) and (16) into (19), power generation becomes a nonlinear function of water discharge and water storage, given by

$$p_{ik} = \alpha_i\, \beta_{f(i)}\, q_{ik}\, v_{f(i)k} - \alpha_i\, \beta_{t(i)}\, q_{ik}\, v_{t(i)k} + \chi_i\, q_{ik} \quad \forall\, i \in I, \quad \forall\, k \in K \tag{20}$$

with

$$\chi_i = \alpha_i\, (l_{f(i)}^0 - l_{t(i)}^0) + \eta_i^0 \quad \forall\, i \in I \tag{21}$$

The proposed MINLP method considers the head change effect in a function (20) of water discharge and water storage. As a new contribution to earlier studies [17]-[18], start-up costs for the hydro units and discontinuous operating regions are considered.

V. Case Study

The proposed MINLP method has been applied on one of the main Portuguese cascaded hydro systems. The model has been developed and implemented in MATLAB and solved using the optimization solver package Xpress-MP. The numerical testing has been performed on a 600-MHz-based processor with 256 MB of RAM.

A. Input Data

The realistic hydro chain has seven cascaded reservoirs and is shown in Fig. 2.

The hydro plants numbered in Fig. 2 as 1, 2, 4, 5 and 7 are run-of-the-river hydro plants. The hydro plants numbered as 3 and 6 are storage hydro plants. Hence, for the storage hydro plants the head change effect is neglected, due to the small head variation during the short-term time horizon. Inflow is considered only on reservoirs 1 to 6. The final water storage in the reservoirs is constrained to be equal to the initial water storage. The hydro units start-up costs have been estimated as a function of its nominal output power, $SU_i = \overline{p}_i \times 2.5$, as in [14]. Also, forbidden zones are considered for the hydro units. These zones result from mechanical vibrations, cavitation and low efficiency level [27].

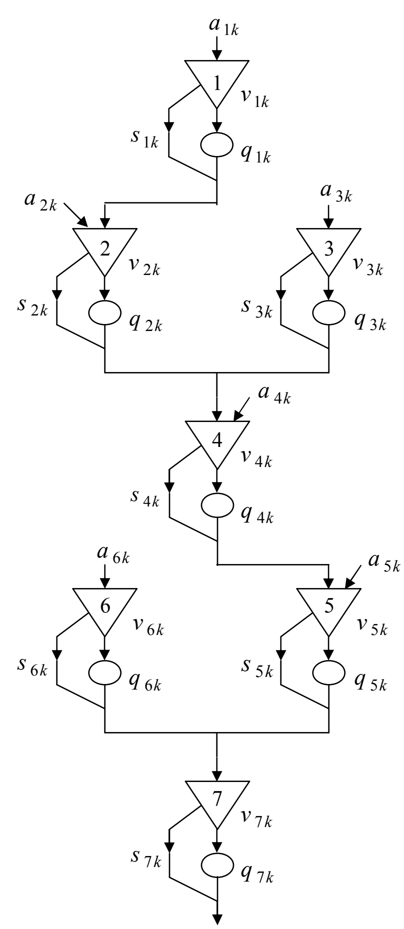

Fig. 2. Cascaded hydro system.

The time horizon is one day divided into 24 hourly intervals. The energy price profile considered over the short-term time horizon is shown in Fig. 3 ($ is a symbolic economic quantity).

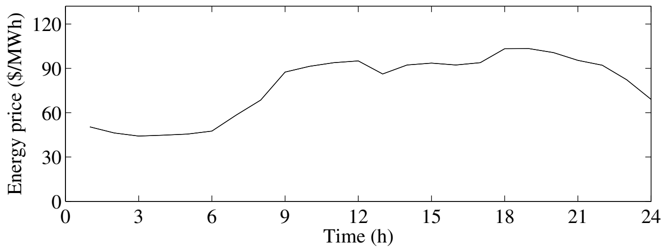

Fig. 3. Energy price profile.

B. Result Analysis

The storage trajectories of the run-of-the-river reservoirs are shown in Fig. 4.

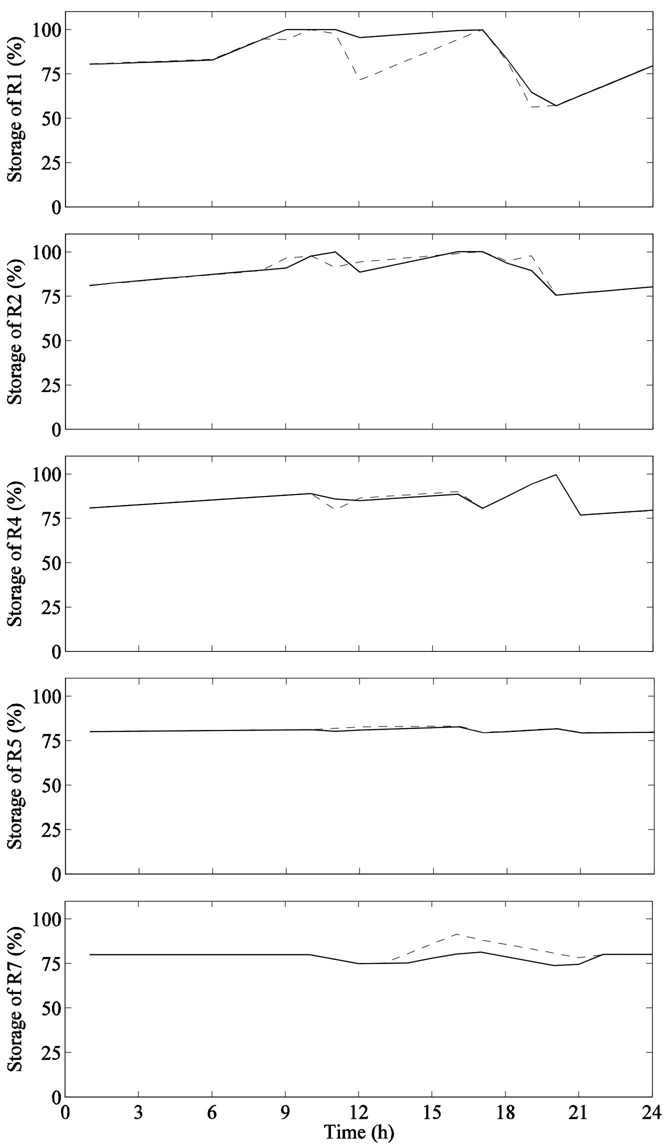

Fig. 4. Storage trajectories of the reservoirs 1, 2, 4, 5 and 7. The solid lines denote MINLP results while the dashed lines denote MILP results.

The comparison of MINLP with MILP results, shown in Fig. 4, reveals the influence of considering the head change effect in the behavior of the reservoirs. The upstream reservoir should operate at a suitable high storage level in order to benefit the power generation efficiency of its associated plant, due to the head change effect. Hence, the storage trajectory of the upstream reservoir is pulled up using the MINLP method. Instead, the storage trajectory of the last downstream reservoir is pulled down using the MINLP method, thereby improving the head for the immediately upstream reservoirs. Hence, a higher efficiency of the last downstream plant is not important for the overall profit in this hydro system.

The discharge profiles for the run-of-the-river reservoirs are shown in Fig. 5.

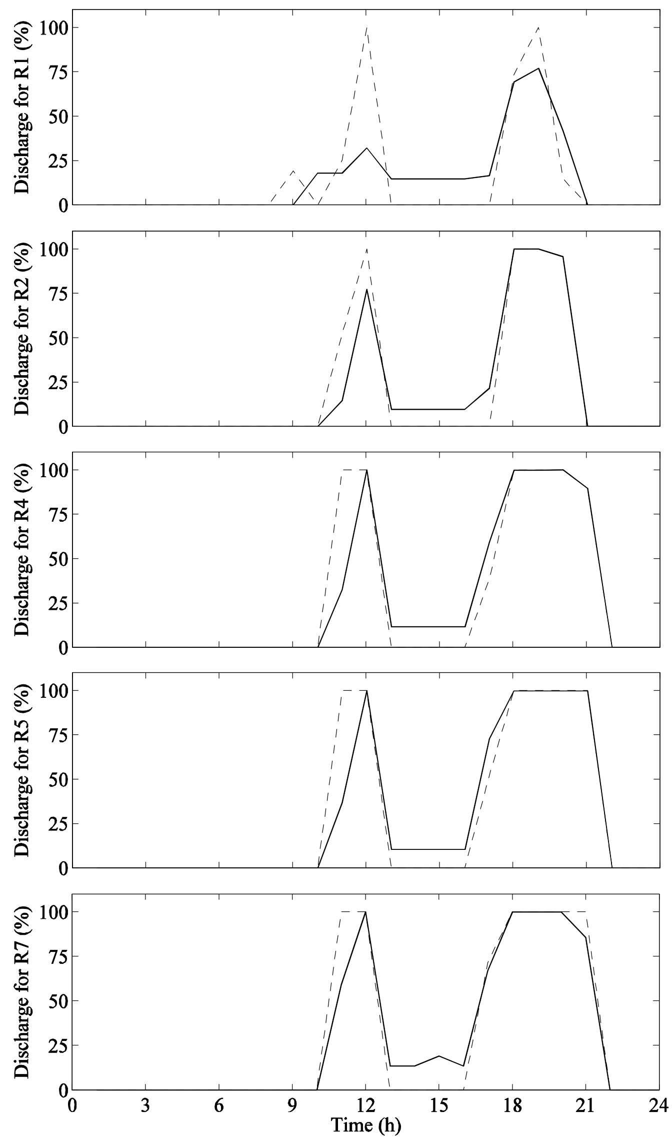

Fig. 5. Discharge profiles for the reservoirs 1, 2, 4, 5 and 7. The solid lines denote MINLP results while the dashed lines denote MILP results.

The comparison of MINLP with MILP results, shown in Fig. 5, reveals that the water discharge changes more quickly from the minimum value to the upper value in the MILP results than in the MINLP results, due to the head change effect.

As a new contribution to earlier studies [17]-[18], the water discharges at forbidden zones are avoided, namely between 0 and $\underline{q}_i$. Also, including start-up costs in the objective function implies a different behavior of the reservoirs: once a hydro unit is committed, it tends to remain on-line during more hours, avoiding frequent start-ups.

The main numerical results for the hydro system are summarized in Table I. Although the average water discharge is as expected the same for both optimization methods, the average storage is superior with the proposed MINLP method, due to the consideration of the head change effect.

TABLE I
COMPARISON OF MINLP WITH MILP RESULTS

Method	Average Discharge (%)	Average Storage (%)	Total Profit ($\$ \times 10^3$)	CPU (s)
MILP	25.00	83.08	718.33	1.75
MINLP	25.00	83.19	747.52	21.95

Thus, a higher total profit is obtained with the proposed MINLP method, about 4%. Moreover, the additional CPU time required is acceptable. Hence, the proposed MINLP method is both accurate and computationally acceptable, providing better results for head-sensitive hydro chains.

VI. CONCLUSIONS

A new MINLP method is proposed for the STHS problem, considering not only head-dependency, but also start/stop of units. The proposed method considers the head change effect in a single function of water discharge and water storage that can be used in a straightforward way. Due to the more realistic modeling presented in this paper, an enhanced STHS is provided, assuring simultaneously an acceptable computation time.

VII. ACKNOWLEDGMENT

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IX. Biographies

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