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# Transmission Loss Allocation of Bilateral Contracts Using Load Flow Permutations Average Method

*Vahid Rasouli Disfani, Farzad Razavi, Babak Kashanizadeh, and Shayan Dargahi*

*Abstract*--**In a deregulated power system, bilateral contracts between suppliers and buyers are allowed and transmission loss due to their power transactions should be allocated firmly. In this paper, we introduce "Load Flow Permutations Average Method" (LFPAM), a novel application of Shapely Value method, to assess the transmission loss allocation of bilateral contracts in an electricity market. This method, in which the load flow technique is carried out in an iterative process, lists all possible permutations of the contracts in load flow analysis and runs shapely value technique in order to evaluate the shares of each bilateral contracts of transmission losses. In the other words, after calculating the all contracts' shares of transmission loss in each permutation, the average of the results determines a firm transmission loss allocation of bilateral contracts. Finally, the method has been presented along with some numerical examples to put emphasis on its accuracy and fairness.**

*Index Terms*—**Bilateral Contracts, Electricity Market, Game Theory, Permutation, Shapely Value, Transmission Loss Allocation**

## I. INTRODUCTION

Competition in power industries is the most important issue that motivates the power systems to replace the traditional vertically integrated structure by a deregulated one. Moreover, since flow of power should be transmitted from generators to loads through transmission lines, the transmission loss is a natural phenomenon in a power system. Therefore, in a deregulated power system, the transmission loss is a continuous issue as it related to a huge amount of money and should be attributed to the power system participants. It is obvious that both sides of bilateral contracts are responsible for compensation of the transmission losses corresponding to their power transaction. Then, suppliers and buyers share the losses allocated to themselves depending on their contract.

A number of works on transmission loss allocation have been recently reported in literature. In the Pro-Rata (PR) technique [1] that ignores the system configuration, losses are globally allocated to producers and customers through a proportional allocation rule. PR is used in mainland Spain to assess the transmission loss allocation. In [2, 3] Z-Bus loss allocation method introduces a loss expression in terms of the Z-bus matrix and injection of current instead of power injection. This method separates the transmission loses among the system buses. In marginal procedures [4], losses are allocated to generators and demands using incremental transmission loss coefficients.

Furthermore, Proportional Sharing Methods [5-8] uses a linear proportional sharing principle plus the results of a power flow process in order to calculate transmission loss allocation. "This principle states that the power flow reaching a bus from any power line splits among the lines evacuating power from the bus proportionally to their corresponding power flows, which is neither provable nor disprovable" [1].

Another method based on unbundling of branch flows has been presented in [9]. To use this method which is modified incremental loss factor method and is applicable on a nodal basis, four methods have been proposed for splitting branch flows. Marginal Transmission Loss Allocation (MTLA) [10] is another method based on Kron's transmission loss expression and results in an iterative process. Incremental Load Flow Approach (ILFA) [10, 11] use modified load flow calculation to assess the transmission loss allocation. In ILFA, contracted load is increased in a discrete step at each load bus while the other contracted loads at other buses kept constant. The resulted differential loss is allocated to the corresponding bilateral contract. In this method, the loads are increased from zero to their respective levels, in alternative sequence and discrete steps to allocate the transmission losses fairly.

However, game theoretic methods are also viable to fairly allocate transmission power losses beside the methods mentioned before. For example, Shapely value, the τ-value, and the average lexicographic value are the most applicable one-point solution concepts for pay-off allocation in cooperative game theory [12]. Of these three methods, Shapely value seems fairer as all contributions of all participants are mentioned; so, it is widely used to compute shares of each participant of a coalition with no discrimination [13].

Since transmission loss allocation of bilateral contracts is a

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continuous and challenging issue in a deregulated system, we discuss the transmission loss allocation of these kinds of contracts. Consequently, a novel application of Shapely value technique is represented in this paper. This method employs the load flow analysis of DIgSILENT software and calculation capabilities of MATLAB to assess the transmission loss allocation.

## II. LOAD FLOW PERMUTATIONS AVERAGE METHOD

Load Flow Permutation Average Method considers all possible permutations of the contracts to carry out the load flow technique in an iterative process. It is obvious that there are different system states in each permutation as many as the number of bilateral contracts in that permutation. For each state, a contract will be added to the power system, a new load flow is run, and the differential transmission loss is attributed to the recently added contract. After evaluation of all states of a permutation, the transmission loss allocation of that permutation is defined. Finally, the average values of all permutation results identify transmission loss allocated to each bilateral contract.

If we assume that $N$ bilateral contracts are signed in the power system, the number of permutations will be equal to $N!$. It means that it is required to run $N \times N!$ load flow calculations in order to allocate the transmission losses because there are n states in each permutation and a load flow calculation is needed for each state. So the number of the required load flow calculations is equal to $N \times N!$, depicted in Table I.

It seems that the number of load flow analyses is the most important challenge and it must be solved.

Table I
The Number of Required Load Flow Calculations

| Number of Signed Bilateral Contracts | Number of required Load Flow analysis |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | 18 |
| 4 | 96 |
| 5 | 600 |
| 6 | 4320 |
| 7 | 35280 |

The main reason of this challenge is that a lot of states in the procedure are repeated. Another way to calculate the loss allocation of permutations is to analyze the system states before defining the permutations. It means that we can list the system states much less than $N \times N!$.

In detail, in a systems state, two conditions for each contract are possible because each bilateral contract may be available or unavailable; so, and .Therefore, there are only $2^N$ system states in a system with $N$ bilateral contracts. Therefore, because the number of system states equals the number of various combinations of signed bilateral contracts, $2^N$ load flow calculations should be analyzed to assess the transmission loss allocation of the system. A comparison between the numbers of the required load flow calculations based on two approaches is shown in Table II.

Table II
The Number of Required Load Flow Calculations

| Number of Signed Bilateral Contracts | Number of required load flow analysis | |
|---|---|---|
| | The First Approach | The Second Approach |
| 1 | 1 | 2 |
| 2 | 4 | 4 |
| 3 | 18 | 8 |
| 4 | 96 | 16 |
| 5 | 600 | 32 |
| 6 | 4320 | 64 |
| 7 | 35280 | 128 |

## III. METHODOLOGY

During investigations, a software package mixed of DIgSILENT and MATLAB has been developed to run load flow analyses and allocation process simultaneously, as described below.

### *A. Load Flow Calculations*

The developed software lists all possible commitment combinations of bilateral contracts i.e. different network states in load flow calculations assuming that all power transactions of power pool mechanism are already completely allocated to the market participants.

Then the combinations listed are applied to power network simulated in DIgSILENT software one after each other, the load flow analysis is run for each state and the network transmission loss $TAL(S_i)$ and $TRL(S_i)$ is recorded in a table the columns of which are shown in Table III.

Table III
Sample Table of load flow results of network states

| $S_i$ | $B_i$ | | | $TAL_i$ | $TRL_i$ |
|---|---|---|---|---|---|
| | $C_{3,i}$ | $C_{2,i}$ | $C_{1,i}$ | | |
| 0 | 0 | 0 | 0 | $TAL_0$ | $TRL_0$ |
| 1 | 0 | 0 | 1 | $TAL_1$ | $TRL_1$ |
| 2 | 0 | 1 | 0 | $TAL_2$ | $TRL_2$ |
| 3 | 0 | 1 | 1 | $TAL_3$ | $TRL_3$ |
| 4 | 1 | 0 | 0 | $TAL_4$ | $TRL_4$ |
| 5 | 1 | 0 | 1 | $TAL_5$ | $TRL_5$ |
| 6 | 1 | 1 | 0 | $TAL_6$ | $TRL_6$ |
| 7 | 1 | 1 | 1 | $TAL_7$ | $TRL_7$ |

The columns depicted in Table III have the following meanings:

$S_i$ shows the identification number of the power network states i.e. various combination of bilateral contracts.

$B_i$ indicates the binary value of $C_i$ an the value $S_i$ clarifies the commitment status of i-th bilateral contract.

$C_{n,i}$ illustrates whether or not the n-th bilateral contracts is committed in the i-th network state.

$TAL_i$ is defined as the active power loss of transmission system obtained from load flow analysis of i-th network state.

$TRL_i$ is also the reactive power loss of transmission system in the same situation.

Owing to the fact that transmission losses are not defined in DC load flow analyses, Only AC load flow analysis is capable to assess the transmission loss of power network. Therefore, DIgSILENT software is employed to run AC load flow analyses in this paper.

### B. Transmission Loss Allocation

After load flow data of all possible combination of bilateral contracts is obtained and sent to MATLAB, the software analyses them to compute transmission loss allocations. In order to discuss the algorithm it is required to review the fundamentals of Shapely value technique. Next, the algorithm used for implementation of Shapely Value Method is presented.

#### 1) Shapely Value Formulation

First, it is necessary to present a brief explanation of formulation of Shapely value technique [13].

Supposing that the number of bilateral contracts is equal to $N$, it is obvious that the set $S = \{c_1, c_2, \cdots, c_N\}$ which contains all contracts have $N$ members and $2^N$ subsets such that $2^{N-1}$ of the subsets contain the contract $c_i$. We name these subsets $S^i{}_k$ where $k = 1,2,\cdots,N-1$. That is, $S^i{}_k$ indicates the k-th subset of $S$ which encompasses $c_i$. The term $S^i$ is also used for the set of $S^i{}_k$ subsets.

So, the transmission loss allocated to the i-th contract, $TLA_i$, is defined as following:

$$TLA_i = \sum_{k=1}^{2^{N-1}} \frac{(N-|S^i{}_k|)!(|S^i{}_k|-1)!}{N!}\left[TL(S^i{}_k) - TL(S^{-i}{}_k)\right]$$

Where

$S^{-i}{}_k = S^i{}_k - \{c_i\}$, $|S^i{}_k|$ means the population of the set $S^i{}_k$ and $TL(S^i{}_k) - TL(S^{-i}{}_k)$ is increased transmission loss due to commitment of contract $c_i$ in load flow analysis.

#### 2) LFPAM Algorithm

The software introduced in this paper employs MATLAB through an algorithm to implement Shapely Value method. As illustrated in Fig. 1, In this part of the software designed, the contributions of each contact are computed in all permutations of bilateral contracts. Then the quantities are stored in a table which consists active and reactive loss contributions of each bilateral contract for all possible permutations as well as the parameter $|S^i{}_k|$ as stated in shapely value formulation. Finally, total allocation of each bilateral contract is easily calculated using the methodology of Shapely Value technique.

The Fig. 1 depicts the flowchart of LFPAM and the whole methodology used in the proposed software.

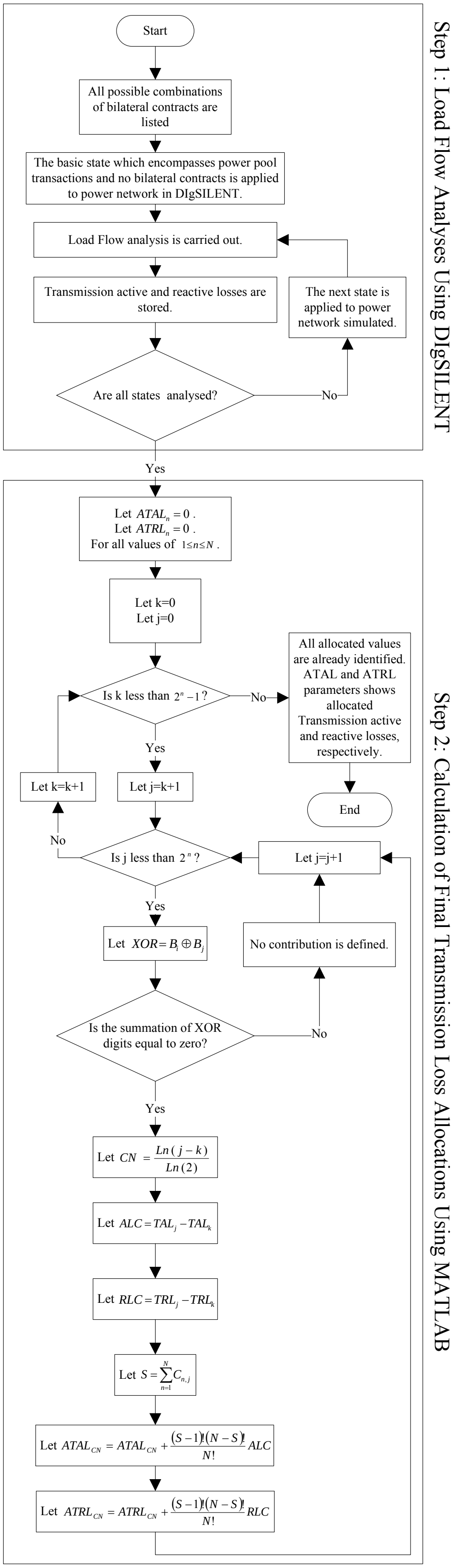

Fig. 1. Flowchart of load flow permutation average method.

The parameters introduced in Fig. 1 have the following meanings:

$ALC$ and $RLC$ show active and reactive loss contribution calculated in each permutation.

$CN$ is used to identify bilateral contract whose power loss contributions are computed.

$ATAL_i$ and $ATRL_i$ mean active and reactive transmission losses allocated to the i-th bilateral contract.

Besides, other parameters have the meaning of items defined previously.

## IV. NUMERICAL EXAMPLES

In order have some case studies, a small 6-bus, 3-generator system shown in Fig. 2 has been considered; further network details are also represented in the appendix. The system consists of two contracted loads and two loads under pool operation.

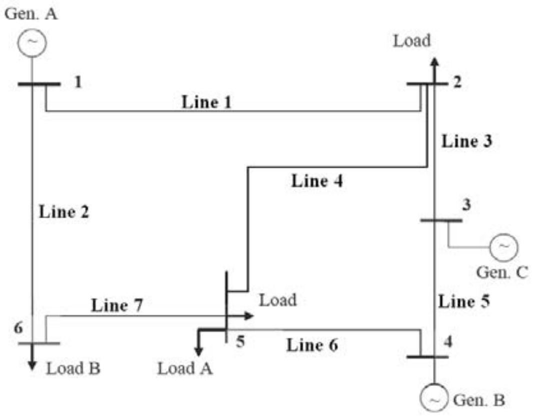

Fig. 2. Six bus system with two bilateral contracts

### A. Case Study 1

The specifications used for bilateral contracts in this case study are the parameters shown in Table IV.

Table IV
The Specifications of the Loads in Case Study 1

| Load Type | | Bus | Real Load (MW) | Reactive Load (MVAR) |
|---|---|---|---|---|
| Pool Operation | | 2 | 110 | 70 |
| | | 5 | 120 | 60 |
| Bilateral Contract | A | 5 | 80 | 35 |
| | B | 6 | 100 | 50 |

If we calculate the load flow of the basic state of the system without any bilateral contracts, the transmission losses of the system is equal to 1.56 MW and 7.56 MVAR. On the other hand, the transmission losses of the final state of the system with both contracts are 6.19 MW and 26.71 MVAR. Based on the load flow results, the transmission losses due to bilateral contracts are equal to 4.63MW and 19.15 MVAR and should be allocated to the bilateral contracts.

In the next stage, we should list all the permutations of the bilateral contracts. In the two bilateral contracts system, we can define two permutations as below.

Permutation 1:

State 1- Basic State (without any bilateral contract i.e. only power pool transactions)

State 2 - State A (only contract A is added)

State 3 - Final State (both contracts are added)

Permutation 2:

State 1- Basic State (without any bilateral contract)

State 2 - State B (only contract B is added)

State 3 - Final State (both contracts are added)

The results of two permutations and the final transmission loss allocation are shown in Table V. Furthermore LFPAM and Pro-Rata results are compared.

Table V
Transmission Loss Allocation Results of Case Study 1

| Transmission Loss Allocation | Contract A | | Contract B | |
|---|---|---|---|---|
| | MW | MVAR | MW | MVAR |
| Permutation 1 | 2.11 | 8.67 | 5.52 | 10.48 |
| Permutation 2 | 2.13 | 8.18 | 2.50 | 10.97 |
| LFPAM Results | 2.12 | 8.425 | 2.51 | 10.725 |
| | 45.79% | 43.99% | 54.21% | 56.01% |
| Pro-Rata Results | 2.06 | 7.89 | 2.57 | 11.26 |
| | 44.44% | 41.18% | 55.56% | 58.82% |

### B. Case Study 2

In the second case study, we assume other specifications for the bilateral contracts, illustrated in Table VI.

Table VI
The Specifications of the Loads in Case Study 2

| Load Type | | Bus | Real Load (MW) | Reactive Load (MVAR) |
|---|---|---|---|---|
| Pool Operation | | 2 | 110 | 70 |
| | | 5 | 120 | 60 |
| Bilateral Contract | A | 5 | 120 | 60 |
| | B | 6 | 130 | 85 |

The load flow calculation of the system final state shows that the transmission losses are 9.84MW and 41.48MVAR. Therefore, the bilateral contracts have caused the transmission losses equal to 8.28MW and 33.92MVAR. The Loss allocation of the system based on LFPAM and Pro-Rata are compared in Table VII.

Table VII
Transmission Loss Allocation Results of Case Study 2

| Transmission Loss Allocation | Contract A | | Contract B | |
|---|---|---|---|---|
| | MW | MVAR | MW | MVAR |
| Permutation 1 | 3.84 | 15.88 | 4.44 | 18.04 |
| Permutation 2 | 4.19 | 18.14 | 4.09 | 15.78 |
| LFPAM Results | 4.015 | 17.01 | 4.265 | 16.92 |
| | 48.49% | 50.13% | 51.51% | 49.87 |
| Pro-Rata Results | 3.97 | 14.04 | 4.31 | 19.88 |
| | 48.00% | 41.38% | 52.00% | 58.62% |

As it cleared from the simulations, the results of the LFPAM are easily computable using the methodology presented. Moreover, The results obtained depict a fair solution for transmission loss allocation o bilateral contracts according to the fact that LFPAM, as a novel method based on Shapely value technique, considers all possible permutations for load flow analyses with no discrimination between bilateral contracts.

## V. CONCLUSION

A novel algorithm "Load Flow Permutations Average Method" (LFPAM), which is based on load flow analyses and shapely value allocation technique, has been presented to assess the transmission loss allocation. The method makes effort to provide firm circumstances for bilateral contracts so that each contract is responsible just for its contribution in transmission loss. LFPAM allocates the transmission loss due to bilateral contracts by consideration of all possible permutations and run of the load flow calculations for each system state. Finally, evaluation of LFPAM has been done for two numerical case studies beside a comparison with other proposed method Pro-Rata. The results depict that, by using LFPAM for transmission loss allocation, fairness has been improved as well as simplicity. That is, Since the LFPAM considers all possible system states and employs the AC load flow analysis regarding to the effect of power network structure and the location of generators and loads, it has an acceptable accuracy to assess the transmission loss allocation fairly.

## VI. APPENDIX

The line data and generation capacity of the system are shown in Table VIII and Table IX respectively.

Table VIII
Transmission Lines Data of Power Network shown in

| Line Number | From Bus | To Bus | R (Ω) | X (Ω) |
|---|---|---|---|---|
| 1 | 1 | 2 | 2.0 | 10 |
| 2 | 1 | 6 | 3.0 | 11 |
| 3 | 2 | 3 | 0.5 | 4 |
| 4 | 2 | 5 | 2.0 | 8 |
| 5 | 3 | 4 | 0.5 | 3 |
| 6 | 4 | 5 | 2.0 | 8 |
| 7 | 5 | 6 | 1.5 | 6 |

Table IX
Generation Capacity of the Power Network shown in

| Generator | $P_{\min}(MW)$ | $P_{\max}(MW)$ |
|---|---|---|
| A | 60 | 270 |
| B | 70 | 220 |
| C | 40 | 150 |

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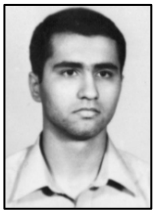
## VIII. BIOGRAPHIES



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