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An Investigation on Optimal Current Mode Control for Boost-Type PWM Rectifiers

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***Abstract*-- Static power converters produce harmonics due to the nature of the conversion process. This paper will investigate the effect on the power factor of a sinusoidal and Fryze current reference on a PWM boost type rectifier. It has been shown in previous papers that a Fryze current ref is best. This paper will develop a mathematical model, based on the single sided Fourier expansion. Using this mathematical model it will be shown that even though a high harmonic content could be present in a Fryze controlled PWM boost type rectifier, the power factor has been optimally improved.**

***Index Terms*-- Control Systems, harmonics, power electronics, reactive power, power factor.**

I. Introduction

Control of harmonic pollution in electrical systems by power converters may be as simple as adding a simple filter [1] or dynamically controlling the current drawn [2]. The control strategy of the power converter is usually developed for ideal balanced supply conditions. As a result a balanced sinusoidal reference may be used. The effectiveness of the method and others will be discussed and simulated. Fig.1 shows a three-phase PWM boost type rectifier.

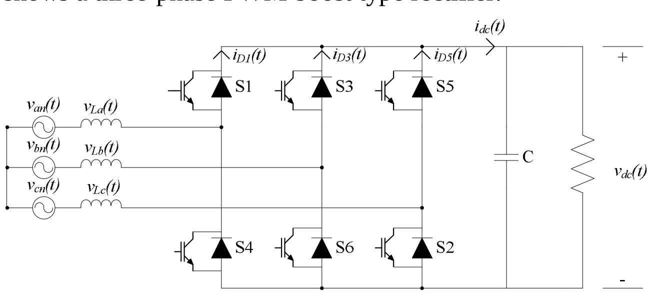

Fig. 1 PWM Boost rectifier

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II. Background

Each combination of IGBT and anti-parallel diode will be considered as a single switch (see Fig.1). If no switching occurs, the configuration of the anti-parallel diodes will function as an uncontrolled rectifier. The converter consists of three phase arms. The switch modules, S1 and S4, are connected to Phase A, switch modules S3 and S6, are connected to Phase B, and the switch modules S5 and S2, are connected to Phase C. It can be said that the converter consists of two halves. The upper half (S1, S3 and S5), which is connected to the positive rail and the lower half (S2, S4 and S6) is connected to the negative rail. The positive rail is more positive, with reference to ground and the negative rail has a negative potential with reference to the ground plane. Each switch module can conduct current in either direction but block voltage in one direction only. This converter is a two-quadrant converter.

Following the methods of [3] and [4]; to simplify the notation let the switches connected to phase A be designated S_a, the switches connected to phase B be designated S_b and the switches connected to phase C be designated S_c. The state of each switch may be further expressed in terms of a binary value, either a ‘1’ or a ‘0’. Then if S_a=1 the upper switch (i.e. S1) is conducting and if S_a=0 (i.e. S4) then the lower switch is conducting. S_b and S_c keep to the same condition.

From three phase circuit theory the following may be written for a balanced system:

$$v_{an}(t) + v_{bn}(t) + v_{cn}(t) = 0 \tag{1}$$

and

$$i_a(t) + i_b(t) + i_c(t) = 0 \tag{2}$$

The following equations may be written for the three phase circuit in Fig. 1:

Phase A:

$$L\frac{di_a(t)}{dt} + Ri_a(t) = v_{an}(t) + \frac{v_{dc}(t)}{3}(S_b - 2S_a + S_c) \tag{3}$$

Phase B:

$$L\frac{di_b(t)}{dt} + Ri_b(t) = v_{bn}(t) + \frac{v_{dc}(t)}{3}(S_c - 2S_b + S_a) \tag{4}$$

Phase C:

$$L\frac{di_c(t)}{dt}+Ri_c(t)=v_{cn}(t)+\frac{v_{dc}(t)}{3}(S_a-2S_c+S_b) \tag{5}$$

The inference of the binary definition of each bi-directional switch is that only one switch in each phase arm will conduct.

III. Control System

The control system must furnish a control signal to facilitate the switching of the converter to draw a near sinusoidal current at unity power factor as well as maintaining a constant DC supply. A potential control method is direct current control [2].

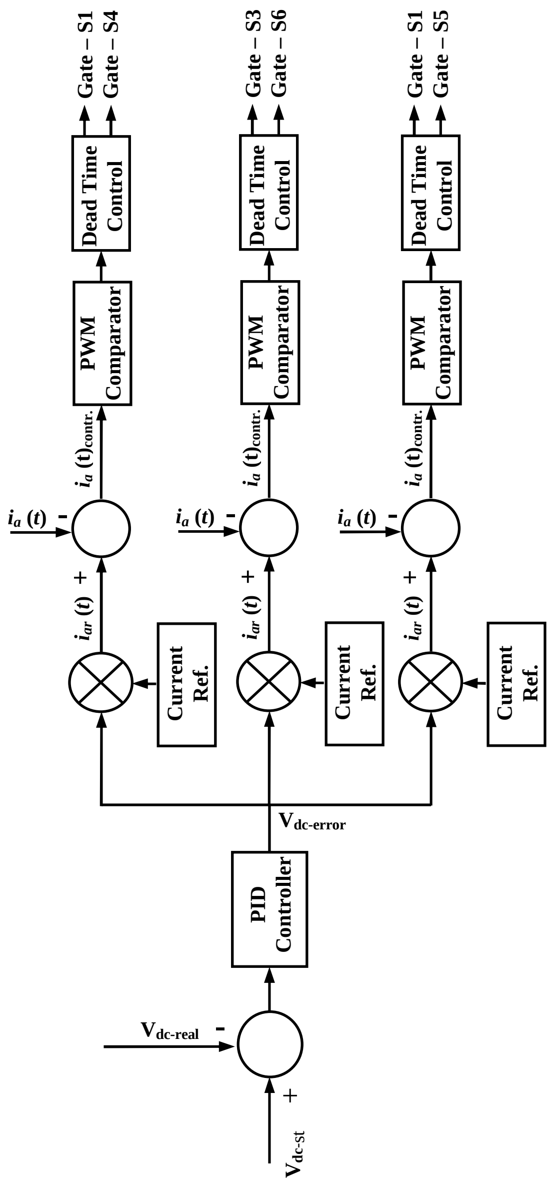

Fig. 2 Proposed Control

This method allows the amplitude and phase of the input current to be accurately controlled. This is done by measuring the instantaneous phase currents and forcing them to follow a predetermined template. This template may be either the Fryze current reference or a sinusoidal reference. Current mode control can be implemented using either hysteresis or PWM.

From the equations (3), (4) and (5), each phase supply will see an effective DC voltage of $\pm(2/3)v_{dc}(t)$, $\pm(1/3)v_{dc}(t)$, or 0 depending on the switching pattern.

According to [3] if a hysteresis tolerance band controller is used the zero state gives no control of the line currents and periodically causes the line currents to exceed the hysteresis limit by a factor of 2. This zero state can cause unnecessary stress on the switching devices.

The switching frequency, which varies continuously, may be determined by considering (3). For simplicity the source and line resistance will be ignored. Eq. (3) can be re-arranged as:

$$\Delta t=\frac{L\Delta i_a(t)}{v_{an}(t)-V_{dc}(t)} \tag{6}$$

The switching frequency is dependent on the line inductance and the DC load voltage. The switching frequency is not constant and varies over a wide range during each half cycle of ac-input voltage. This varying switching frequency results in varying high-order current harmonics which can be difficult to filter from the line current. A PWM control method is thus more desirable. By operating the converter at a fixed switching frequency, the high order harmonics can easily be filtered out. The proposed control method will employ a PWM method, having a fast response time and eliminating the problem of a variable switching frequency. A PWM switching scheme does generate fixed high order harmonics, which may be filtered from the line, if required.

The object is to provide a constant DC voltage. In a closed loop control system, feedback is required to adjust the system parameters to bring the output in line with the desired set point. The difference between the set point and the actual value is termed the error. The difference between '$\mathbf{V_{dc\text{-}Real}}$' and '$\mathbf{V_{dc\text{-}Set}}$' provides the difference signal. This difference is passed through a PI controller to minimize the error as rapidly as possible with little or no overshoot. This signal is termed the '$\mathbf{V_{dc\text{-}error}}$'. The signal conditioned by the PI controller may be expressed in the complex s plane as:

$$(V_{dc-set}-V_{dc-real})\cdot(K_p+\frac{K_i}{s})=V_{dc-error} \tag{7}$$

This signal (7) is multiplied against a current reference template, to yield the current error, $i_{ae}(t)$. This value is subtracted from the actual current value to yield $i(t)_{control}$ which is compared against a triangular signal at a fixed frequency to produce a PWM signal.

Briefly, when the DC load increases, the capacitor voltage decreases, increasing the **'$V_{dc\text{-}error}$'** signal. This increases the current error signal $i(t)_{control}$ increasing the input current drawn by the converter. A larger current injects more power to the DC capacitor, increasing the DC voltage. If the DC voltage is too large the procedure will be reversed.

IV. Harmonic definition using a single sided Fourier expansion

When a linear circuit is subjected to a forcing function, the complete circuit response will consist of a transient response and a forced response. Steady state linear AC circuit theory is derived from the forced response of circuits when subjected to a sinusoidal forcing function. This concept may be expanded to accommodate definitions for circuits subjected to multi-frequency forcing functions i.e. distortion.

For a sinusoidal single-frequency system, *v* and *i* are both time dependant functions and may be written as:

$$v(t) = \sqrt{2}V\cos(\omega t + \alpha) \tag{8}$$

$$i(t) = \sqrt{2}I\cos(\omega t + \beta) \tag{9}$$

Equations (6) and (7) may be written in a generalized complex function:

$$v(t) = \sqrt{\mathbf{2}}V\mathrm{e}^{\mathrm{j}\alpha}.\mathrm{e}^{\mathrm{j}\omega t} \tag{10}$$

$$i(t) = \sqrt{\mathbf{2}}I\mathrm{e}^{\mathrm{j}\beta}.\mathrm{e}^{\mathrm{j}\omega t} \tag{11}$$

The complex function is now expressed in two separate parts. The first is a complex constant and the second is a function of time that implies rotation in the complex plane. The complex phasor $\boldsymbol{V} = \sqrt{2}Ve^{j\alpha}$ is termed the transform of $v(t)$. The same is valid for (11). Multi-frequency systems imply non-linearity and do not lend themselves to the definition of a phasor. If the multi-frequency forcing function is periodic then the Fourier analysis produces discrete responses in the frequency domain. Each of these responses is sinusoidal with unique phase and magnitude. Each of these discrete responses may be defined with a phasor. The single sided complex Fourier series of a distorted periodic voltage or current waveform may be expressed as:

$$\boldsymbol{v}(t) = \sum_{n=0}^{\infty}\sqrt{2}V_n\mathrm{e}^{\mathrm{j}n\omega t} \tag{12}$$

$$\boldsymbol{i}(t) = \sum_{n=0}^{\infty}\sqrt{2}I_n\mathrm{e}^{\mathrm{j}n\omega t} \tag{13}$$

A distorted voltage can be analyzed with Fourier to obtain the discrete sinusoidal components. The composite voltage or current profile may be obtained by summing the individual harmonic time dependent components.

$$\boldsymbol{v}(t) = V_1\,\mathbf{sin}\,\omega t + \sum_{n=3,5,\dots}^{\infty} V_n\,\mathbf{sin}\left(n\omega t + \alpha_n\right) \tag{14}$$

$$\boldsymbol{i}(t) = \sum_{n=1,3,5}^{\infty} I_n\left[\mathbf{sin}\left(n\omega t + \beta_n\right) + \mathbf{cos}\left(n\omega t + \beta_n\right)\right] \tag{15}$$

The IEEE STD defines distortion factor (sometimes referred to as harmonic factor) as: The ratio of the root mean square of the harmonic content to the root mean square value of the fundamental quantity, expressed as a percent of the fundamental. *Total Harmonic Distortion* (THD) is a term which has come in to common usage to define either voltage or current 'distortion factor' [5]. The IEEE STD defines distortion factor (*DF*) and total harmonic distortion as one and the same. In other words it is a measure of the closeness in shape between a waveform and its fundamental component:

$$DF = \sqrt{\frac{\sum_{n=2}^{k}|V_n|^2}{|V_1|^2}} \tag{16}$$

Individual harmonic distortion is a measure of the contribution of an individual harmonic frequency contribution to the distortion and may be defined as

$$IHF_{\mathrm{n}} = \frac{|V_n|}{|V_1|} \tag{17}$$

Power factor is the measure of how effectively a load draws real power. Power factor is a dimensionless quantity. A power factor of 1.0 is ideal as the imaginary power component is zero. As defined by the IEEE STD, power factor is the ratio of the total power input, in watt, to the total Volt-Ampere input.

Displacement power factor is the ratio of the active power of the fundamental, in watt, to the apparent power of the fundamental.

V. Reference Current

The PWM boost rectifier is design to generally draw a sinusoidal current and in phase with the input voltage. When selecting a current reference for current mode control of the boost-type PWM rectifiers, it could be either a sinusoidal (18) or Fryze type (19):

$$i_{ref-s}(t) = K_iV_1\,\mathbf{sin}\left(\omega t\right) \tag{18}$$

$$i_{ref-F}(t) = K_i \sum_{n=1,3,5,...}^{\infty} V_n \sin\left(n\omega t + \varphi_n\right) \quad (19)$$

where K_i is a conversion constant.

The choice for the reference current between sinusoidal or Fryze is further analyzed using the power factor (PF_s) defined as the ratio of active power (P) over apparent power (S).

The active power in case of distorted input voltage (14) with sinusoidal reference (18) can be written as:

$$P_s = \frac{1}{2\pi}\int_0^{2\pi}\left[\sum_{n=1,3,..}^{\infty} V_n \sin\left(n\omega t + \varphi_n\right)\right]\cdot\left[K_i V_1 \sin\left(\omega t\right)\right] d\left(\omega t\right) \quad (20)$$

After proper mathematical manipulation, the active power is:

$$P_s = K_i \frac{V_1^2}{2} \sum_{n=1,3,..}^{\infty} \frac{V_n}{V_1} \quad (21)$$

and the power factor becomes:

$$PF_s = \frac{P_s}{V_{rms} I_{rms}} = \frac{V_{1-rms}^2}{V_{rms}^2} \sum_{n=1,3,..}^{\infty} \frac{V_n}{V_1} \quad (22)$$

Further, the active power in case of distorted input voltage (14) with Fryze reference (19) can be written as:

$$P_F = \frac{1}{2\pi}\int_0^{2\pi}\left\{\begin{matrix}\left[\sum_{n=1,3,5,...}^{\infty} V_n \sin\left(n\omega t + \varphi_n\right)\right] \\ \left[K_i \sum_{n=1,3,5,...}^{\infty} V_n \sin\left(n\omega t + \varphi_n\right)\right]\end{matrix}\right\} d\left(\omega t\right) \quad (23)$$

$$P_F = K_i \sum_{n=1,3,...}^{\infty} \left(V_n^2 / 2\right) = K_i V_{rms}^2 \quad (24)$$

The current drawn using Fryze reference has the rms value different from the supply voltage by the same constant K_i and:

$$PF_F = \frac{P_F}{V_{rms} I_{rms}} = \frac{K_i V_{rms}^2}{K_i V_{rms}^2} = 1 \quad (25)$$

It can be noticed that the power factor when using Fryze reference is unity, while for a sinusoidal reference current it depends on the distortion of the supply voltage.

VI. Simulation Results

A boost-type PWM rectifier and the proposed control method were simulated in PSIM ver. 7.1. The supply was unbalanced and distorted. In the simulation, the amplitude unbalance and phase displacement values of the supply were:

$$\begin{bmatrix} v_r \\ v_w \\ v_b \end{bmatrix} = \begin{bmatrix} 310\cos(\omega t + 0^\circ) \\ 310\cos(\omega t + 115^\circ) \\ 300\cos(\omega t - 135^\circ) \end{bmatrix} \quad (26)$$

In the first instance a Fryze current reference was used. This is where the real, instantaneous magnitude and phase displacement of the phase voltage is used as a template for the current as mentioned before.

Figure 3 shows the simulated distorted phase voltage of the blue phase. The voltage notching is clearly visible. This voltage profile will form the template for the phase current in the blue phase i.e. Fryze Current reference. In a similar fashion the red phase voltage profile and the white phase voltage profile will be the red and white phase currents templates respectively.

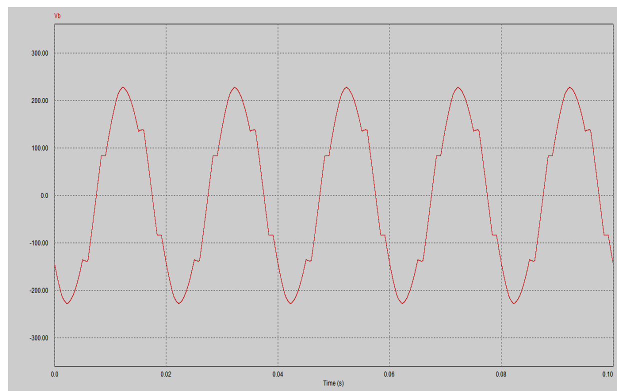

Fig. 3 Distorted Phase Voltage

Figure 4 shows the phase currents as a result of the phase voltage templates. As a result, each phase current will be in phase with the respective phase voltage, improving the power factor. Figure 5 shows the magnified profile of the current and voltage profile; the power factor is improved to 0.998, despite the voltage distortion. However the trade-off is that the phase current will have a relative high content of harmonics. The FFT of the simulated phase current is shown in figure 6.

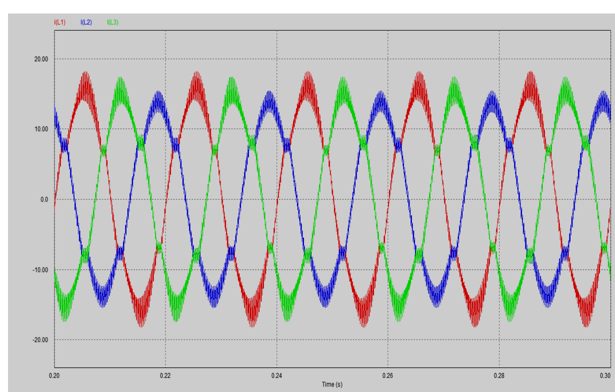

Fig. 4 Phase Currents

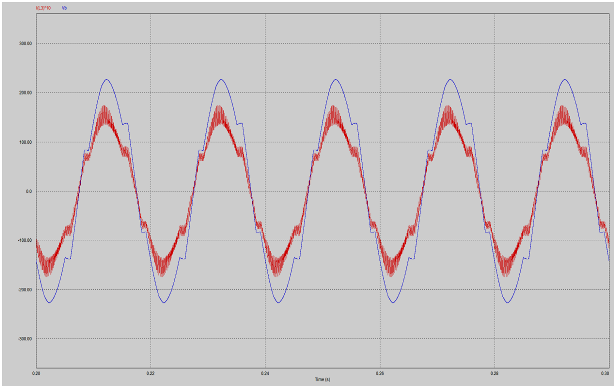

Fig. 5 Magnified current with Voltage profile

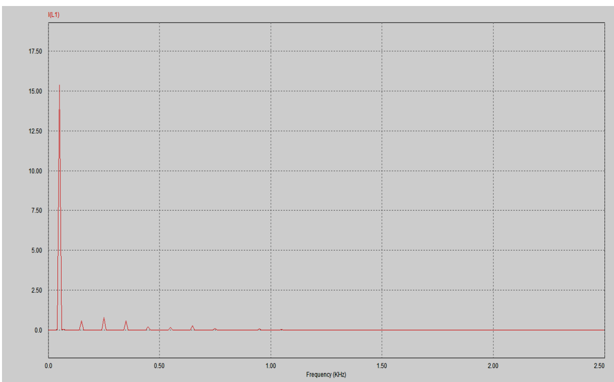

Fig. 6 FFT analysis of the Fryze Current

If the current reference in the control loop is replaced with a balanced three phase sinusoidal supply, the simulation results are as follows:

The phase current, as expected, will be sinusoidal in nature with limited harmonic content (figure 7); however the power factor of 0.98 is a little bit lower than for the sinusoidal reference. Figure 8 shows the current FFT of the phase current.

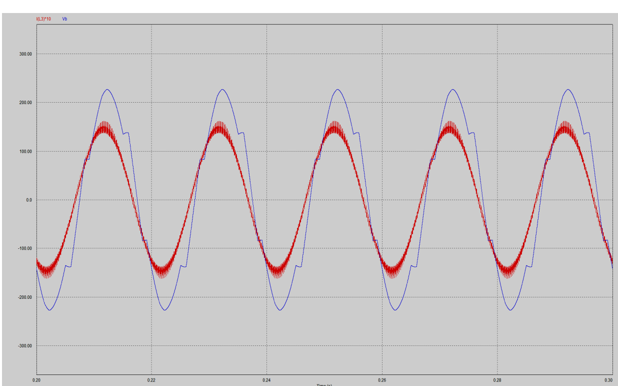

Fig. 7 Phase current using a sinusoidal current reference

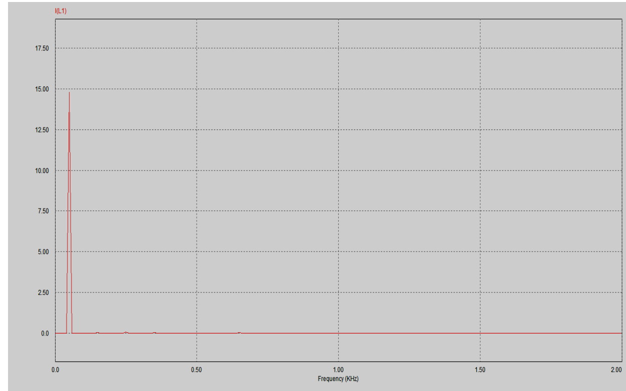

Figure 8 FFT of the Phase Current

VII. Conclusions

When selecting a current reference for current mode control for boost-type PWM rectifiers, the ideal current reference should be the Fryze current reference. With a Fryze current reference the power factor will have a higher value than any other method as per IEEE definition of power factor. It is still not perfect unity because of the effect of switching frequency which can be seen as noise. A current which is proportional to the input voltage (Fryze reference) is better for the individual load and the power supply; it practically creates a “linear/resistive load”. However, the harmonic content is higher than when using a sinusoidal reference.

Using the balanced sinusoidal reference, the current profile will be sinusoidal in nature, but the power factor is not optimally improved. A poor factor will result in in-efficient use of the electrical capacity of the electrical device.

VIII. References

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