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A Comparative Analysis of *FBD*, *PQ* and *CPT* Current Decompositions – Part I: Three-Phase Three-Wire Systems

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Abstract--**This paper investigates the main similarities and discrepancies among three important current decompositions proposed for the consideration of unbalanced and/or non linear three-phase three-wire power circuits. The considered approaches were the so called FBD Theory, the pq-Theory and the Conservative Power Theory (CPT), recently presented by Tenti *et al.*. Such decompositions and related definitions may influence the power measurement techniques, revenue metering, instrumentation technology and also power conditioning strategies. The three methods have been summarized, discussed and compared by means of computational simulation. Although the three methods are based on different concepts, the results obtained under ideal conditions are very similar. The main differences appear in the presence of unbalanced and non linear load conditions. Under linear unbalanced conditions, both FBD and pq-Theory suggest that the some current components contain a third-order harmonic. Besides, neither pq-Theory nor FBD method are able to provide accurate information for reactive current under unbalanced and distorted conditions, what can be done by means of the CPT-Theory. The paper tries to explain the causes of these differences in terms of the decomposition's foundations and the resulting waveforms and spectra.**

Index Terms-- **FBD theory, pq-Theory, Homo-variables, harmonics, current decomposition, power theory.**

I. Introduction

THE worldwide search for a generalized power theory, applicable for power systems under non-sinusoidal and/or unbalanced conditions, has mostly been motivated by the ever increasing of power electronic converters utilization. It point out the major requirement of improvement and adaptation of reactive/harmonic compensators technology and revenue metering techniques under such conditions.

In this sense, numerous new power theories have been defined, and several of them are based on the frequency domain to describe suitable power and current terms under non-sinusoidal and unbalanced conditions [1-7]. On the other hand, giving special emphasis to instantaneous quantities, other important current decompositions and power definitions have been presented [8-17].

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However, despite of the enormous efforts already spent, there is still no complete agreement on several current decompositions and related power definitions. Most of the misunderstanding is probably caused since several authors usually had presented their contributions directed to a specific application, instead of discussing a general applicable power theory [18].

Thus, considering just the time domain approaches, one could call attention to the proposals of Depenbrock (FBD) [10,11], Akagi *et al.* (*pq*-Theory) [12,13] and Tenti *et al.* (CPT) [14,15], which are strongly related to power conditioning applications.

Given that the *pq*-Theory is very well-known and accepted by the power electronics community, some authors tend to consider it a theoretical tool not only for active filter control [16,19-21], but also for power properties' definitions and understanding [22,23], regardless of all the misunderstanding about physical phenomena under non sinusoidal and unbalanced conditions [18,31-34].

Willems [24] had already verified that the *pq*-Theory faces some conceptual problems. More recently, Depenbrock *et al.* [25] have investigated the original and modified pq-Theory for three-phase *four-wire* systems, but it was Czarnecki [26,27], who investigated how the properties of three-phase systems are described by means of the *pq*-Theory and discuss why such theory should not be used for understanding the power properties of the load instantaneously.

Considering three-phase three-wire circuits, this paper will demonstrate that the FBD method exactly matches the *pq*-Theory, in case of sinusoidal and balanced PCC (Point of Common Coupling) voltages, and both can jeopardize the power properties' analysis under unbalanced or non linear loads. These two theories are compared to the proposal of Tenti *et al*. and it will be demonstrated that this last one can be very consistent and useful for current decompositions and power phenomena explanation.

Thus, next sections present a concise review of the investigated power theories and related current decompositions. Then, simulation results for three main cases are discussed and compared in order to point out their major similarities and discrepancies. The discussions are not directed from the point of view of their applications, but as current decompositions for power properties elucidation.

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II. THE FBD THEORY

The FBD (Fryze-Buchholz-Depenbrock) method is an extension of the Fryze [8] and Buchholz [9] theories, on which Depenbrock makes use of Fryze's current decomposition and Buchholz's instantaneous and RMS collective values for the definition of new current decompositions. Such currents were also applied to the calculation of novel power components and for the proposition of compensation strategies [10,11].

With the correct considerations, the FBD method can be applied in any multiphase power circuit, which can be represented by a uniform circuit on which none of the conductors is treated as an especial conductor. In this uniform circuit, the voltages in the m-terminals are referred to a virtual star point "*". The single prerequisite is that Kirchhoff's laws must be valid for the voltages and currents at the terminals [11]. Here, it is important to point out that the measured voltages to the virtual star point may not represent the RMS or the instantaneous values of the voltages over the load terminals [28], especially under unbalanced conditions.

Considering multiphase power circuits, the FBD-method uses multidimensional voltage and current vectors ($\boldsymbol{v}_*, \boldsymbol{i}$) and their instantaneous collective values (v_Σ, i_Σ), defined respectively as:

$$\boldsymbol{v}_* = \begin{bmatrix} v_{a*} \\ v_{b*} \\ v_{c*} \\ \vdots \\ v_{m*} \end{bmatrix} \quad ; \qquad \boldsymbol{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \\ \vdots \\ i_m \end{bmatrix} \tag{1}$$

$$i_\Sigma = \sqrt{\sum_{\mu=1}^{m} i_\mu^2} \quad ; \qquad v_\Sigma = \sqrt{\sum_{\mu=1}^{m} v_{\mu*}^2}, \tag{2}$$

where, "m" indicates the number of conductors (wires) and bold letters indicate vector representation. Thus, the collective instantaneous power results from the inner (dot) product:

$$p_\Sigma = \boldsymbol{v}_* \cdot \boldsymbol{i} \tag{3}$$

Under periodic conditions the collective RMS value of the currents and of the voltages can be calculated as:

$$I_\Sigma = \sqrt{\frac{1}{T}\int_0^T i_\Sigma^2 dt} \quad ; \qquad V_\Sigma = \sqrt{\frac{1}{T}\int_0^T v_\Sigma^2 dt}\,, \tag{4}$$

and the collective "active" power results from:

$$P_\Sigma = \frac{1}{T}\int_0^T p_\Sigma dt. \tag{5}$$

Thus, according FBD method, the instantaneous current through each phase of the system (i_μ) is decomposed in some components, proportional and orthogonal to the voltages:

Active currents ($i_{a\mu}$): responsible for the transference of average energy to the load. This current was introduced by Fryze [8] for single-phase circuit and expanded for Buchhloz [9] to polyphase circuit. These definitions are generally valid under periodical condition and $i_{a\mu}$ is responsible for the same active power as the current i_μ.

$$i_{a\mu} = \frac{P_\Sigma}{V_\Sigma^2} v_{\mu*} = G_a v_{\mu*}. \tag{6}$$

Nonactive currents ($i_{n\mu}$): associated to any type of disturbance and oscillations that affect the instantaneous power, but do not transfer average energy to the load.

$$i_{n\mu} = i_\mu - i_{a\mu}. \tag{7}$$

Power currents ($i_{p\mu}$): responsible for the instantaneous power, including possible oscillations related with harmonic and unbalances.

$$i_{p\mu} = \frac{p_\Sigma}{v_\Sigma^2} v_{\mu*} = G_p v_{\mu*}. \tag{8}$$

Powerless currents ($i_{z\mu}$): they do not contribute for the energy conveyance and can be compensated instantaneously without the necessity of energy storage elements.

$$i_{z\mu} = i_\mu - i_{p\mu}. \tag{9}$$

Variation currents ($i_{v\mu}$): responsible for the oscillation of the instantaneous equivalent conductance G_p around its average value G_a, or also, variations of p_Σ around P_Σ.

$$i_{v\mu} = i_{p\mu} - i_{a\mu} = i_{n\mu} - i_{z\mu}. \tag{10}$$

Thus, the orthogonal currents decompositions proposed by FBD results that:

$$\|i_\mu\|^2 = \|i_{a\mu}\|^2 + \|i_{n\mu}\|^2 = \|i_{a\mu}\|^2 + \|i_{v\mu}\|^2 + \|i_{z\mu}\|^2\,. \tag{11}$$

III. THE PQ-THEORY

The instantaneous power theory proposed by Akagi *et al.* [12,] is usually known as *pq*-Theory. This theory is based on the Clarke transformation of voltages and currents in three-phase systems (*a,b,c*) into (α,β,0) orthogonal coordinates.

The *pq*-Theory describes the power properties of three-phase three-wire systems by means of two main instantaneous power components: the *instantaneous real power p,* and the *instantaneous imaginary power q.* The proposal could also be applied for three-phase four-wire system by introducing the instantaneous zero-sequence power p_0 [13,22]. This theory was originally proposed as a mathematical tool, specially directed for the control of active power filters and this was one of the most important motivations for its great dissemination during the last two decades. It is worth mention that Akagi *et al.* spread the concept of reactive compensation without energy storage elements.

Thus, using the Clarke's Transformation, the phase voltages in the α, β and *0* coordinates has the form:

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \boldsymbol{C_1} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{12}$$

Similarly, the instantaneous currents (i_a, i_b and i_c) can be transformed to the α, β and *0* coordinates. Note that in the case of three-phase three-wire systems, the measured phase voltages are referred to a virtual star point (as in the FBD) and with four-wire systems, the voltages are referred to the return conductor.

Therefore, considering three or four-wire circuits, the authors define three instantaneous power components: the **zero-sequence power** "p_0" (just in case of four-wire system), the **real power** "p" and the **imaginary power** "q" as:

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} v_0 & 0 & 0 \\ 0 & v_\alpha & v_\beta \\ 0 & v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}. \tag{13}$$

The sum of p_0 and p results in the **traditional instantaneous power** of three-phase systems:

$$p_{3\phi} = p + p_0 = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 \ , \tag{14}$$

which can be simplified in case of three-wire systems as:

$$p_{3\phi} = p = v_\alpha i_\alpha + v_\beta i_\beta \tag{15}$$

Then, the authors introduced the concept of the **instantaneous imaginary power** "q", defined as:

$$q = v_\beta i_\alpha - v_\alpha i_\beta. \tag{16}$$

Using these two instantaneous power, p and q, the orthogonal currents i_α and i_β can be decomposed into **instantaneous active** ($i_{\alpha p}$ and $i_{\beta p}$) and **reactive** ($i_{\alpha q}$ and $i_{\beta q}$) **currents**, as follows:

$$i_{\alpha p} = \frac{v_\alpha}{v_{\alpha\beta}^2} p \ ; \ i_{\beta p} = \frac{v_\beta}{v_{\alpha\beta}^2} p, \tag{17}$$

$$i_{\alpha q} = \frac{v_\beta}{v_{\alpha\beta}^2} q \ ; \ i_{\beta q} = -\frac{v_\alpha}{v_{\alpha\beta}^2} q, \tag{18}$$

where $v_{\alpha\beta}^2 = v_\alpha^2 + v_\beta^2$.

Accordingly, the instantaneous zero-phase sequence, active and reactive currents can be calculated in the original coordinates, by means of the inverse Clarke transformation:

$$\begin{gathered} \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_0 \\ 0 \\ 0 \end{bmatrix} = \mathbf{C_2} \begin{bmatrix} i_0 \\ 0 \\ 0 \end{bmatrix}, \\ \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \mathbf{C_2} \begin{bmatrix} 0 \\ i_{\alpha p} \\ i_{\beta p} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} = \mathbf{C_2} \begin{bmatrix} 0 \\ i_{\alpha q} \\ i_{\beta q} \end{bmatrix}. \end{gathered} \tag{19}$$

So, the instantaneous three-phase currents (a, b and c) can be decomposed on the following components:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} + \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} + \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix}. \tag{20}$$

According to [17], the powers of (13) could also be decomposed into:

$$p = \bar{p} + \tilde{p}, \quad q = \bar{q} + \tilde{q}, \tag{21}$$

where $\bar{p}$ and $\tilde{p}$ represent the average and oscillating component of p; and where $\bar{q}$ and $\tilde{q}$ represent the average and oscillating component of q. It was assumed that the oscillating parts of p and q are related to the occurrence of unbalanced and/or distorted voltages and currents.

Consequently, the instantaneous active current can also be decomposed into **average** ($\bar{x}$) and **oscillating** ($\tilde{x}$) **component**.

$$\begin{gathered} i_{\alpha p} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} \bar{p} + \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} \tilde{p} = i_{\alpha \bar{p}} + i_{\alpha \tilde{p}} \ ; \\ i_{\beta p} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \bar{p} + \frac{v_\beta}{v_\alpha^2 + v_\beta^2} \tilde{p} = i_{\beta \bar{p}} + i_{\beta \tilde{p}} \ . \end{gathered} \tag{22}$$

Resulting in the (*a,b,c*) coordinates:

$$\begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} = \mathbf{C_2} \begin{bmatrix} 0 \\ i_{\alpha \bar{p}} \\ i_{\beta \bar{p}} \end{bmatrix} + \mathbf{C_2} \begin{bmatrix} 0 \\ i_{\alpha \tilde{p}} \\ i_{\beta \tilde{p}} \end{bmatrix} = \begin{bmatrix} i_{a \bar{p}} \\ i_{b \bar{p}} \\ i_{c \bar{p}} \end{bmatrix} + \begin{bmatrix} i_{a \tilde{p}} \\ i_{b \tilde{p}} \\ i_{c \tilde{p}} \end{bmatrix} \ . \tag{23}$$

Finally, the instantaneous phase currents yield:

$$\begin{gathered} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} + \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} + \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} = \\ \begin{bmatrix} i_{a\bar{p}} \\ i_{b\bar{p}} \\ i_{c\bar{p}} \end{bmatrix} + \begin{bmatrix} i_{a\tilde{p}} \\ i_{b\tilde{p}} \\ i_{c\tilde{p}} \end{bmatrix} + \begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix} + \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix}. \end{gathered} \tag{24}$$

However, neither the interpretation of these current components, nor their association with specific physical phenomena was directly treated by the authors of the *pq*-Theory.

IV. The CPT Framework

The third considered approach (Conservative Power Theory, CPT) was recently proposed by Tenti *et al.* [14] and it is based on the definition of instantaneous complex power under non-sinusoidal conditions and it represents an extension of the usual complex power, defined for sinusoidal conditions. Even though detailed discussion has been directed to single-phase systems, this theory is also easily extended to multiphase systems [29].

The authors had introduced the so-called homo-variables (integral and derivate) which can be defined under periodic conditions and are homogeneous to the current, voltage and power terms. Since homo-voltages and homo-currents satisfy the Kirchhoff's Laws, the corresponding homo-powers are conservative in any electric network, what allows introducing the concept of conservation of the complex power under non-sinusoidal conditions. In addition, a current decomposition was proposed, on which every term is related to a specific physical phenomenon (power absorption P, energy storage Q, voltage and current distortion D). Moreover, it has been discussed its application to harmonic and reactive compensation, for local or distributed devices [15,30].

Assuming multidimensional systems, the following definitions make use of the same symbols applied to the FBD method, it means, bold variables to vector representation and the index "μ" for each m-phase variable. Regarding to the voltage referential, the authors suggest using the return conductor in case of its existence and the virtual point unless [29]. Thus, the homo-integrals of the voltages and currents are defined as:

$$\begin{gathered} \hat{v}_\mu(t) = \omega(v_{\mu_\int}(t) - \bar{v}_{\mu_\int}) \\ \hat{\imath}_\mu(t) = \omega(i_{\mu_\int}(t) - \bar{\imath}_{\mu_\int}) \end{gathered} \tag{25}$$

where: $v_\int(t) = \int_0^T v(\tau)\,d\tau,\ i_\int(t) = \int_0^T i(\tau)\,d\tau$, are time integral of voltages v_μ and currents i_μ, and $\bar{v}_\int$, $\bar{\imath}_\int$ are the average value of each $v_\int$ and $i_\int$, over period *T*.

Note that $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{\imath}}$ are dimensionally homogeneous to voltage and current respectively. This means that the operation of integration does not influence the amplitude of the resultant signals, since they are multiplied by the angular frequency.

Likewise, the homo-derivatives of the voltages and currents are given by:

$$\check{v}_\mu(t) = \frac{1}{\omega}\frac{dv_\mu(t)}{dt}; \qquad \check{\imath}_\mu(t) = \frac{1}{\omega}\frac{di_\mu(t)}{dt} \tag{26}$$

As well as $\hat{\boldsymbol{v}}$ and $\hat{\boldsymbol{\imath}}$ the variables $\breve{\boldsymbol{u}}$ and $\breve{\boldsymbol{\imath}}$ are also dimensionally homogeneous to the original voltages and currents, respectively. In this case the derivative is multiplied by the inverse of the angular frequency.

In addition, the authors have demonstrated some important properties of the defined homo-variables. Considering periodical quantities, with period T and fundamental angular frequency $\omega = 2\pi/T$, it is well-known that the internal product of voltage and current vectors is defined as:

$$\langle \boldsymbol{v}, \boldsymbol{i} \rangle = \langle \boldsymbol{i}, \boldsymbol{v} \rangle = \frac{1}{T}\int_0^T \boldsymbol{v}(t).\boldsymbol{i}(t)dt \tag{27}$$

and in the same way, the voltage and current norms are:

$$\|\boldsymbol{v}\| = \sqrt{\langle \boldsymbol{v}, \boldsymbol{v} \rangle}\,, \quad \|\boldsymbol{i}\| = \sqrt{\langle \boldsymbol{i}, \boldsymbol{i} \rangle}\,, \tag{28}$$

which results equal to the RMS collective values from the FBD method (V_Σ, I_Σ). For single-phase circuits or considering each phase of an m-dimensional system:

$$\|v_\mu\| = \sqrt{\frac{1}{T}\int_0^T v_\mu^2\,(t)\,dt} = V_\mu\ ,\quad \|i_\mu\| = \sqrt{\frac{1}{T}\int_0^T i_\mu^2(t)dt} = I_\mu\,,$$

yields the RMS phase voltages and currents.

The homo-variables defined in (25) and (26) have the following properties:

$$\begin{gathered} \boldsymbol{v} = \breve{\boldsymbol{\imath}} \Leftrightarrow \hat{\boldsymbol{v}} = \boldsymbol{i}; \qquad \boldsymbol{v} = \hat{\boldsymbol{\imath}} \Leftrightarrow \breve{\boldsymbol{v}} = \boldsymbol{i} \\ \langle \hat{\boldsymbol{v}}, \breve{\boldsymbol{v}} \rangle = -\|\boldsymbol{v}\|^2; \qquad \langle \hat{\boldsymbol{\imath}}, \breve{\boldsymbol{\imath}} \rangle = -\|\boldsymbol{i}\|^2 \\ \langle \boldsymbol{v}, \hat{\boldsymbol{v}} \rangle = \langle \boldsymbol{v}, \breve{\boldsymbol{v}} \rangle = 0 = \langle \boldsymbol{i}, \hat{\boldsymbol{\imath}} \rangle = \langle \boldsymbol{i}, \breve{\boldsymbol{\imath}} \rangle \\ \langle \hat{\boldsymbol{v}}, \boldsymbol{i} \rangle = -\langle \boldsymbol{v}, \hat{\boldsymbol{\imath}} \rangle \\ \langle \breve{\boldsymbol{v}}, \boldsymbol{i} \rangle = -\langle \boldsymbol{v}, \breve{\boldsymbol{\imath}} \rangle \\ \langle \hat{\boldsymbol{v}}, \breve{\boldsymbol{\imath}} \rangle = \langle \breve{\boldsymbol{v}}, \hat{\boldsymbol{\imath}} \rangle = -\langle \boldsymbol{v}, \boldsymbol{i} \rangle \end{gathered} \tag{29}$$

Moreover, if v_μ and i_μ are sinusoidal quantities with RMS value respectively equal to V and I and phase angle equal to φ, the following properties are valid:

$$\begin{gathered} \|v_\mu\| = \|\hat{v}_\mu\| = \|\breve{v}_\mu\| = V_\mu; \\ \|i_\mu\| = \|\hat{\imath}_\mu\| = \|\breve{\imath}_\mu\| = I_\mu \\ \langle v_\mu, i_\mu \rangle = V_\mu.I_\mu.cos\varphi = P_\mu\,, \\ \langle \hat{v}_\mu, i_\mu \rangle = V_\mu.I_\mu.sin\varphi = Q_\mu\,, \end{gathered} \tag{30}$$

which exactly matches with the traditional definition of single-phase active and reactive fundamental powers.

Under the assumption of periodic behavior, the following quantities have been defined, which are valid for both sinusoidal and distorted, balanced or unbalanced conditions:

Active power: that represents the conveyed average power. This definition is identical to the conventional active power (Steinmetz, Budeanu, Fryze, Buchholz).

$$P = \langle \boldsymbol{v}, \boldsymbol{i} \rangle = \frac{1}{T}\int_0^T \boldsymbol{v}(t).\boldsymbol{i}(t)dt \tag{31}$$

Reactive power: that represents the average energy stored in the network and was defined as:

$$Q = \langle \hat{\boldsymbol{v}}, \boldsymbol{i} \rangle = \frac{1}{T}\int_0^T \hat{\boldsymbol{v}}(t).\boldsymbol{i}(t)dt \tag{32}$$

The physical meaning of this and other power terms are widely discussed in [14]. Following, the original currents are split into some parcels, regarding to their association to the power terms. The **active current** is the minimum current (i.e., with minimum norm) conveying active power P to the load and it is defined as:

$$i_{a\mu} = \frac{P}{\|\boldsymbol{v}\|^2}.v_\mu = G_e.v_\mu\,. \tag{33}$$

The **reactive current** is the minimum current transferring reactive power Q, and it is related to the average energy being exchanged through the circuit:

$$i_{r\mu} = \frac{Q}{\|\hat{\boldsymbol{v}}\|^2}\hat{v}_\mu = B_e\hat{v}_\mu\,. \tag{34}$$

Both the active and reactive currents have an explicit physical meaning. They are associated with the presence of the active and reactive powers, P and Q, and are related to the *load average equivalent* conductance G_e, and susceptance, B_e.

The **void current** is the remaining current (residual term), since it does not convey active P nor reactive Q power:

$$i_{v\mu} = i_\mu - i_{a\mu} - i_{r\mu}. \tag{35}$$

According to the authors, the void currents may exist only in presence of current distortion, however it will be demonstrated following that it is also influenced by current unbalances. Further details of each current component and its physical meaning can be found in [14].

By definition, all current terms are orthogonal:

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i_a}\|^2 + \|\boldsymbol{i_q}\|^2 + \|\boldsymbol{i_v}\|^2\ . \tag{36}$$

V. Simulation Results: Comparison and Discussion

In the view of the previous definitions, and considering just the three-phase three-wire power circuits, next sections demonstrate the main similarities and differences among the proposals. The main goal is to compare the resulting current components by means of each method. For that, three different conditions were simulated and analyzed.

In order to make easier the comparisons, the following under scripts were applied: FBD, pq, CPT.

A. *Case I: Unbalanced resistive load – small line impedance*

Figure 1 shows the power circuit for Case I, while the Table 1 presents the values of grid voltages, line impedance and load phase resistances. Figure 2 shows the measured PCC voltages (v) and currents (i).

Table 1 – Voltages and impedances for Case I.

Source	Line	Load (Y)
Va = 127∠0° Vrms	*R_{La} = 1mΩ L_{La}= 10 μH*	*Ra = 9,3405Ω*
Vb = 127∠-120° Vrms	*R_{Lb} = 1mΩ L_{Lb}=10μH*	*Rb = 6,2270Ω*
Vc = 127∠120° Vrms	*R_{Lc} = 1mΩ L_{Lc}=10μH*	*Rc =3,1135Ω*

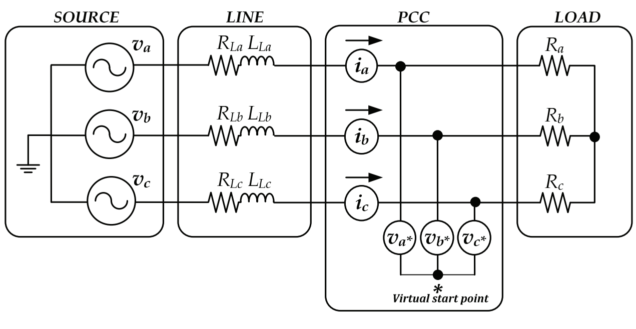


Figure 1: Power circuit for Case I – unbalanced resistive load.

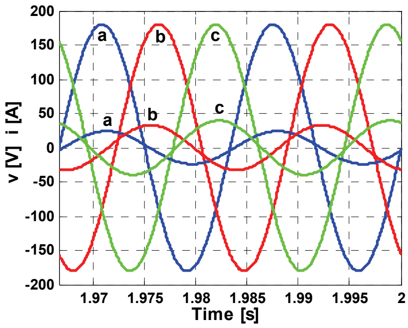


Figure 2: PCC voltages and currents for Case I.

Note they are not exactly in-phase, since the voltages are measured to a virtual point, which in practice represents that the voltages are referred to the power source central point, instead of the load central point. Besides, the voltages results balanced, since this kind of measuring filters out the homopolar components of the load voltages.

Figure 3 shows the current decomposition by means of the FBD (a), the pq-theory (b) and CPT (c). In this case, the active current based on the FBD and CPT are exactly the same, since they are based on the Fryze's definition. Besides, they match with the average part of the Akagi's active current ($i_{\bar{p}(pq)}$) and are in-phase with the respective voltages, since equations (6), (33) and the left side of (22) are equivalents under sinusoidal and balanced voltages ($i_{\bar{p}(pq)} = i_{a(CPT)} = i_{a(FBD)}$).

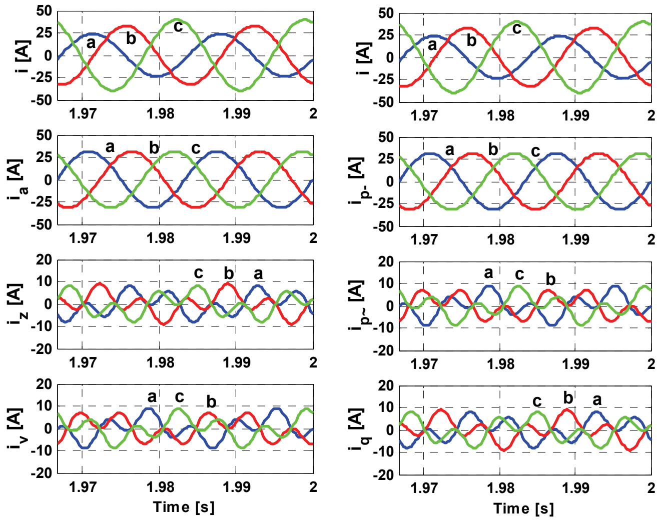
However, some confusing results can be pointed out, e.g.: the components $i_{z(FBD)}$ and $i_{v(FBD)}$ results distorted, even without any voltage distortion or non linear load, what could indicate that they are not good representation of the power phenomena in such condition. In addition, the oscillating part of the Akagi's active current ($i_{\tilde{p}(pq)}$) is also distorted, what means that the total active current in this case is not sinusoidal and even considering a linear pure resistive load, with no capacitors or inductors, the decomposition indicates the existence of reactive current (Fig. 3-b).

Finally, Fig. 3-c shows the current decomposition based on the CPT. Note that the resulting reactive currents are equal to zero ($i_{r(CPT)} = 0$), indicating the absence of energy storage elements, and the void currents are sinusoidal ($i_{v(CPT)}$) and represents the resistive load unbalances.

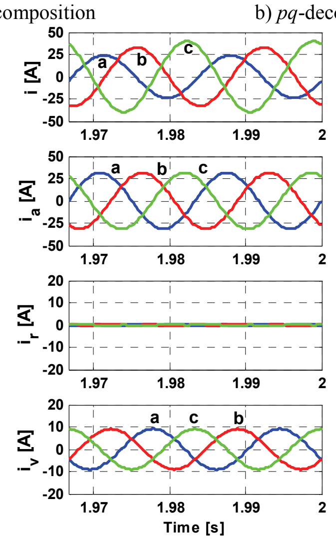
Moreover, considering the distorted current components, the comparisons of their spectra and waveform (Fig. 4) allow identifying the following relations:

$$i_{z(FBD)} = i_{q(pq)} \qquad i_{v(FBD)} = i_{\tilde{p}(pq)}$$
$$i_{v(CPT)} = i_{z(FBD)} + i_{v(FBD)} = i_{q(pq)} + i_{\tilde{p}(pq)}$$

Czarnecki [26,27] has demonstrated that the total active and reactive currents from the *pq*-Theory present a third harmonic content, originated by the decomposition itself. From Figs. 3 and 4, one can observe that the harmonic content comes from the oscillating active current ($i_{\tilde{p}(pq)}$) and the reactive current ($i_{q(pq)}$). On the other hand, it has never been discussed in literature the equivalence with the FBD method, under such conditions. Fig. 4 shows that either the powerless current or the variation current also presents the same third harmonic content.

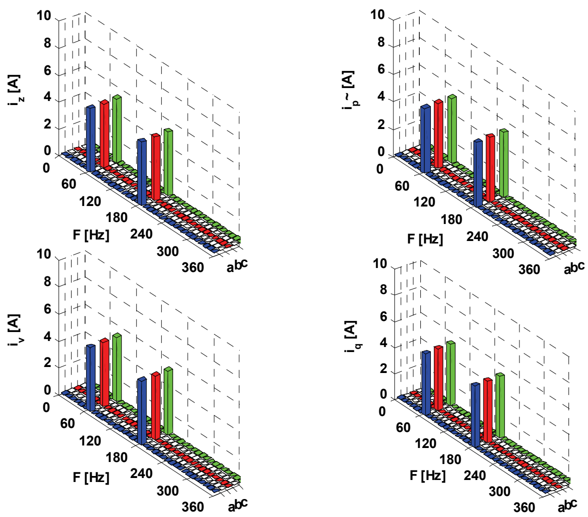


a) *FBD*-decomposition b) *pq*-decomposition



c) *CPT*-decomposition

Figure 3: Current decompositions for Case I.



a) *FBD*-decomposition b) *pq*-decomposition

Figure 4: Spectra of some *FBD* and *pq* current components - Case I.

In terms of physical phenomena interpretation, such results suggest that the Tenti's approach could br more appropriate and consistent, if considering the traditional definitions for sinusoidal systems. The load unbalance is represented by means of the sinusoidal void current components. Note that the phase sequence of the decomposed currents ($i_{v(CPT)}$) is negative if compared to the active current. According the Tenti et al. [14], such current ($i_{v(CPT)}$) does not contribute neither to the active nor to the reactive power phenomena.

Thus, if ($i_{v(CPT)} = i_{z(FBD)} + i_{v(FBD)} = i_{q(pq)} + i_{\tilde{p}(pq)}$), it is proved that the harmonic components are introduced by the *FBD* and *pq* decompositions and even though the differences in names or in the decomposition process, their results are very similar in this first case.

B. Case II – Two non linear and one linear load – small line impedance

Considering the same power source and line impedances, Figure 5 shows the circuit for Case II, while Table 2 presents the different load impedances.

Over again, the results from *FBD* and *pq*-Theory are equivalent (Fig. 6):

$$i_{z(FBD)} = i_{q(pq)} \qquad i_{v(FBD)} = i_{\tilde{p}(pq)}$$
$$i_{a(FBD)} = i_{\bar{p}(pq)} = i_{a(CPT)}$$

The active components are in-phase and with the same waveform of the PCC voltages. The powerless and variation current from *FBD* are distorted and unbalanced (Fig. 6-a), as well as the oscillating component of the active current and the reactive component from *pq*-theory (Fig. 6-b), mixing the effects caused by the load nonlinearities and unbalances.

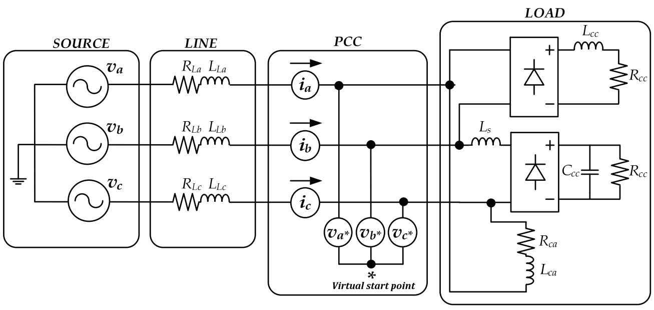


Figure 5: Power circuit for Case II – unbalanced non linear load.

Table 2 – Load impedances for Case II.

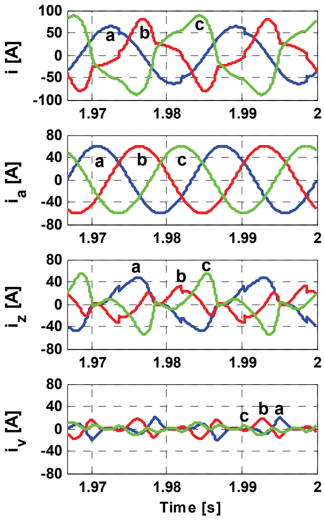
RL Rectifier	RC Rectifier	RL
$L_{CC} = 8mH \ R_{CC} = 5\Omega$	$C_{CC} = 8m\Omega \ R_{CC} = 4\Omega$	$Lac = 8mH \ Rac = 5\Omega$

In this case, the differences among the CPT and the other two methods are still more significant, since its decomposition identifies the active current ($i_{a(CPT)}$), related to the active power consumption (resistors or equivalent average conductance), the reactive current ($i_{r(CPT)}$), related to the capacitors and inductors present in the power circuit (equivalent average susceptance) and the void current, related to the load unbalances and nonlinearities ($i_{v(CPT)}$). Note that ($i_{a(CPT)}$) and ($i_{r(CPT)}$) are balanced and practically sinusoidal (Fig. 6-c), since the line impedances are very small.

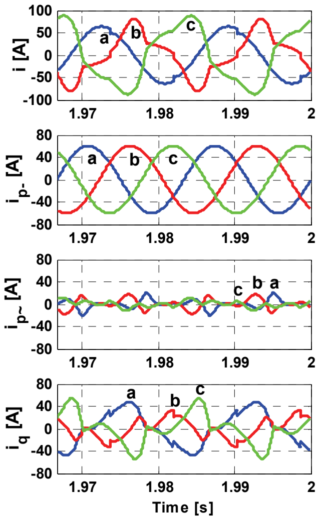
Further, the following similarities can be drawn:

$$i_{r(CPT)} + i_{v(CPT)} = i_{z(FBD)} + i_{v(FBD)} = i_{q(pq)} + i_{\tilde{p}(pq)}$$

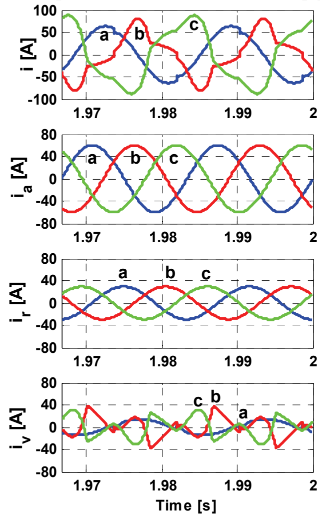
$$\begin{aligned} i_{a(CPT)} + i_{r(CPT)} + i_{v(CPT)} &= i(t) \\ &= i_{a(FBD)} + i_{z(FBD)} + i_{v(FBD)} \\ &= i_{q(pq)} + i_{\tilde{p}(pq)} + i_{\bar{p}(pq)} \,. \end{aligned}$$



a) *FBD*-decomposition



b) *pq*-decomposition



c) *CPT*-decomposition

Figure 6: Current decompositions for case II.

C. Case III - Two non linear and one linear load - high line impedance

Assuming the same circuit of Fig. 5, but changing the line impedance to $R_L = 10m\Omega$ and $L_L = 2mH$, which represents a week PCC condition, Fig. 7 indicates the PCC voltages and currents. Note that in this case, the voltages are distorted and unbalanced, because of the larger line impedance. This was not observed in the previous two cases.

This is a critical case. From Figs. 8 and 9 it is possible to conclude the following relations:

$$i_{a(CPT)} = i_{a(FBD)} \neq i_{\bar{p}(pq)}$$
$$i_{z(FBD)} = i_{q(pq)} \,; \qquad i_{v(FBD)} \neq i_{\tilde{p}(pq)}$$
$$i_{a(CPT)} + i_{r(CPT)} + i_{v(CPT)} = i_{a(FBD)} + i_{z(FBD)} + i_{v(FBD)} =$$
$$i_{\bar{p}(pq)} + i_{\tilde{p}(pq)} + i_{q(pq)} = i(t) \,.$$

Observe that the average part of the Akagi's active current ($i_{\bar{p}(pq)}$) does not agree with FBD and Tenti's active current. It happens since in case of nonsinusoidal or unbalanced voltages, the denominator of (6) and (33) are equal, but they do not correspond to (v_α^2+v_β^2) in (22). It means that even if the numerators from the three equations are equal, the ratios are not. Indeed, the ratio in (6) and (33) are constant over a fundamental period (average equivalent conductance), while the ratio in (22) is not (and does not represent any physical phenomena).

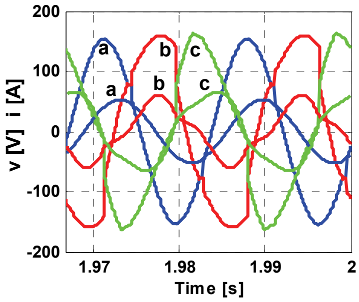
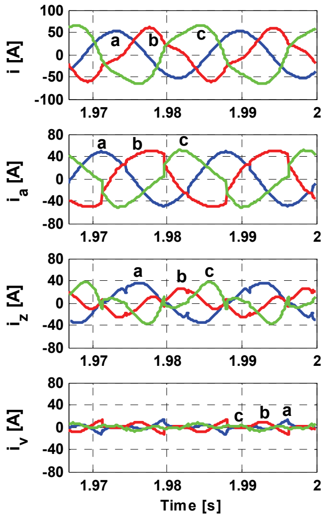
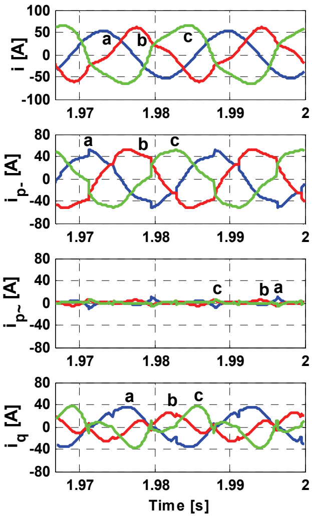


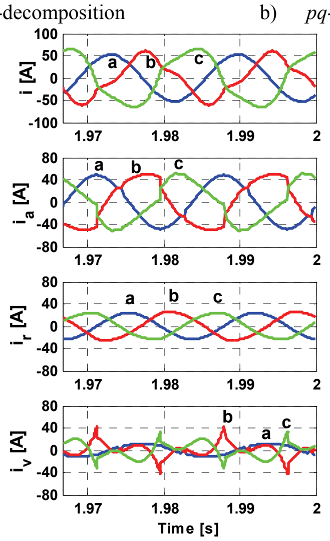
Figure 7: PCC voltages and currents for Case III.



a) *FBD*-decomposition



b) *pq*-decomposition

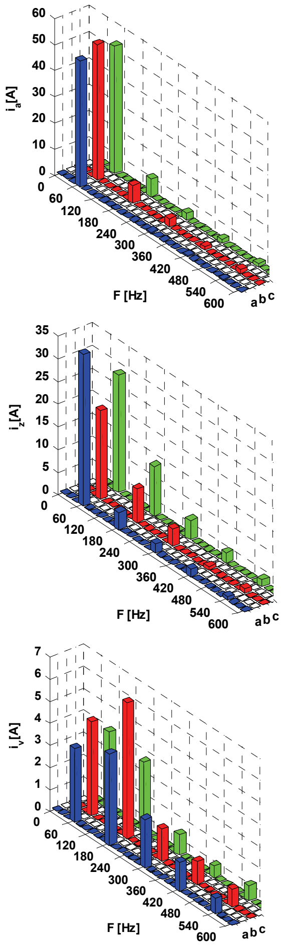


c) *CPT*-decomposition

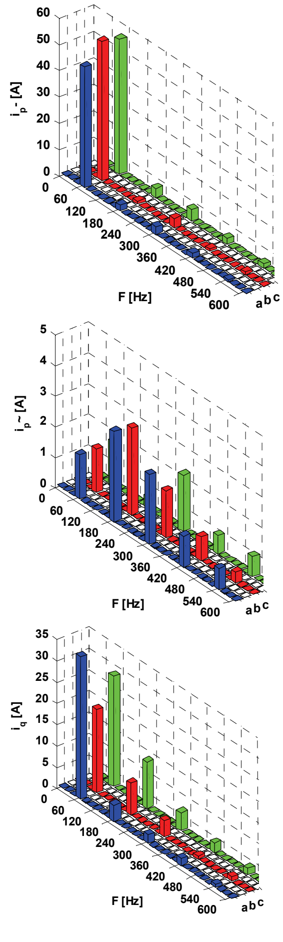
Figure 8: Current decompositions for case III.

Another important observation is that the Tenti's reactive currents are slightly distorted (see Fig. 9). Since they represent the portion of current relative to energy storage elements and their calculation is based on an integral (34) function, it is expected some attenuation in the harmonic content if compared to the voltages or to the active currents.

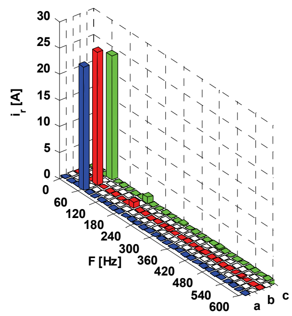
These last two observations may be valid even for Case II, however in that conditions, the voltages were practically sinusoidal and balanced (small line impedance).



a) *FBD*-decomposition



b) *pq*-decomposition



c) *CPT*-decomposition

Figure 9: Spectra of some FBD, PQ and CPT current components - Case III.

VI. Conclusions

Notwithstanding the basis of the *FBD* method seems to be more suitable to the understanding of different power phenomena, if compared to the *pq*-Theory, especially because it does not uses any axis transformation and it is based on the extension of well-accepted Fryze's definitions, their results are very similar in the case of three-phase three-wire circuits.

In [26,27], the author's analysis were directed to the instantaneous active and reactive current components from *pq*-Theory. In this paper, cases I and II show that under some conditions, the interpretation of either the *FBD* and pq-Theory can lead to invalid conclusions, e.g., considering a third harmonic content of a linear resistive load circuit. Case III shows that under significant voltage deterioration (week PCC condition), even the average active current from *pq*-decomposition is useless for the interpretation of the circuit. Besides, it has been demonstrated that the active current from FBD and CPT methods match for three-wire circuits.

Considering that the current decompositions and related power components could be useful to revenue metering, power conditioning, active filtering, power quality monitoring and so on, the results suggest that the proposal from *Tenti et al*. seems to be a very interesting alternative to the analysis, control and regulation of distinct power circuits, from the case of traditional sinusoidal and balanced voltage and current signals, to nonlinear load circuits with deteriorated voltages.

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