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# A Comparative Analysis of *FBD*, *PQ* and *CPT* Current Decompositions – Part II: Three-Phase Four-Wire Systems

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*Abstract*--**This paper investigates the major similarities and discrepancies among three important current decompositions proposed for the interpretation of unbalanced and/or non linear *three-phase four-wire* power circuits. The considered approaches were the so-called FBD Theory, the pq-Theory and the CPT. Although the methods are based on different concepts, the results obtained under ideal conditions (sinusoidal and balanced signals) are very similar. The main differences appear in the presence of unbalanced and non linear load conditions. It will be demonstrated and discussed how the choice of the voltage referential and the return conductor impedance can influence in the resulting current components, as well as, the way of interpreting a power circuit with return conductor. Under linear unbalanced conditions, both FBD and pq-Theory suggest that the some current components contain a third-order harmonic. Besides, neither pq-Theory nor FBD method are able to provide accurate information for reactive current under unbalanced and distorted conditions, what can be done by means of the CPT.**

*Index Terms*—***FBD*-theory, *pq*-theory, Homo-variables, harmonics, current decomposition, power theory.**

## I. INTRODUCTION

As discussed in the first part of this paper, the search for a general applicable power theory, for the analysis, monitoring, designing, revenue metering or power conditioning under non linear and unbalanced circuits has been an intriguing subject during, at least, the last 100 years.

Although considering the great number of important contributions from several authors [1-17], there is not a final agreement on the current decompositions and related power definitions, which should be adopted. If there are still some doubts regarding to single-phase definitions [1,5,6,8,14,20,21], the confusion and discrepancies are even greater in three-phase power systems, especially those with return or neutral conductor [5,10,18-24,26-29].

Thus, assuming the three current decomposition methods summarized, compared and discussed in the Part I of this paper (for three-phase three-wire circuits), the Part II of the paper will demonstrate the similarities and discrepancies among them, when considering four-wire circuits.

As it will be discussed, some of the misunderstanding can be explained in terms of the choice of the voltage referential and also in terms of the return conductor impedance.

By means of three different load and PCC (Point of Common Coupling) conditions, it will be demonstrated that the analysis of the physical phenomena based on the FBD or on the pq-Theory can be very confusing, since different effects caused by the loads are mixed up in the resulting decomposed currents. On the other hand, the proposal of Tenti *et al.* (CPT) seems to be a very helpful approach for current decompositions and power phenomena explanation.

So, considering the review of the investigated power theories and current decompositions, presented in Part I, next section brings additional discussions and comparisons of the resulting current decompositions for different conditions. The discussions are directed to the interpretation and understanding of the physical phenomena by means of the proposed current components.

## II. SIMULATION RESULTS: COMPARISON AND DISCUSSION

Taking into consideration the FBD, pq-Theory and CPT current decompositions, and assuming just the three-phase *four-wire* power circuits, next sections demonstrate the main similarities and differences among the proposals. The main goal is to compare the resulting current components by means of each method. In order to do that, three different conditions were simulated and analyzed. Note that there are basically the same conditions applied in Part I of the paper, however in this case, the single-phase loads were feed with phase-voltages instead of line-voltages.

In order to make easier the comparisons, the following under scripts were applied: FBD, pq, CPT for the considered approaches and $\mu$ indicating phase-variables.

### A. Case I: Unbalanced resistive load – small line impedance

Figure 1 shows the power circuits for Case I. Note that the voltage referential changes accordingly to the evaluated proposal (Figs. 1-a and 1-b). Table 1 depicts the values of grid voltages, line impedance and load phase resistances. Figures 2-4 show the measured PCC voltages ($v$) and currents ($i$) and the resulting current components from each method.

As expected, the load unbalance causes current flowing by

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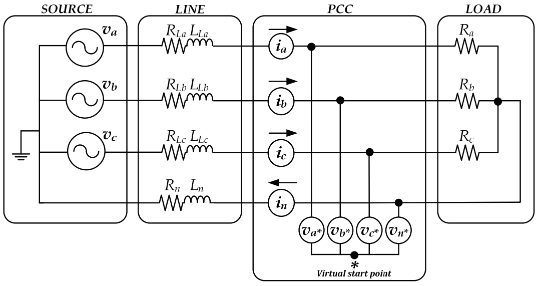
the return conductor (*n-neutral*) and accordingly to the FBD method, such conductor should be treated as equal in an m-conductor polyphase methodology [10-11]. Thus, assuming a three-phase four-wire (*m=4*) circuit, four voltages and four currents are measured as shown in Fig. 1-a. The other two methods are based on *m-1* voltage and current measures [25].

Differently from the three-wire case (Part I), in this case the measured voltages and currents from FBD are practically in-phase, since the return conductor ensures that the load central point matches the source central point (Fig. 2 – left side top). Indeed, it is applicable as long as the impedance of the return conductor were small. In such a case, the measured voltages to the virtual star point, which represents the source central point, will also reflect the load phase voltages and the return to virtual star point voltage will be practically zero ($v_{n*(FBD)} \cong 0$). For the same reason, the decomposed active currents yields in-phase and have the same waveform of the voltages ($i_{a\mu(FBD)}$), excepted for the neutral active current ($i_{an(FBD)} \cong 0$), as shown in Fig. 2 (left side bottom).

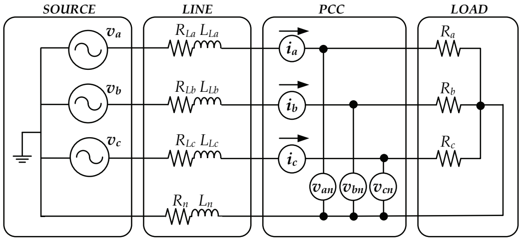
From Figs. 2-4 it is possible to observe that the active currents based on the FBD and CPT methods are equal to the average part of the Akagi's active current ($i_{a\mu(CPT)} = i_{a\mu(FBD)} = i_{\bar{p}\mu(pq)}$) .

Table 1 – Voltages and impedances for Case I.

| Source | Line | Load (Y) |
|---|---|---|
| *Va = 127∠0° Vrms* | $R_{La}$ = 1mΩ $L_{La}$= 10 μH | *Ra = 9,3405Ω* |
| *Vb = 127∠-120° Vrms* | $R_{Lb}$ = 1mΩ $L_{Lb}$=10μH | *Rb = 6,2270Ω* |
| *Vc = 127∠120° Vrms* | $R_{Lc}$ = 1mΩ $L_{Lc}$=10μH | *Rc =3,1135Ω* |
|  | $R_N$ = 1mΩ $L_N$=10μH |  |



a) Voltages measured to a virtual star point (*): FBD Method.



b) Voltages measured to the return conductor: *pq*-Theory and CPT.

Figure 1: Power circuits for Case I – unbalanced resistive load.

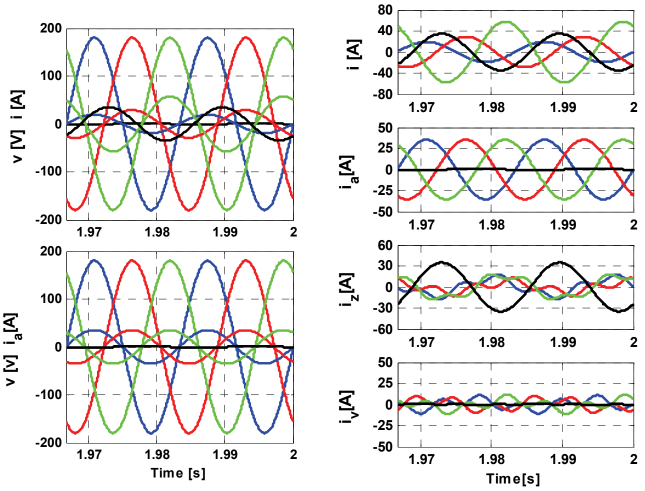


Figure 2: Measured PCC voltages and currents and resulting components for Case I – FBD Method.

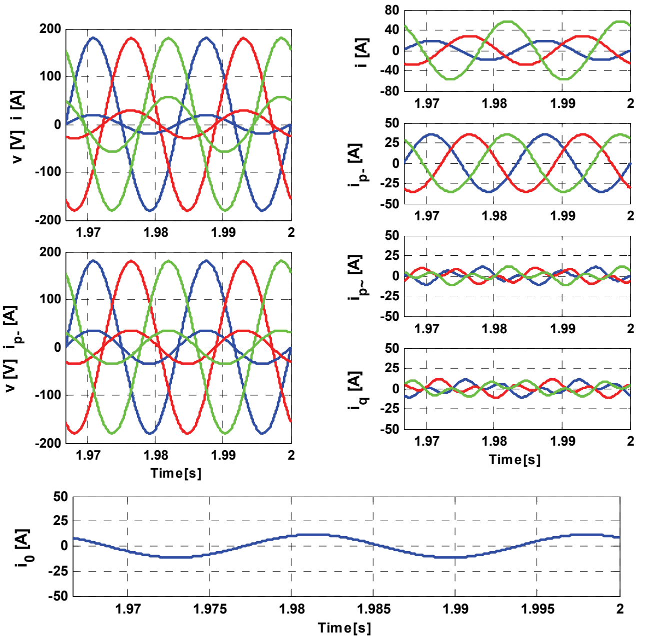


Figure 3: Measured PCC voltages and currents and resulting components for Case I – *pq*-Theory.

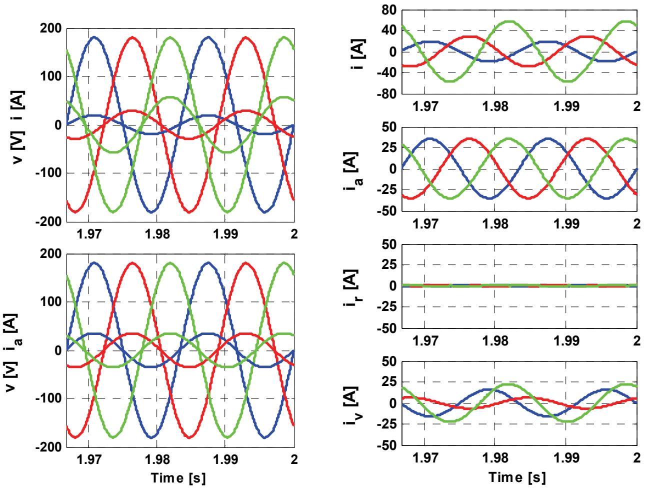


Figure 4: Measured PCC voltages and currents and resulting components for Case I – CPT method.

Regarding to the FBD method, Fig. 2 shows that the neutral active and variation currents are null ($i_{an(FBD)} = i_{vn(FBD)} = 0$), however, the powerless current results equal to the original neutral current ($i_{zn(FBD)} = i_n$). This fact indicates that the consideration of the return conductor as a homogeneous phase (source) conductor should be dealt with very much attention, since the information related to it is very different from the other conductors.

On the other hand, the parcels of powerless current ($i_{z\mu(FBD)}$), variation current ($i_{v\mu(FBD)}$), oscillating active current ($i_{\tilde{p}\mu(pq)}$) and reactive current ($i_{q\mu(pq)}$) indicates a third order harmonic content, which is not expected in case of linear resistive loads. Besides, the FBD variation current is equal to the PQ oscillating active current ($i_{v\mu(FBD)} = i_{\tilde{p}\mu(pq)}$), as in the three-wire case, but the powerless current is not equal to the reactive current from Akagi's ($i_{z\mu(FBD)} \neq i_{q\mu(pq)}$), differently from the three-wire case. In order to compare and find the similarities between FBD and Akagi's methods it is possible to demonstrate that:

$$i_{z\mu(FBD)} = i_{q\mu(pq)} + i_{0\mu(pq)} = i_n \; .$$

It is evident from Fig. 4 that the CPT approach seems to be more suitable than the other two methods for the analysis of Case I. In such method, the load unbalance is reflected just in the sinusoidal void current component ($i_{v\mu(CPT)}$) and the reactive component results null ($i_{r\mu(CPT)} = 0$), since there are not energy storage elements on the circuit. Thus, the power phenomena seem to be interpreted more coherently with the traditional concepts.

To provide evidence of the current components relations, the final version of the paper will bring the spectra of some current components. However, the following associations can be summarized:

$$i_{a\mu(FBD)} = i_{\bar{p}\mu(pq)} \; ;$$
$$i_{v\mu(FBD)} = i_{\tilde{p}\mu(pq)} \; ;$$
$$i_{z\mu(FBD)} = i_{q\mu(pq)} + i_{0\mu(pq)} \; ;$$

$$i_{an(FBD)} = i_{vn(FBD)} = 0 \; ;$$
$$i_{zn(FBD)} = i_n \; ;$$

$$i_{v\mu(CPT)} = i_{z\mu(FBD)} + i_{v\mu(FBD)} = i_{\tilde{p}\mu(pq)} + i_{q\mu(pq)} + i_{0\mu(pq)} \; ;$$

$$i_{a\mu(CPT)} + i_{v\mu(CPT)} = i_\mu \; ;$$
$$i_{a\mu(FBD)} + i_{z\mu(FBD)} + i_{v\mu(FBD)} = i_\mu \; ;$$
$$i_{\bar{p}\mu(pq)} + i_{\tilde{p}\mu(pq)} + i_{q\mu(pq)} + i_{0\mu(pq)} = i_\mu \; .$$

### *B. Case II – Two non linear and one linear load – small line impedance*

Considering the same power source and line impedances of the Case I, Fig. 5 shows the power circuit for Case II, while Table 2 presents the different load impedances. Observe that the same loads that were used in the Part I of the paper, fed by phase to phase voltages, here are fed by phase to neutral voltages.

The comparisons of the waveforms of Figs. 6-8 and their spectra results in very similar conclusions to those at the end of last section, expect that in this case, the presence of energy storage elements leads to the occurrence of reactive current from the CPT method ($i_{r\mu(CPT)} \neq 0$). Observe that while the line impedance were small, the voltage distortion will be small, as well as in the components: $i_{a\mu(CPT)}$, $i_{r\mu(CPT)}$, $i_{a\mu(FBD)}$, $i_{\bar{p}\mu(pq)}$.

Due to the occurrence of reactive current, in this case the following relationship is applicable:

$$\begin{aligned} i_{r\mu(CPT)} + i_{v\mu(CPT)} &= i_{z\mu(FBD)} + i_{v\mu(FBD)} \\ &= i_{\tilde{p}\mu(pq)} + i_{q\mu(pq)} + i_{0\mu(pq)} . \end{aligned}$$

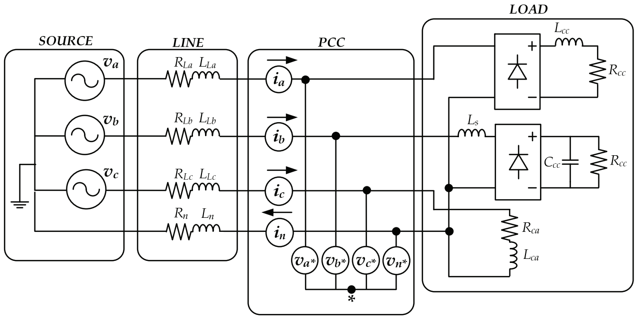
Note that the CPC method isolates the reactive component ($i_{r\mu(CPT)}$) related to the average equivalent susceptance of the circuit, from the void current ($i_{v\mu(CPT)}$), related to load unbalances and nonlinearities.

The following relations are also valid:

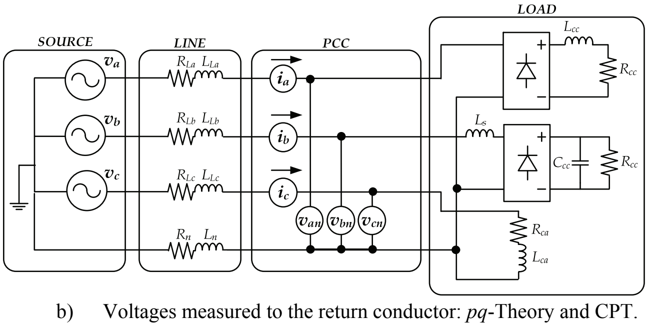
$$i_{a\mu(CPT)} = i_{a\mu(FBD)} = i_{\bar{p}\mu(pq)};$$
$$i_{v\mu(FBD)} = i_{\tilde{p}\mu(pq)};$$
$$i_{z\mu(FBD)} = i_{q\mu(pq)} + i_{0\mu(pq)};$$

$$\begin{aligned} & i_{a\mu(CPT)} + i_{r\mu(CPT)} + i_{v\mu(CPT)} \\ &\quad = i_{a\mu(FBD)} + i_{z\mu(FBD)} + i_{v\mu(FBD)} \\ &\quad = i_{\bar{p}\mu(pq)} + i_{\tilde{p}\mu(pq)} + i_{q\mu(pq)} + i_{0\mu(pq)} \\ &\quad = i_\mu \end{aligned}$$

$$i_{an(FBD)} = i_{vn(FBD)} = 0 \; ;$$
$$i_{zn(FBD)} = i_n \; .$$



a) Voltages measured to a virtual star point (*): FBD Method.



b) Voltages measured to the return conductor: *pq*-Theory and CPT.

Figure 5: Power circuit for Case II – unbalanced non linear load.

Table 2 – Load impedances for Case II.

| RL Rectifier | RC Rectifier | RL |
|---|---|---|
| $L_{CC} = 8mH$ $R_{CC} = 5\Omega$ | $C_{CC} = 8m\Omega$ $R_{CC} = 4\Omega$ | $Lac = 8mH$ $Rac = 5\Omega$ |

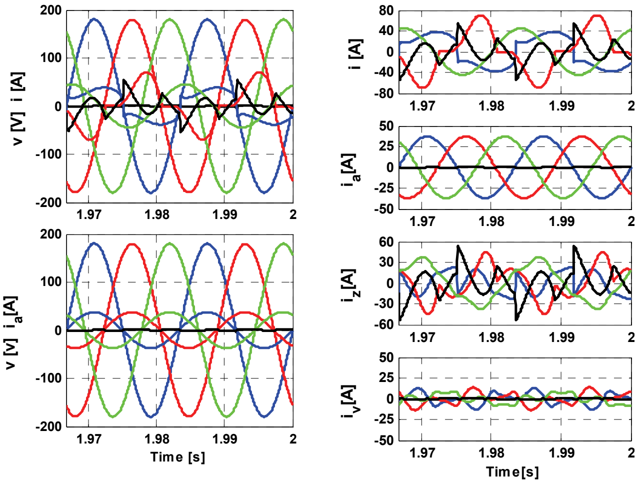


Figure 6: Measured PCC voltages and currents and resulting components for Case II – FBD Method.

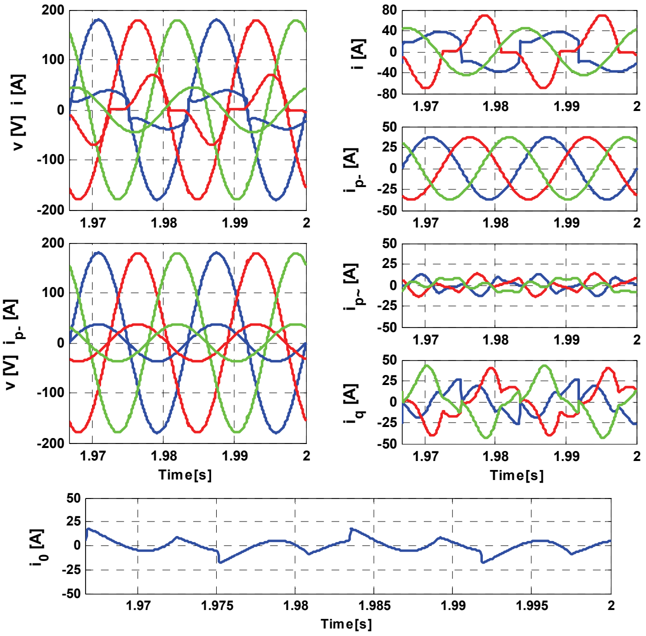


Figure 7: Measured PCC voltages and currents and resulting components for Case II – *pq*-Theory.

### C. Case III - Two non linear and one linear load – high line impedance

Assuming the same circuit of Fig. 5, but changing the line impedance to $R_L = 10m\Omega$ and $L_L = 2mH$ (including in the return conductor), which represents a week PCC condition; Figs. 9-11 show the resulting current components for each method.

Note that in this case, the voltages are distorted and unbalanced, because of the larger line impedance. This was not observed in the previous two cases. Besides, based on the distinct voltage referential, the FBD voltages do not coincide with PQ and CPT methods, which are equal.

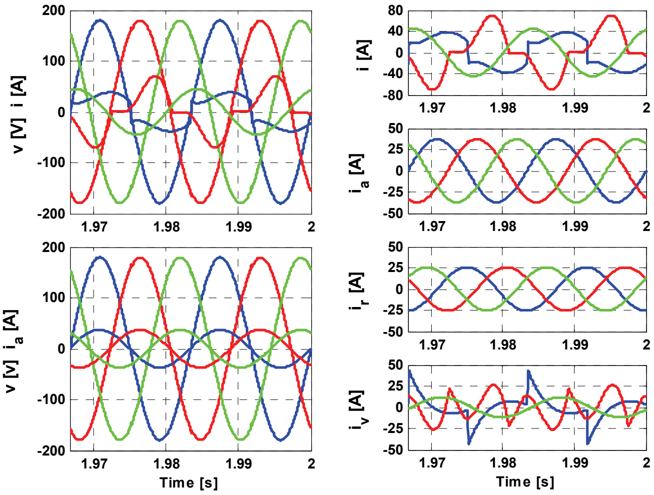


Figure 8: Measured PCC voltages and currents and resulting components for Case II – CPT method.

The analysis of this case is even more complicated than that of Case III in Part I, on which at least the voltage references were the same.

Based on the Blakesley Theorem [30], the authors have discussed in details the influence of such referential [23], and it is possible to conclude that from the point of view of power measurements or calculation, both can be applied (virtual star point or return conductor). However, very much attention should be considered if one is interested on the resulting current components for physical phenomena interpretation, power quality monitoring or even to power conditioning.

In this case:

$$v_{\mu n} \neq v_{\mu *} \quad ,$$
$$v_{n*} \neq 0 \quad ,$$

what leads to the following conclusions:

- The FBD active currents ($i_{a\mu(FBD)}$) are in-phase with and have the same waveform of the respective voltages to the star point ($v_{\mu *}$);
- The CPT active currents ($i_{a\mu(CPT)}$) are in-phase with and have the same waveform of the respective voltages to the return (neutral) conductor ($v_{\mu n}$);
- The average parts of the active currents from the pq-Theory are in-phase with the voltages to the neutral conductor ($v_{\mu n}$), but they have not the same waveform.

Thus, different from all previous case (Part I and II), here it is demonstrated that under such conditions, even the FBD and CPT active currents cannot match:

$$i_{a\mu(CPT)} \neq i_{a\mu(FBD)} \; .$$

But one could ask, why since both are based on the Fryze's definition? The answer regards to the fact that the virtual referential reflects the voltages from the PCC to the source central point, including the voltage drops across the line impedances. On the other hand, the neutral referential reflects the voltages from the PCC to the load central point or the voltages over the phase equivalent load impedances.

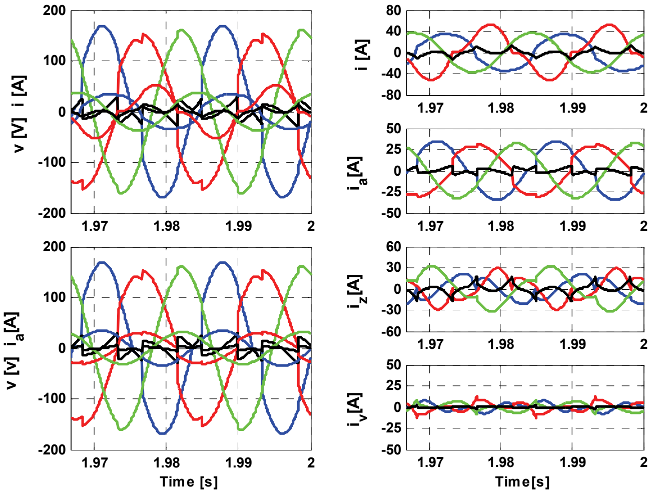


Figure 9: Measured PCC voltages and currents and resulting components for Case III – FBD Method.

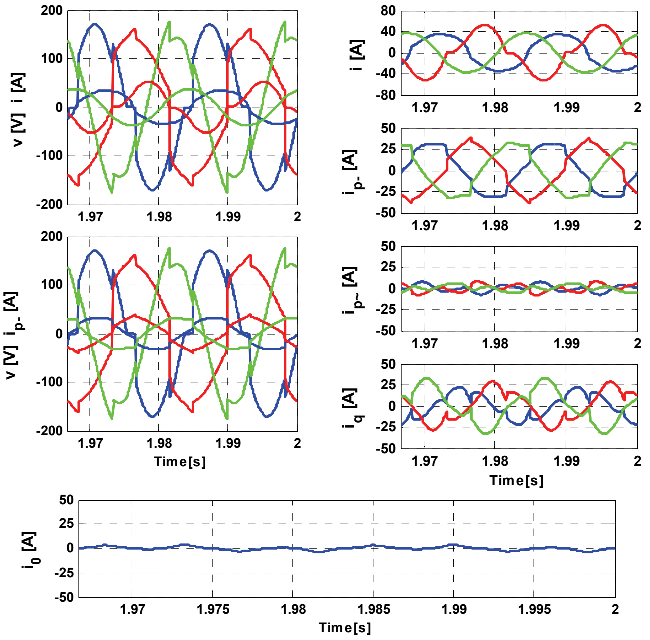


Figure 10: Measured PCC voltages and currents and resulting components for Case III – *pq*-Theory.

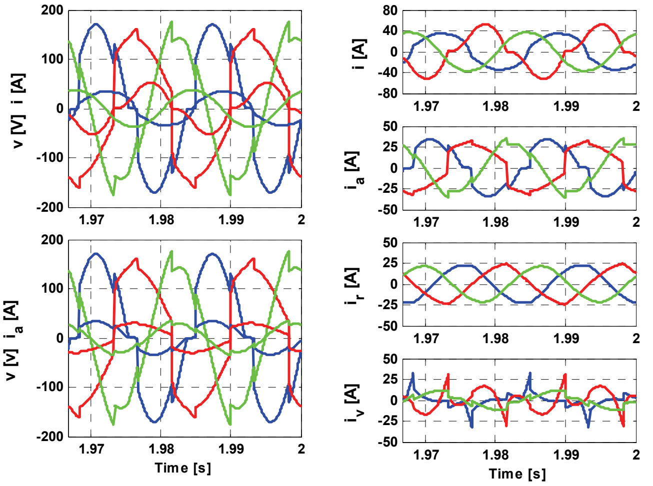


Figure 11: Measured PCC voltages and currents and resulting components for Case III – CPT method.

Considering the Akagi's average active currents ($i_{\bar{p}\mu(pq)}$), they have not the same waveform of the voltages, for the same reasons discussed in Case III of Part I. If the ratio on equation (22) is not constant, the currents have not the same shape of the voltages. Such ratio will be constant only in case of balanced and sinusoidal voltages.

Thus, the active currents from the three methods have no relation in this case:

$$i_{a\mu(CPT)} \neq i_{a\mu(FBD)} \neq i_{\bar{p}\mu(pq)}.$$

Particularly regarding to the differences between the FBD and CPT active currents, one could apply the correction method proposed in [23], which simply consist in adjusting the voltages measured using the virtual star point, so that they match the voltages measured to the return conductor, on the load side.

From the waveforms of Figs. 9-11 and the respective spectra, it is also possible to conclude that:

$$\begin{gathered} i_{z\mu(FBD)} \neq i_{q\mu(pq)} \\ i_{v\mu(FBD)} \neq i_{\tilde{p}\mu(pq)} \\ i_{z\mu(FBD)} \neq i_{q\mu(pq)} + i_{0\mu(pq)} \end{gathered}$$

$$\begin{gathered} i_{a\mu(CPT)} + i_{r\mu(CPT)} + i_{v\mu(CPT)} = i_{\mu} \\ i_{0\mu(pq)} + i_{\bar{p}\mu(pq)} + i_{\tilde{p}\mu(pq)} + i_{q\mu(pq)} = i_{\mu} \\ i_{a\mu(FBD)} + i_{z\mu(FBD)} + i_{v\mu(FBD)} = i_{\mu} \end{gathered}$$

$$i_n = i_{an(FBD)} + i_{zn(FBD)} + i_{vn(FBD)}$$

$$\begin{aligned} i_{r\mu(CPT)} + i_{v\mu(CPT)} &\neq i_{z\mu(FBD)} + i_{v\mu(FBD)} \\ &\neq i_{\tilde{p}\mu(pq)} + i_{q\mu(pq)} + i_{0\mu(pq)} \end{aligned}$$

## III. CONCLUSIONS

In addition to the important conclusions of the Part I, these last examples demonstrate that the differences among the resulting current components can be even bigger for four-wire circuits. In these cases, particularly in Case III, the voltage referential has great influence on the results.

It was possible to observe that the physical phenomena interpretation is not an easy assignment under such conditions. Over again, the CPT proposal seems to be a more suitable for the task, since the resulting active currents represent the currents drawn by a balanced equivalent resistive load and the reactive currents represent the currents drawn by a balanced equivalent reactance. The residual components (the void currents) represent the load unbalances and the nonlinearities of the power circuit.

The authors of this paper are now working on how to split the influence of unbalances and harmonic distortions on the CPT void current component [4], as well as, evaluating the need of identifying or not fundamental (60/50 Hz) current components, as proposed by [5]. Future papers will deal with these questions.

## IV. REFERENCES

[1] C. I. Budeanu, “Puissances Reactives et Fictives” *Instytut Romain de l'Energie*, Pub. No. 2, Bucharest, Rumania, 1927 (in Romanian).

[2] N. L. Kuster and W. J. More, “On the Definition of Reactive Power under Non-Sinusoidal Condition”, *IEEE Trans. on Power Apparatus and Systems*, PAS-99, pp.1845-1854. 1980

[3] W. Shepherd, P. Zakikhani “Suggested Definition of Reactive Power for Non-sinusoidal Systems”. *Proc. Inst. Elec. Eng*., vol. 119, pp.1361-1362, Sept. 1972.

[4] L. S. Czarnecki, “Orthogonal Decomposition of the Currents in a 3-Phase Nonlinear Asymmetrical Circuit with a Nonsinusoidal Voltage Source”, *IEEE Trans. on Instrumentation and Measurements*, vol.IM-37, n.1, pp.30-34, March 1988.

[5] *IEEE Trial-Use Standard Definitions for the Measurement of Electric Power Quantities under Sinusoidal, Non-sinusoidal, Balanced or Unbalanced Conditions*, IEEE Standard 1459-2000, June, 2000.

[6] L.S. Czarnecki, “Currents’ Physical Components (CPC) in Circuits with Non-sinusoidal Voltages and Currents. Part 1: Single-phase linear circuits,” *Electrical Power Quality and Utilization Journal*, Vol. XI, No. 2, 2005, pp. 3-14.

[7] L.S. Czarnecki, “Currents Physical Components (CPC) in Circuits with Non-sinusoidal Voltages and Currents. Part 2: Three-phase three-wire linear circuits,” *Electrical Power Quality and Utilization Journal*, Vol. XII, No. 1, 2006, pp. 3-13.

[8] S. Fryze, “Wirk-, Blind- und Scheinleistung in Elektrischen Stromkreisen mit Nichtsinusförmigem Verlauf von Strom und Spannung”, *Elektrotechnische Zeitschrift*, vol. 53, No. 25, 1933, pp. 596-599, 625-627, 700-702.

[9] F. Buchholz, Das Begriffsystem Rechtleistung, *Wirkleistung, totale Blindleistung*, Selbstverlag München, 1950.

[10] M. Depenbrock, “The FBD-Method, a Generally Applicable Tool For Analyzing Power Relations”, *IEEE Trans. on Power Systems*, vol. 8, n. 2, 381-387, May 1993.

[11] M. Depenbrock, Quantities of a Multiterminal Circuit Determined on the Basis of Kirchhoff's Laws, *ETEP European Trans. on Electrical Power Engineering* , Vol. 8, 1998, No. 4, pp. 249- 257.

[12] H. Akagi, Y. Kanazawa, A. Nabae, “Generalized Theory of the Instantaneous Reactive Power in Three-Phase Circuits”, in *Proc.* 1983 JIEE-IPEC *Conf. Power Electronics. Conf.*, pp. 1375-1386.

[13] H. Akagi, A. Nabae, The p-q Theory in Three-Phase Systems under Non-Sinusoidal Conditions”, *ETEP European Trans. on Electrical Power Engineering*, Vol. 3, No. 1, Jan./Feb. 1993, pp. 27-31.

[14] P. Tenti and P. Mattavelli, “A Time-Domain Approach to Power Term Definitions under Non-sinusoidal Conditions”, in *Proc. of 2003 Sixth International Workshop on Power Definitions and Measurements under Non-Sinusoidal Conditions*.

[15] P. Tenti, E. Tedeschi, P. Mattavelli, “Optimization of Hybrid Filters for Distributed Harmonic and Reactive Compensation”, *in Proc. 2005 IEEE International Conference on Power Electronics and Drive Systems PEDS*.

[16] F. Peng, G. Ott, and D. Adams, “Harmonic and Reactive Power Compensation Based on the Generalized Instantaneous Reactive Power Theory for 3-phase 4-wire Systems”, *in Proc. 1997 IEEE Power Electronics Specialists Conference PESC*, vol. 2, pp. 1089-1095.

[17] J. L. Willems, “Instantaneous Sinusoidal and Harmonic Active and Deactive Currents in Three-phase Power Systems”, *ETEP European Trans. on Electrical Power Engineering*, Vol. 4, No. 5, pp. 335-343. 1994.

[18] F. P. Marafão, Analysis and Control of Electrical Energy by means of Digital Processing Techniques. PhD Thesis. University of Campinas, Brazil, 2004. (*in Portuguese*).

[19] E. H. Watanabe, R. M. Stephan, and M. Aredes, “New Concept of Instantaneous Active and Reactive Powers in Electrical Systems with Generic Loads”. *IEEE Trans. Power Delivery* Vol. 8, pp. 697–703, 1993.

[20] H. Kim, F. Blaabjerg, and B. Bak-Jensen, “Spectral Analysis of Instantaneous Powers in Single-phase and Three-phase Systems with use of p-q-r Theory”. *IEEE Transaction on Power Electronics* Vol. 17, No. 5, pp. 711-720, 2002.

[21] J. L.Willems, “A New Interpretation of the Akagi-Nabae Power Components for Nonsinusoidal Three-phase Situations,” *IEEE Trans. on Instrumentation and Measurement*, vol. 41, no. 4, pp. 523–527, Aug. 1992.

[22] M. Depenbrock, V. Staudt and H. Wrede, “Theoretical Investigation of Original and Modified Instantaneous Power Theory Applied to Four-Wire Systems”. *IEEE Transaction on Industrial Application*, Vol. 39, No. 4, pp. 1160-1168, 2003.

[23] F P Marafão, S. M. Deckmann, and H. K. Morales P. “The influence of the Voltage Referential to Power Quality Evaluation”, *IEEE Latin America Magazine*, Vol. 6, No. 1, PP. 81-88, 2008. (*in Portuguese*).

[24] E. Tedeschi and P. Tenti, “Cooperative Design and Control of Distributed Harmonic and Reactive Compensators”, *in Proc*. 2008 *International School on Nonsinusoidal Currents and Compensation (ISNCC)*.

[25] A. Blondel, “Measurement of Energy of Polyphase currents”, *in Proc. International Electrical Congress,* Chicago, USA, pp. 112-117, 1893.

[26] A. E. Emanuel, “Reflections on the Effective Voltage Concept”, in *Proc. 2003 International Workshop on Power Definitions and Measurements under Non-sinusoidal Conditions*, pp. 1-8.

[27] J. L. Willems and J. A. Ghijselen, “The Choice of the Voltage Reference and the Generalization of the Apparent Power”, in *Proc. 2003 International Workshop on Power Definitions and Measurements under Non-sinusoidal Conditions*, pp. 9-18.

[28] J. L. Willems, J. A. Ghijselen and A. E. Emanuel, “The Apparent Power Concept and the IEEE Standard 1459-2000”, *IEEE Trans. Power Delivery*, vol. 20, no. 2, pp. 876-884, Apr. 2005.

[29] J. Willems, “Critical Analysis of Different Current Decomposition and Compensation Schemes”, *in Proc*. 2008 *International School on Nonsinusoidal Currents and Compensation, (ISNCC)*.

[30] T. H. Blakesley, “A New Electrical Theorem”, *in Proc. of the Physical Society of London*, vol.13, pp. 65-67, 1894.

## V. BIOGRAPHIES

**Helmo Kelis Morales Paredes,** was born in Puno-Perú. He received his B.S. degree (2002) in Electrical Engineering from Arequipa National University (UNSA), Perú. In 2006 he received his Master degrees from University of Campinas (UNICAMP), Brazil. Currently, he is a PhD student at UNICAMP. His current interests are mainly concerned with power quality evaluation and power definitions under multiphase nonlinear systems.

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