Strategic Bidding in Pay as Bid Power Market By Combined Probabilistic and Game Theory Procedures

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***Abstract*— The strategic bidding problem for generation companies (GENCOs) in a pay as bid market is considered with respect to risk analysis for different participant types, in which GENCOs suffer from lack of complete information about other opponents bidding strategies. According to Iran power market characteristics, in which participants should declare their maximum available energy to the ISO, Bertrand model is adopted for strategic bidding in the power market. Then, by transforming incomplete data that each GENCO has from others into complete information using game theory and bayesian Nash equilibrium, a proper method is proposed considering the power market characteristics, for maximization of GENCOs' profit according to other GENCOs' bidding behaviors and power system operating conditions. because of uncertainty of sources in a power system such as strategic behavior of opponent GENCOs, demand fluctuations and forced outage of network components, calculation of equilibrium point is not enough in proper strategic bidding, so the problem utilized so that risk factor is taken into account. The method is tested on three of participants in Iran power market. The numerical results shows the efficiency of the proposed method for strategic bidding of GENCOs participate in pay as bid markets.**

***Index Terms*—Optimal strategic bidding, GENCO competition, complete and incomplete games, Bayesian Nash equilibrium, Pay As Bid auction, Bertrand model, Risk Management.**

I. INTRODUCTION

The electricity market in a deregulated environment is leading in a way to increase competition among participants, comprising generation companies (GENCOs), distribution companies (DISCOs), transmission line owners and other participants, while managing to reduce net cost of power system. In this environment, each generation company will try to maximize its own profit, by managing its bidding behavior, considering other GENCOs' bidding information, participate in market. Therefore it is important for a GENCO to adopt a suitable bidding strategy, while estimating other GENCOs' behavior at the moment, to achieve maximum profit.

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Strategic bidding, including the the amount of power to supply and its offered price, refers not only to optimization of a GENCO behavior, but also to other GENCOs behaviors, which are unknown or known partially to other GENCOs. There are game based methods and non game procedures for transformation of incomplete information to complete information. Game theory based strategy for submitting GENCOs' bids to the ISO, is recognized as a proper method in this study.

It implements Nash equilibrium for strategic bidding of participated GENCOs in electricity market, in which the revenue of each GENCO depends on other GENCOs' bids, risk management by company, and market's fluctuating prices at the moment.

In [1], authors proposed a probability based method, for submitting optimal bids, in which participants' revenue defined as an objective function.

Bayesian Nash equilibrium, was employed by D. Fang et al. in [2], using Cournot market model and by assumption of complete information in its algorithm. In [3] and [4], market clearing price calculated base on game theory method. The proposed method in [5-10] investigated bidding behaviors of GENCOs in centralized electricity market. There are some non-game base methods proposed for developing proper bidding strategies of GENCOs, for example an usual optimization method was implemented in [11], to find a suitable bidding strategy of generation companies. A bidding strategy that proposed in [12] is based on economic principles which are known as cobweb theorem. This approach calculates the maximum bidding and generation level of GENCOs by means of an iterative procedure using GENCOs' residual demand curve. A stochastic optimization method introduced in [13], considering *Monte-Carlo* simulation to find optimal bidding strategy, and a six generator example was presented to illustrate the method. A detailed review of bidding strategies was investigated in [14].

Meanwhile, the problem of GENCOs' bidding with regard to incomplete information in *pay as bid* markets, such as Iran's power market, had not discussed in literature.

There are two pricing rules, Uniform Price Auction (UPA) and Pay-as-Bid (PAB) auction. Payments by the auctioneer to the producers differ across these two auction forms; under UPA all producers which bid below or at the MCP obtain this price, whilst under PAB, producers are paid their bids, as long as this is below or equal to the MCP. According to this, it is observed that decision making in GENCOs' bidding strategy is more important in PAB auction than UPA.

In this paper base on data provided by Iran power market ISO, such as: calculating hourly probability density function (PDF) of market clearing price in previous day, maximum and minimum of accepted market prices, and load forecasting data

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which are announced by ISO to participants each day, in addition to incomplete information that GENCO has from it opponents such as maximum and minimum limits for GENCOs' supply power, an objective function is proposed to find optimal bidding strategy of GENCO according to probability density function of accepted price ranges, which calculate first by determination of the probability that submitted GENCO bids are in the market final price range, then as a result of it, GENCOs' bidding strategies will classify in a row. Finally, optimal bidding strategy will be calculated using game theory based method to maximize GENCOs' revenue, assuming Bertrand market model, and considering risk analysis for different participant types associated to different uncertainty sources in the environment.

II. Different Models for Optimization Problem

Assuming a quadratic cost function for generation companies as follow:

$$C_i = C(P_i) = a_i P_i^2 + b_i P_i + c_i, \quad i = 1 \ldots n_g \tag{1}$$

In which P_i is generator power, and a_i, b_i, c_i are cost function coefficients of generators.

There are some economic models for strategic bidding, that we can have a brief review on them:

- Bertrand model: In this model each GENCO submits bids at its marginal cost at Nash equilibrium point, and all GENCOs will participate in market based on offered price of generation level.
- Cournot Model: The model assumes GENCOs' power homogenously and price for demand side response, and market clearing price (MCP), is the point which aggregated supply curve intersects with demand curve. In this model GENCOs offer their generation level and associated prices simultaneously to ISO, and their profit maximize through expectation mode of competitors.
- Stackelberg Model: This model is similar to Cournot model. However the competitors do not offer bids simultaneously.
- Supply Function Equilibrium (SFE): This model developed by Klemperer and Meyer for calculating the competitors' maximum profit equilibriums with uncertain demand.

The Bertrand model is implemented in this paper, because of its efficiency to use in pay as bid market and adoptability to characteristics of Iran power market. In this model, it is assume that GENCOs participate in market by regulating their bids, and ISO determines accepted generation level and associated prices, and the fact that GENCOs should submit bids lower than accepted price.

On the other hand, if some GENCO's offer bids more than accepted price, it can be opportunity for other for others to win in market competition, by lower bids. Because the loser GENCO, can offer a lower bids after this stage, so in Bertrand model the equilibrium point is achievable when cost of generation be equal to market price.

III. Game Theory

One of dominant characteristics of Game theory is that opponents decisions can be predicted, in which decisions considering critical conditions are formulated to maximize mathematical expectation,

In strategic games, a game with incomplete information is a game in which the GENCO is not aware of its opponent strategy, therefore it should be predicted by incomplete information available from them. There are two assumptions in using game theory in this area; first, the decisions are made base on logical process. Second, each GENCO can estimate other opponent's behavior. Suppose, B is a subset of decisions within all possible decisions A, which are chosen as best decisions. A logical decision is to choose a^* from set B which is optimal from GENCO's point of view, in which $g(a^*) \geq g(a), \forall a \in B$.

In other words, the GENCO has an optimization problem as following: $\max_{a \in B} U(g(a))$

Nash equilibrium is one of most practical methods in game theory, which is in a form of state space equations. In Which each participant has an estimation about other opponents, and adjust its strategy according to these estimations. In fact, Nash equilibrium in game theory can be define as possible decisions, In which the following inequality satisfies.

$$(a_{-i}^*, a_i^*) \underset{i}{\geq} (a_{-i}^*, a_i) \tag{2}$$

Thus a^* is Nash equilibrium point, when there is no other decisions better than a_i^*, for participant (i). Assuming each participant (j) choose the optimal decision. In other words, a Genco cant make more profitable decision among others.

IV. Strategic Bidding Problem Formulation:

Assume a Genco participate in an auction that participants can bid their prices up to 10 steps. The revenue is a function of Genco's bid and marginal cost function. Assuming MCP is available as probability density function (PDF) of market clearing price, and each participant has an equal portion chance in market. The revenue of each participant in a pay as bid market can be formulated as:

$$profit = f(\rho_m, \overline{\rho}, \overline{G}) = (\rho_i - C_i) G_i \tag{3}$$

In which ρ_i is bid price, C_i is average supply cost, and G_i is bid supply value. $\overline{G}, \overline{\rho}$ are vectors of supply value and price. According to market clearing method, which is base on pay as bid procedure, if Genco's bid price was lower than maximum clearing price of the market, its revenue will be significant, otherwise the revenue is zero.

Since the revenue can not calculated by deterministic methods, it should be define by probabilistic variables. Therefore the mathematical expectation of the revenue can be defined as:

$$E(f_i(\rho_m, \rho_i, G_i)) = EP_i(\rho_m, \rho_i, G_i) \tag{4}$$

$$= \int_{\rho_i}^{\rho_{\max}} (\rho_i - C_i) G_i f_{\rho_m}(\rho_m) d\rho_m = (\rho_i - C_i) G_i (1 - F_{\rho_m}(\rho_i))$$

The maximum revenue can be achieved when the Genco's bidding price calculated as follows:

$$\rho_p^* = \frac{1 - F_{\rho_m}(\rho_p^*)}{f_{\rho_m}(\rho_p^*)} + C_{\rho_m} \qquad (5)$$

The equations which discussed so far were only base on probability of acceptance of each participant's bid, and not base on other Genco's behavior. But in fact the revenue of a Genco in market is a function of other Genco's bidding strategies. By choosing proper strategy by opponents, the Genco will lose is opportunity to supply.

Using game theory, each participant prefers to make a decision, which can grantee its success in market competition.

Therefore participants try to make their best in decision making to maximize their profit according to incomplete information that they have about other opponents.

In Iran power market the information about minimum and maximum bid prices, minimum and maximum accepted prices in market, and weighted average of accepted prices in market, and its amplitude announce by ISO each day. Figure (1) shows a sample of weighted average of market accepted prices for week given from Iran power market in 2008.

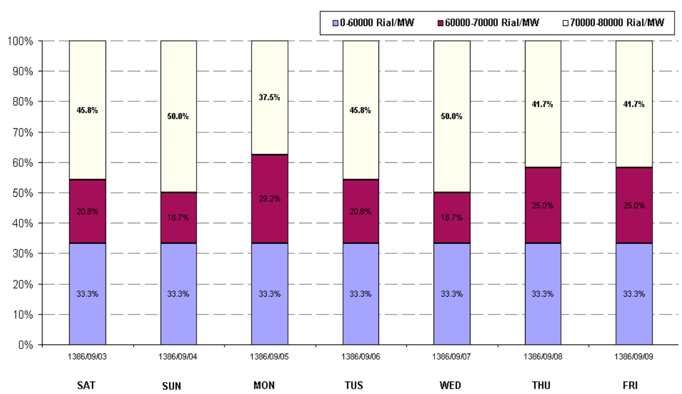


Fig. (1): A sample of weighted average of accepted prices given from Iran power market weekly reports.

According to the figure (1), 99 percent of bid prices are in the range of 30000 to 60000 [Rial/MWh]. The probability of bidding outside this range is very low.

It should be noted that the maximum and minimum limits for bidding prices in Iran power market is between 8000 [Rial/MWh] and 60000 [Rial/MWh].

Statistics in Iran power market show that more than 90 percent of participants bid their prices up to three steps.

The result of statistics is shown in figure (2).

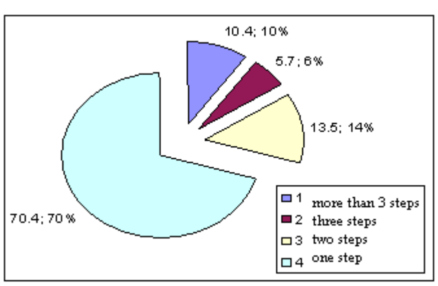


Fig. (2): percentage of bided price in different steps in Iran power market 2008.

In which, in first step the participant bid the value of $P_{\min} + 10\%(P_{\max} - P_{\min})$ in lowest price range of daily reports of bid price in market. In second step the participant bids the value of $P_{\min} + 80\%(P_{\max} - P_{\min})$ by higher prices than first step, but lower than weighted average price in market daily reports. Finally in third step, participant bids its maximum power ($P_{\max}$) in the price range between weighted average price and maximum price limit.

In Iran power market all of participants should bid up to their maximum power ($P_{\max}$). Because participants bid their prices according to probability density function of ISO daily reports, thus opponents strategy is considered indirectly in bidding strategy of each Genco. According to the fact that Genco's objective is to maximize their own profit, Therefore strategic bidding problem in each step can formulated as,

$$\begin{aligned}
&\underset{\overline{\rho}_P, \overline{G}_P}{Max}\ E_p(\rho_m, \overline{P}_p, \overline{q}_p) = \sum_{i=1}^{n} (P_{p,i} - C_{p,i}) q_{p,i} (1 - F_{\rho_m}(P_{p,i})) \\
&s.t.: (P_{p,i} - C_{p,i}) = h_p(P_{p,i}) \qquad i = 1,2,\cdots,n \qquad (6) \\
&h_p(p) = \frac{1 - F_{\rho_m}(P)}{f_{\rho_m}(P)} \\
&C_{p,i} = \frac{C(\sum_{j=1}^{j=i} q_{p,j}) - C(\sum_{j=1}^{j=i-1} q_{p,j})}{q_{p,i}} \qquad i = 1,2,\cdots,n \\
&\sum_{k=1}^{n} q_{p,k} = q_{\max} \\
&P_1 \le P_p \le P_2
\end{aligned}$$

By solving above optimization problem, the bid value and its price will be available, by which the payoff of each participant and corresponding revenue can be determined. Assuming there are (*N*) participants in market, and each has (*n*) types, then total bidding strategies will be (*n*)(*N-1*). Therefore GENCO's profit can be shown in a matrix form as follows:

$$S = \begin{bmatrix} S_{11} \cdots & S_{1n} \\ \vdots \quad \cdots & \vdots \\ S_{m1} \cdots & S_{mn} \end{bmatrix} \qquad (7)$$

Elements in each column shows the revenue of each participant which is determined by different price levels and base on equation (5), and each row is related to different type of participant. According to the fact that the type of participant is not known for other opponents, thus cost function and revenues of other Gencos are not known. Therefore market competition can not model completely, and these incomplete information should transform to complete information in order to model the problem and then finding Nash equilibrium point by game theory.

V. TRANSFORM INCOMPLETE INFORMATION TO COMPLETE INFORMATION :

In a game, participants should capable of modeling bidding strategies of other opponents to find optimal bidding strategy for themselves. Important parameters in this procedure are cost function coefficients *a,b,c* in a quadratic function format as follows:

$$C(p) = aP^2 + bP + c \qquad (8)$$

Which the function can vary depend on different fuel type. To reach maximum profit, participants try to keep these information hidden from other opponents. Therefore opponents should estimate others bidding strategy using incomplete information they have. It is assumed that others are aware of minimum and maximum generation level and the type of fuel that each participant use to supply energy.

Fuel cost assumed a quadratic function as follows:

$$F(p) = \alpha P^2 + \beta P + \gamma \qquad (9)$$

Which is base on [MJ/h] or [MBtu/h].

Operational cost is comprise of utility own cost, maintenance and other constant costs. Since each Genco consumes different fuel type, thus coefficients of fuel cost function α, β, γ are different for them. In this study the probability of using different fuel types determine according to fuel types. The same probability can be considered for uncertainties modeling in the problem. Therefore the probability of using a special type of fuel can be calculated, and Nash equilibrium point can be achieved according to Bertrand market model.

VI. MATHEMATICAL EXPECTATION MATRIX OF GENCO's REVENUE

The profit achieved by each participant is closely depend on opponent's strategy. Therefore each participant's profit defined by a conditional probability. Because known information about other opponents bidding strategy is not complete, thus participants make their strategy by considering mathematical expectation of others profit. According to Iran power market regulatory rules, each Genco should present its maximum power ($P_{\max}$) in market. Therefore mathematical expectation of each participant's profit can be model by search method as follows: (10)

$$S_{new} = \begin{bmatrix} \theta_1^1(1,\ldots,1).P_1^1[w - P_{sum\le}].S_{11} & \cdots & \theta_1^n.P_1^n[w - P_{sum\le}].S_{1n} \\ \vdots & \vdots & \vdots \\ \theta_m^1.P_m^1[w - P_{sum\le}].S_{m1} & \cdots & \theta_m^n.P_m^n[w - P_{sum\le}].S_{mn} \end{bmatrix}$$

Where $P_{sum\le}$, is the sum of all participant supply power which their bidding price are lower than the price of the participant. If the value of $w - P_{sum\le}$, was higher than maximum supply power ($P_{\max}$) by the participant, then $[w - P_{sum\le}]$, given equal to $P_{\max}$.

P_m^n is the probability of using a special strategy by the participant. The similar procedure can be implemented to calculate matrix of participant's profit. When all of profit matrixes defined, the Nash equilibrium point can be find by search method.

VII. STRATEGIC BIDDING PROBLEM CONSIDERING RISK IN MATRIX GAMES:

Since there are different uncertainties in system such as changes in fuel, load,…, suitable bidding strategy requires not only considering mathematical expectation of participants profit, but also considering risk aversion in bidding strategy formulation. This is because of the fact that investment in uncertain environments threats investor's revenue. In this study, the probability density function of market clearing price used to determine the probability of bid price acceptance in a pay as bid auction.

The probability of not acceptance in market competition can be formulated as:

$$P_{notaccept}(\rho_{p,i}) = P(\rho_m \le \rho_{p,i}) = \int_{-\infty}^{\rho_{p,i}} f_{\rho_m}(\rho_m) d\rho_m = F_{\rho_m}(\rho_{p,i}) \qquad (11)$$

Therefore strategic bidding with respect to risk management in pay as bid market can be formulated as follows:

$$\underset{\bar{\rho}_p, \bar{G}_p}{Max}\, E_p(\rho_m, \bar{\rho}_p, \bar{G}_p) = \sum_{i=1}^{n} (\rho_{p,i} - C_{p,i}) G_{p,i} (1 - F_{\rho_m}(\rho_{p,i})) \qquad (12)$$

$$s.t. \qquad \sum_{i=1}^{n} G_{p,i} = G_{p,t}$$

$$1 - F_{\rho_m}(\rho_{p,i}) \ge \alpha_{p,i} \qquad 0 \le \alpha_{p,i} \le 1 \qquad i = 1,2,\ldots,n$$

Assuming similar step, the above problem, can be written as follows:

$$\underset{\bar{\rho}_p}{Max}\, E_p(\rho_m, \bar{\rho}_p, \bar{G}_p) = \sum_{i=1}^{n} (\rho_{p,i} - C_{p,i}) G_{p,i} (1 - F_{\rho_m}(\rho_{p,i})) \qquad (13)$$

$$s.t. \qquad 1 - F_{\rho_m}(\rho_{p,i}) \ge \alpha_{p,i} \qquad 0 \le \alpha_{p,i} \le 1 \qquad i = 1,2,\ldots,n$$

The term $(1 - F_{\rho_m}(\rho_{p,i}))$ is associate to probability of acceptance. Where $\alpha_{p,i}$ is given as risk aversion. Assuming that the variables of probability density function of market clearing price remain unchanged by increasing $\alpha_{p,i}$ from zero, feasibility region of problem (10) gradually limits until activation of the constraint. As a result, we can write:

$$(\rho_{p,i}^* - C_{p,i}) = \frac{1 - F_{\rho_m}(\rho_{p,i}^*)}{f_{\rho_m}(\rho_{p,i}^*)} \qquad (14)$$

$$1 - F_{\rho_m}(\rho_{p,i}^*) = \alpha_{p,i}^* \Rightarrow \rho_{p,i}^* = F_{\rho_m}^{-1}(1 - \alpha_{p,i}^*)$$

The critical value $\alpha_{p,i}^*$, which cause the constraint to be activated, can be determined by:

$$F_{\rho_m}^{-1}(1 - \alpha_{p,i}^*) - C_{p,i} = \frac{\alpha_{p,i}^*}{f_{\rho_m}(F_{\rho_m}^{-1}(1 - \alpha_{p,i}^*))} \qquad (15)$$

Therefore, when $\alpha_{p,i}$ was smaller than $\alpha_{p,i}^*$, the constraint isn't activated and optimal solution will satisfy the constraint to maximize mathematical expectation of revenue. If $\alpha_{p,i}$ be larger than $\alpha_{p,i}^*$, then the constraint will be activated, and optimal solution will be achieved by solving problem (11).

According to equations (10-12) the solution of the problem for each step will be in a form of following equations:

$$\rho_{p,i}^{**} = \begin{cases} \rho_{p,i}^{*} : \rho_{p,i}^{*} - C_{p,i} = \dfrac{1 - F_{\rho_m}(\rho_{p,i}^{*})}{f(\rho_{p,i}^{*})} & \alpha_{p,i} \le a_{p,i}^{*} \quad i = 1,2,\ldots,n \\ F_{\rho_m}^{-1}(1 - \alpha_{p,i}^{*}) & \alpha_{p,i} \ge a_{p,i}^{*} \end{cases} \qquad (16)$$

$\rho_{p,i}^{**}$ is an optimal price which can be calculated by changing risk aversion. However after constraint activation, calculated price is acceptable if it isn't smaller than average supply cost.

VIII. VERIFICATION OF PROPOSED PROCEDURE :

Results of proposed algorithm investigated on a system comprise of three Gencos, namely Montazeri, Neka, and Modhej in Iran power market. Minimum and Maximum supply limits for these three Gencos is shown in table (1). It is assumed that system loading is 700 MW.

Table (1): Genco's minimum and maximum supply limits.

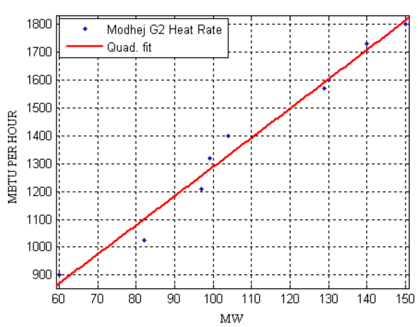
Genco	P_{max}	P_{min}
Neka	435	230
Montazeri	200	145
Modhej	145	80

Type of fuel for these utilities is as follows:
A. Neka and Modhej use only gas.
B. Montazeri use both gasoline and gas.

Heat rates of these three utilities is shown in figure (3-a to 3-d) .

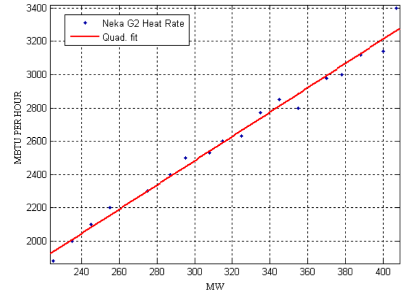
Convection of gas assumed 34266 $[KJ/M^3]$ and for gasoline, 38590$[KJ/Litre]$. Gas fuel cost assumed 13.832 $[Rial/kWh]$, and for gasoline 27.663 $[Rial/kWh]$.

Therefore supply cost is known for each utility. According to data given from power market, and using estimation techniques using statistics, the market clearing price is determined by a normal probability density function (PDF). The normal PDF has following characteristics in this study:
A normal probability distribution function with average of $\mu_m = 30000[Rial/MWh]$, and standard deviation of 4000$[Rial/MWh]$.



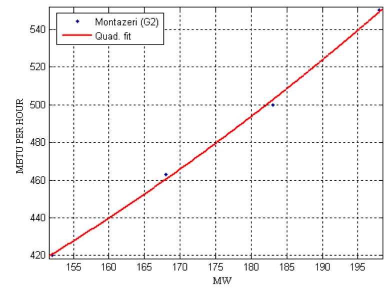
(3-a) Modhej (unit G2)

Quad. Fit: $Y = -0.004X^2 + 11.182X + 228.82$



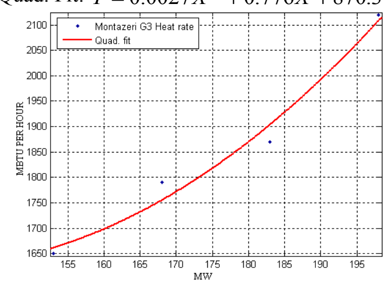
(3-b) Neka (unit G2)

Quad. Fit: $Y = -0.001X^2 + 7.927X + 222.75$



(3-c) Montazeri (unit G2)

Quad. Fit: $Y = 0.0027X^2 + 0.776X + 870.37$



(3-d) Montazeri (unit G3)

Quad. Fit: $Y = 0.0072X^2 + 0.201X + 225.44$

Fig. (3): Cost functions of Gencos.

The probability of using particular fuel type is given proportional to fuel cost. Therefore the probability of using gas is determined as 0.66, and using gasoline as 0.34. Fuel type is given as an uncertainty in this problem. These uncertainties in operational costs is defined in table (2).

Table (2): operational cost as an uncertainty.

Genco	Fuel Type	Type prob.
Neka	Gas	0.47
Montazeri	Gas	0.48
	Gasoline	0.52
Modhej	Gas	0.67

According to above assumptions, the mathematical expectation of the probability of using all Gencos at the same time (case (a) in table (3)), is as follows:

Table (3): probability of using all Gencos at the same time

Case (a)	probability
π_{111}	0.204
π_{121}	0.337

In which 1 and 2 are associate to using gas and gasoline.
According to above results, the the proability of GENCO's is determined as shown in table (4),

Table (4): Conditional probability of each participant revenue

$\theta_2^1(1,1)$	0.355
$\theta_2^2(1,1)$	0.195

Bided supply power and their prices in three steps are calculated by optimization problem defined in equation (6), as shown in table (5).

Table (5): Bid power values and their prices

Neka	Power [MW]	230	394	435
	Price [Rials]	35784	48763	59876
Montazeri	power [MW]	145	189	200
	Price [Rials]	37369	46785	58694
Modhej	Power [MW]	80	132	145
	Price [Rials]	37694	46569	57691

Now, Since bided supply power and their price are known, thus the payoff of each Genco is determined as shown in table (6), and the resulted revenue according to equilibrium point is shown in table (7).

Table (6): Each Genco's payoff.

Genco	Payoff [Rials]
Neka	15496007
Montazeri	7477045
Modhej	5437108

Table (7): Each Genco's Revenue.

Genco	Revenue [Rials]
Neka	15115112
Montazeri	7278595
Modhej	5298508

IX. CONCLUSION

A novel procedure introduced in this paper for strategic bidding of Gencos in power market. The method is fully compatible to pay as bid markets . The market model assumed as Bertrand model, and each participant make decision by transforming other Gencos' behaviors from incomplete to complete information by game theory based procedure and base on weekly and daily reports provided by Iran power market ISO and probability density function of market clearing price in a day ahead market.

The profit achieved by each participant is closely depend on opponent's strategy. Therefore each participant's profit defined by a conditional probability. Because known information about other opponents bidding strategy is not complete, thus participants make their strategy considering other's profit mathematical expectation.

Efficiency of the method investigated and verified on a case study including three of participants in Iran power market environment. The results shows that the method is fully compatible to Iran power market characteristics.

Because objective of introduced method was compatibility of strategic bidding problem with Iran power market characteristics and there was no other study on the market, so the method didn't compare with other approaches, but can be a proper case for future studies.

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