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Identifying Critical Sets in State Estimation Using Gram Matrix

Madson C. de Almeida, Eduardo N. Asada and Ariovaldo V. Garcia

***Abstract*— This paper presents a numerical algorithm for the identification of critical measurements and critical sets in power system state estimation. The proposed algorithm is based on the use of Gram matrix constructed considering the rows of the measurement Jacobian matrix as vectors. This paper shows some features of the Gram matrix that can be useful in the optimal planning of metering systems for power system state estimation. Numerical examples with a 6-bus system and also with IEEE-14 system are used for testing the proposed algorithm.**

***Index Terms*— critical measurements, critical sets, Gram matrix, redundancy analysis, state estimation.**

I. INTRODUCTION

IN power system state estimation, if a single bad data exists, its identification is compromised if the erroneous measurement is part of a critical set (minimally dependent set). This is the reason why critical sets of measurements are referred in the literature also as Bad Data Groups [1]. Moreover, if the erroneous measurement is critical, then the gross errors are undetectable [2].

A measurement is defined as critical if its suppression from the measurement set makes the system unobservable. A set of noncritical measurements is defined as a critical set if the removal of any measurement from this set makes the remaining measurements of the set critical [1]. In this context, this paper presents a numeric algorithm for the identification of critical measurements and critical sets in power system state estimation and some new features of the Gram matrix that can be useful to the planning of metering systems. Another application of Gram matrix in state estimation and its relation with the minimum norm formulation can be found in [3]

II. GRAM MATRIX PROPERTIES

The Gram matrix (A) associated with the "vectors" h_i; $i = 1, \ldots, p$ is formed by scalar products (or inner products) of these vectors as follows:

$$A = HH' \tag{1}$$

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The $A(i, j)$ element is the inner product of the vectors h_i and h_j, or $A_{ij} = h_i' h_j$. In the state estimation problem, the Gram matrix A indicates the interrelation of the measurements.

From linear algebra properties, matrix A is positive semi-definite Hermitian [4] and $rank(A) = rank(H)$, so that matrix A is non-singular if and only if $h_i, i = 1, ..., p$ are linearly independent, or similarly, if the p measurements are non-redundant. Thus, analogously to the numerical observability analysis, a rank deficiency of the Gram matrix caused by redundant measurements will result in a number of null pivots during factorisation. The null pivot can be obtained during Gauss elimination, which also corresponds to the Gram-Schmidt orthogonalisation of the rows of H [5].

Suppose that A consists only of linear independent rows. It can be decomposed as:

$$HH' = LU \tag{2}$$

where L is a lower triangular matrix with unity diagonal elements and U is an invertible upper triangular matrix. Suppose that the Gauss elimination is carried out on the augmented matrix $[HH'|H]$(with dimension $m \times (m+n)$), m is the number of measurements and n is the number of state variables. The augmented matrix will result in $[U|Q]$, where $Q = (L^{-1})H$. Now consider:

$$QQ' = L^{-1}HH'(L^{-1})' = U(L^{-1})' \tag{3}$$

Since QQ' is symmetrical and $U(L^{-1})'$ is an upper triangular matrix, QQ' must be a diagonal matrix, therefore the rows of Q are orthogonal. For a better understanding, consider the following cases with a 3-Bus system shown in Fig. 1 (all reactances are equal to one).

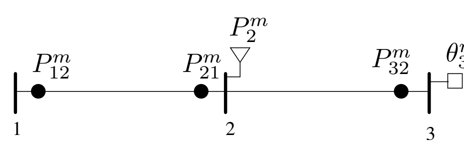

Fig. 1. 3-Bus system

Case 1: Two non-redundant line-flow measurements: P_{12}^m and P_{32}^m

$$H = \begin{matrix} & \theta_1 & \theta_2 & \theta_3 \\ P_{12}^m & 1 & -1 & \\ P_{32}^m & & -1 & 1 \end{matrix}; \quad A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

A is a non-singular matrix, and Gauss elimination will not lead to zero pivots. Eliminating the element A_{21} and also performing

Gauss elimination on H, results in:

$$(\tilde{A}:\tilde{H}) = \left(\begin{array}{cc:ccc} 2 & 1 & 1 & -1 & 0 \\ 0 & 3/2 & -1/2 & -1/2 & 1 \end{array}\right)$$

The diagonal element $\tilde{A}_{22}$ is not zero and both rows of $\tilde{H}$ are results of the Gram-Schmidt orthogonalisation without normalising.

Case 2: Redundant measurements

Adding the measurements P_{21}^m and P_2^m :

$$H = \begin{array}{c} \\ P_{12}^m \\ P_{32}^m \\ P_{21}^m \\ P_2^m \end{array} \begin{array}{c} \begin{array}{ccc} \theta_1 & \theta_2 & \theta_3 \end{array} \\ \left(\begin{array}{ccc} 1 & -1 & \\ & -1 & 1 \\ -1 & 1 & \\ -1 & 2 & -1 \end{array}\right) \end{array}; \; A = \left(\begin{array}{cccc} 2 & 1 & -2 & -3 \\ 1 & 2 & -1 & -3 \\ -2 & -1 & 2 & 3 \\ -3 & -3 & 3 & 6 \end{array}\right)$$

$$(A:H) = \left(\begin{array}{cccc:ccc} 2 & 1 & -2 & -3 & 1 & -1 & 0 \\ 1 & 2 & -1 & -3 & 0 & -1 & 1 \\ -2 & -1 & 2 & 3 & -1 & 1 & 0 \\ -3 & -3 & 3 & 6 & -1 & 2 & -1 \end{array}\right)$$

After Gauss elimination of A and with the application of linear operations in H, we have:

$$(\tilde{A}:\tilde{H}) = \left(\begin{array}{cccc:ccc} 2 & 1 & -2 & -3 & 1 & -1 & 0 \\ 0 & 3/2 & 0 & -3/2 & -1/2 & -1/2 & 1 \\ 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 \end{array}\right)$$

The zero pivots correspond to measurements P_{21}^m and P_2^m which are redundant with P_{12}^m and P_{32}^m. As seen in the previous example, the rows of matrix $\tilde{H}$ are orthogonal. If a new measurement is redundant with the previous ones (a linearly dependent equation is introduced), then its pivot is zero in the Gauss elimination. It can be shown, as in the case of the gain matrix [6], that a zero pivot will be followed by all entries of the row equal to zero (row of matrix U equal to zero), i.e, when a zero pivot appears in the i-diagonal ($\tilde{A}_{ii} = 0$), then $\tilde{A}_{ij} = 0, j > i$ and $\tilde{A}_{ji} = 0, j > i$.

Summarizing, if $\tilde{A}$ is the factorized Gram matrix, the following is true [7]:

- If the pivot $\tilde{A}_{jj}$ is zero, then the set of the vectors h_i, $i = 1, ..., j$ is linearly dependent. Or, the measurements y_1 to y_j form a redundant set;
- If there are $n-1$ nonzero pivots in $\tilde{A}$, the system is observable and the measurements associated to the nonzero pivots forms a basis;
- If the order of measurements in the Jacobian matrix is changed, the set of measurements that form the basis can change;

A. Redundancy Analysis

Let an n-bus observable system containing m measurements. Observing the earlier example, it is possible to identify $n-1$ measurements that form a basis and, consequently, it is possible to create a Jacobian matrix of the nonredundant measurements (H_b). Thus, the row of the Jacobian matrix (h'_j) corresponding to a new measurement y_j can be written as linear combination of the $n-1$ non-redundant rows of the Jacobian matrix H_b, according to (4). As matrix H_b is non-singular and non-symmetric, in order to simplify the calculation of vector α^j, one can simply post multiply (4) by H'_b obtaining (5), where the Gram matrix $A_b = H_b H'_b$ is non-singular and symmetric.

$$h'_j = (\,\alpha_1^j \quad \ldots \quad \alpha_i^j \quad \ldots \quad \alpha_{n-1}^j\,) \begin{pmatrix} h'_1 \\ \ldots \\ h'_i \\ \ldots \\ h'_{n-1} \end{pmatrix} = \alpha^j \; H_b \qquad (4)$$

$$\alpha^j A_b = h'_j H'_b \qquad (5)$$

In this context, the measurement y_j can be written as linear combination of the measurements y_1 to y_{n-1} as,

$$-y_j + \alpha_1^j y_1 + \ldots + \alpha_i^j y_i + \ldots + \alpha_{n-1}^j y_{n-1} = 0 \qquad (6)$$

B. Example

Consider the six bus system of Fig. 2. The measurements are processed in the following order $P_{1,6}$, $P_{1,4}$, $P_{3,2}$, $P_{2,5}$, P_6, P_1, P_2 and $P_{6,4}$. All reactances values are unitary.

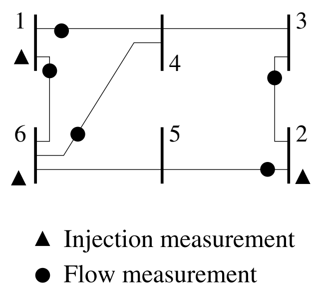

Fig. 2. The 6-Bus system with measurements

The corresponding H matrix and the Gram matrix (A) for this system are:

$$H = \begin{array}{c} \\ h_{P_{1,6}} \\ h_{P_{1,4}} \\ h_{P_{3,2}} \\ h_{P_{2,5}} \\ h_{P_6} \\ h_{P_1} \\ h_{P_2} \\ h_{P_{6,4}} \end{array} \begin{array}{c} \begin{array}{cccccc} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \end{array} \\ \left(\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 & -1 & 3 \\ 2 & 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{array}\right) \end{array} \qquad (7)$$

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 & -4 & 3 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 & 3 & 0 & -1 \\ 0 & 0 & 2 & -1 & 0 & 0 & -3 & 0 \\ 0 & 0 & -1 & 2 & 1 & 0 & 3 & 0 \\ -4 & 0 & 0 & 1 & 12 & -4 & 1 & -4 \\ 3 & 3 & 0 & 0 & -4 & 6 & 0 & 0 \\ 0 & 0 & -3 & 3 & 1 & 0 & 6 & 0 \\ 1 & -1 & 0 & 0 & -4 & 0 & 0 & 2 \end{pmatrix} \qquad (8)$$

The Gauss elimination of Gram matrix A results in the following matrix with three zero pivots occurring in positions corresponding to measurements P_1, P_2 and $P_{6,4}$.

$$\tilde{A} = \begin{pmatrix} 2.0 & 1.0 & 0 & 0 & -4.0 & 3.0 & 0 & 1.0 \\ & 1.5 & 0 & 0 & 2.0 & 1.5 & 0 & -1.5 \\ & & 2.0 & -1.0 & 0 & 0 & -3.0 & 0 \\ & & & 1.5 & 1.0 & 0 & 1.5 & 0 \\ & & & & 0.667 & 0 & 0 & 0 \\ & & & & & \boxed{0} & 0 & 0 \\ & & & & & & \boxed{0} & 0 \\ & & & & & & & \boxed{0} \end{pmatrix} \tag{9}$$

The zero pivots correspond to measurements that are redundant with the rest. Therefore, the measurement related to nonzero pivots form a basis of nonredundat measurements.

According to the factorized Gram matrix the system is observable and the measurements $P_{1,6}$, $P_{1,4}$, $P_{3,2}$, $P_{2,5}$ and P_6 form a basis. The remaining measurements (P_1, P_2 and $P_{6,4}$) are redundant and can be written, according to (5), as a linear combination of the nonredundant measurements, as the following,

$$\begin{cases} +P_{1,6} + P_{1,4} - P_1 = 0 \\ -P_{3,2} + P_{2,5} - P_2 = 0 \\ -P_{1,6} + P_{1,4} - P_{6,4} = 0 \end{cases} \tag{10}$$

These equations can be written in a *tableau* form, which rows are associated to the redundant measurements P_1, P_2 and $P_{6,4}$. Note that the portion of the *tableau* associated to the redundant measurements forms a negative identity matrix.

$P_{1,6}$	$P_{1,4}$	$P_{3,2}$	$P_{2,5}$	P_6	P_1	P_2	$P_{6,4}$
1	1	0	0	0	-1	0	0
0	0	-1	1	0	0	-1	0
-1	1	0	0	0	0	0	-1

It can be observed in the *tableau* that the column related to measurement P_6 is null, which means that it is a critical measurement. Moreover, if the measurement P_2 is lost, then the second row of the *tableau* is removed and the basic measurements $P_{3,2}$ and $P_{2,5}$ become critical.

III. Finding Critical Sets

The critical sets are composed by: one redundant measurement plus one or more basic measurements (type 1); and two or more basic measurements (type 2). In the proposed *tableau*, it is possible to identify the critical sets that contain redundant measurements. In order to identify the critical sets that contain only basic measurements (nonredundant measurements), it is necessary to remove from the measurements set, all basic noncritical measurements that do not belong to a critical set. Thus, it is necessary to remove measurements from the basis or change the basis.

A. Change of the Basis

Considering the earlier example, if we want to modify the basis by removing measurement $P_{1,4}$, then another measurement that provides the same qualitative information must be included. In this example we have chosen the measurement $P_{6,4}$, because this measurement is redundant with $P_{1,4}$.

In order to remove measurement $P_{1,4}$, linear operations are applied to the *tableau* (the column related to $P_{1,4}$) in order to obtain a negative unitary value in the corresponding column as follows,

$P_{1,6}$	$P_{1,4}$	$P_{3,2}$	$P_{2,5}$	P_6	P_1	P_2	$P_{6,4}$
2	0	0	0	0	-1	0	1
0	0	-1	1	0	0	-1	0
1	-1	0	0	0	0	0	1

Finally, the measurement $P_{1,4}$ is exchanged by $P_{6,4}$ resulting in the following *tableau*

$P_{1,6}$	$P_{6,4}$	$P_{3,2}$	$P_{2,5}$	P_6	P_1	P_2	$P_{1,4}$
2	1	0	0	0	-1	0	0
0	0	-1	1	0	0	-1	0
1	1	0	0	0	0	0	-1

After these operations, measurement $P_{1,4}$ does not belong to the basis anymore. If one wants to remove it from the measurements set, simply remove the third row (the row associated to $P_{1,4}$) of the new *tableau*.

B. Algorithm for identification of critical sets

Step 1: Mount and factorize the Gram matrix. The $n-1$ measurements associated to the nonzero pivots are basic measurements (nonredundant measurements). The remaining ones are redundant measurements;

Step 2: Using (5), calculate the vectors α associated to the redundant measurements and form the initial *tableau*;

Step 3: Find the critical measurements. They are basic measurements associated to null columns in the initial *tableau*;

Step 4: Remove one redundant measurement from the set. If new critical measurements appear, form a critical set containing these new critical measurements plus the removed measurement. Return to the initial *tableau*. Repeat this step until all redundant measurements have been tested;

Step 5: Remove from the measurements set one noncritical basic measurement that does not belong to any critical set. If there are new measurements classified as critical, form a critical set containing the new critical measurements plus the removed measurement. Return to the initial *tableau*. Repeat this step until all noncritical basic measurements that does not belong to any critical set be tested.

Remarks:

- In the step 4, the algorithm identifies the critical sets that contain one off-basis measurement (Type 1 critical set)

- The step 5 identifies the critical sets that are part of the basis of the initial *tableau*.

IV. Tests and Results

The results obtained for the 6-Bus system and for IEEE 14-Bus system are presented in the following sections.

A. 6-Bus System

The six bus system with the measurement set indicated in the Figure 2 is used in this first test to illustrate better the algorithm. In the initial *tableau*, the measurement P_6 is identified as critical. According to Step 4, redundant measurements are removed in the test. In order to remove measurement P_1, the first row of the *tableau* is removed. In this case, no critical measurements are detected and, therefore, no new critical set is formed. Removing measurement P_2 (second row of the initial *tableau*), critical measurements $P_{3,2}$ and $P_{2,5}$ are identified and, therefore, a critical set formed by P_2, $P_{3,2}$ and $P_{2,5}$ is identified. Finally, after the removal of the measurement $P_{6,4}$, which means the removal of the third row, no new critical measurements are identified and, therefore, no new critical set is formed. Thus, after execution of **Step 4**, one critical set of Type 1 has been identified.

In order to identify the critical sets of Type 2, according to the **Step 5**, it is necessary to remove, one by one, all noncritical basic measurement that does not belong to any identified critical set. For instance, as P_6 is a critical measurement and $P_{3,2}$ and $P_{2,5}$ forms a critical set, only $P_{1,6}$ and $P_{1,4}$ need to be removed from the basis. The removal of $P_{1,6}$ requires moving it out from the basis. As $P_{1,6}$ is redundant with the nonbasic measurements P_1 and $P_{6,4}$, it is possible to exchange it with any of the previous measurements. For example, if $P_{6,4}$ is selected the following *tableau* is obtained.

$P_{6,4}$	$P_{1,4}$	$P_{3,2}$	$P_{2,5}$	P_6	P_1	P_2	$P_{1,6}$
-1	2	0	0	0	-1	0	0
0	0	-1	1	0	0	-1	0
-1	1	0	0	0	0	0	-1

According to the previous *tableau*, the removal of $P_{1,6}$ results in no critical measurements, therefore, no new critical set is formed. Finally, in order to remove $P_{1,4}$, measurement P_1 or $P_{6,4}$ can be used instead. For example, if $P_{6,4}$ is selected, the following *tableau* is obtained.

$P_{1,6}$	$P_{6,4}$	$P_{3,2}$	$P_{2,5}$	P_6	P_1	P_2	$P_{1,4}$
2	1	0	0	0	-1	0	0
0	0	-1	1	0	0	-1	0
1	1	0	0	0	0	0	-1

After removing $P_{1,4}$ from the original tableau (removing the third row of the new tableau) no new critical measurements are identified, therefore, no new critical set is formed. Thus, in the example no critical sets of type 2 are identified. A summary of the results for the 6-bus system is shown in Table I

TABLE I
Summary of the results for 6-bus system

	Measurements
Critical Measurements	P_6
Critical Set of type 1	P_2, $P_{3,2}$, $P_{2,5}$
Critical Set of type 2	-

V. Tests with IEEE-14 Bus System

As an example the IEEE 14-bus system is used with the measurement set indicated in Figure 3. This example is the same presented in reference [1]. The measurements are processed in the following order: P_3, P_6, P_9, P_{10}, P_{12}, $P_{1,5}$, $P_{1,2}$, $P_{2,3}$, $P_{2,5}$, $P_{4,7}$, $P_{4,9}$, $P_{7,8}$, $P_{6,11}$, $P_{6,12}$, $P_{6,13}$, $P_{12,13}$, $P_{9,10}$ and $P_{9,14}$. According to the factorized Gram matrix this system is observable and the measurements P_3, P_6, P_9, P_{10}, P_{12}, $P_{1,5}$, $P_{1,2}$, $P_{2,3}$, $P_{4,7}$, $P_{4,9}$, $P_{7,8}$, $P_{6,11}$ and $P_{6,12}$ form a basis. The remaining measurements are redundant and, therefore, can be written as a linear combination of the basic measurements. These linear combinations are used to form a tableau as in the earlier example.

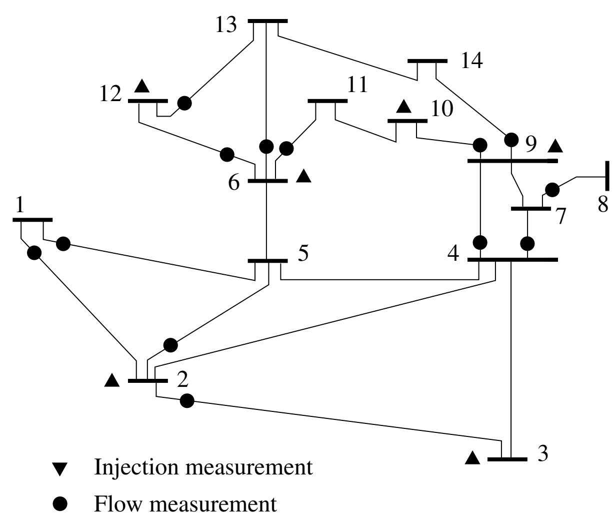

Fig. 3. IEEE 14-Bus system with measurements

According to the tableau, the measurement $P_{7,8}$ is critical, since the column associated to $P_{7,8}$ contains only zeros. In order identify the critical sets of type 1 all nonbasic measurements are removed from original tableau, one at a time. After removing $P_{9,14}$, two new critical measurements, P_9 and $P_{4,7}$, are identified and, therefore, a critical set is formed by $P_{9,14}$, P_9 and $P_{4,7}$. Removing the remaining nonbasic measurements no new critical sets of type 1 are identified. Finally, in order to identify the critical sets of type 2, the noncritical basic measurements that does not belong to any critical set are removed, one at a time. After the removal of measurement $P_{1,2}$, the measurement $P_{1,5}$ is identified as a new critical measurement and, therefore, the measurements $P_{1,2}$ and $P_{1,5}$ form a critical set of type ii. After the removal of measurement P_3, the measurements P_6, P_{10},$P_{2,3}$ and $P_{6,11}$ are identified as new critical measurements and, therefore, the measurements P_3, P_6, P_{10},$P_{2,3}$ and $P_{6,11}$ form a critical set of type 2. The remaining noncritical basic measurements that does not belong to any critical set are P_{12}, $P_{4,9}$ and $P_{6,12}$, after their removal no new critical sets are identified. This results are the same presented in the reference [1]. The summary with the results for IEEE-14-Bus system is shown in Table II.

TABLE II
SUMMARY OF THE RESULTS FOR IEEE-14 BUS SYSTEM

	Measurements
Critical Measurements	$P_{7,8}$
Critical Set of type 1	P_9, $P_{9,14}$, $P_{4,7}$
Critical Set of type 2	$P_{1,2}$, $P_{1,5}$
Critical Set of type 2	P_3, P_6, P_{10}, $P_{2,3}$,$P_{6,11}$

VI. Conclusion

This paper presents a numerical algorithm for the identification of critical measurements and critical sets in power system state estimation based on the use of Gram matrix. This algorithm is part of a more general application of Gram matrix in state estimation, which can be applied in whole state estimation process. The complexity of Gram matrix approach is no higher than the standard approach such as those based on least squares methods. The identification of critical measurements and critical sets can be useful in the planning of optimal metering systems.

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