

# Loss Reduction as a Mixed Integer Optimization Problem

Stéphane Fliscounakis, Fabrice Zaoui, Marie-Pierre Houry and Emilie Milin

**Abstract**—This paper deals with the influence of network topological changes on the resistive losses for a significant period. It is based on a previous work [1] where the goal was to reduce power losses on a peak load situation. The second part of the work, presented here, attempts to reduce the energy losses of transmission network by implementing a cyclic search of topology changes and phase shifting adjustments over a week time period. As the whole problem for such a long period is not tractable in a Mixed Integer Non-Linear Programming approach, a piecewise-linear formulation is presented in order to find a convenient relation between active losses and real flows for each branch. Subsequently, a mixed integer linear formulation is developed to find an optimal topology that tends to minimize the active losses. Considered topological configurations concern the splitting or merging of electric buses and the line switching. Once the new topology has been identified, the interactions between real and reactive power are handled to adjust the taps of phase shifting transformers through a Mathematical Program with Equilibrium Constraints (MPEC). A practical example based on several hundreds of observations (snapshots) of the French national grid is presented. Obtained results confirm the interest of the method.

## I. INTRODUCTION

The French Transmission System Operator RTE is responsible for purchasing energy to compensate for system losses on the French High Voltage Grid. These system losses, around 12 TWh each year, represent a significant cost. So the challenge for the company is to manage system losses cost by controlling purchase cost, establishing good losses forecasts and increasing network efficiency. The method described in this paper is a possible way to improve efficiency by reducing system losses on the network with a cyclic search of network topological changes and phase shifting adjustments applicable along a week time period.

To find a more suitable topology, authors propose a Mixed Integer Linear Programming (MILP) formulation to reduce the active power flows that are mainly responsible for resistive losses. Two steps are defined to reach this objective. Firstly the goal is to identify the links between active power losses and flows on each circuit with the consideration of a sufficient large enough number of achieved data (snapshots). Once those links are established thanks to a linear optimizer, the search for an optimal topology that would reduce losses, for the snapshots period, is initialized.

Topology changes are modeled with Boolean variables for the optimization process. They concern the splitting or

merging of electric buses, the multiple connection of a circuit (line switching) [1]. The security is taken into account with constraints on the flow and short-circuit current limits [4,5].

Tap adjustments require a mixed integer non-linear optimization when on-load tap changers (OLTC) are concerned. Mathematical program with equilibrium constraints (MPEC) approach [6] makes the problem tractable when the topology is fixed, due to the small number of discrete variables involved.

## II. METHODS TO REDUCE RESISTIVE LOSSES

### A. Power loss reduction based on an estimated state

The substation topology changes are commonly defined off-line by operators and engineers using their expertise. To cope with the possible limits violation of short-circuit currents by bus merging, the balanced faults are simulated in any substation candidate for switching. The proposed method aims at minimizing the active power losses while satisfying security constraints for the base and contingency situations. A topology candidate is obtained through a DC optimization whose objective function is described in section III. With respect to this topology, the AC optimization detailed in section IV adjusts the phase shifting transformer tap positions.

**do:**

- **min** approximate losses (bus merging, line switching)
- s.t.** [1], [5]:
  - DC approximation of flow limit constraints for the base case and a harmful contingencies (HC) subset
  - short-circuit current constraints for a balanced faults (BF) subset
  - the number of bus merging is limited
  - the number of line switching is limited
- AC optimization of the phase shifting transformer taps, based on the candidate topology
- AC security assessment and balanced fault analysis

**while** modifying subsets HC and/or BF is required.

An AC security and balanced fault analysis are processed on the network solution. Modifying HC and/or BF subsets is required if the contingencies and/or balanced faults which result in limit violations have never been identified. If no limit violation occurs, the algorithm is successful.

### B. Energy loss reduction on a period

We wish to investigate if a unique switching strategy holds for a period (for instance a week), i.e. meets the security rules and leads to substantial energy loss savings. It appears that a very good candidate is the most frequent set of topological or

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transformer modifications obtained from the above process applied to the snapshots of the period. The storage of snapshots, which is an automatic task in Energy Management Systems, provides here a crucial tool.

### III. ACTIVE LOSSES AND FLOWS

A relation between active power losses and flows is searched for each circuit. This relation is an approximation based on a piecewise-linear formulation [2]. Even if the underlying relation is naturally quadratic, the piecewise-linear approach can also give a pretty good approximation considering a sufficient number of slopes.

In addition, slopes are calculated with the help of a large number of observations (network snapshots coming from a state estimator) for robustness. Typically several hundreds of snapshots are considered to get significant relations between active losses and flows.

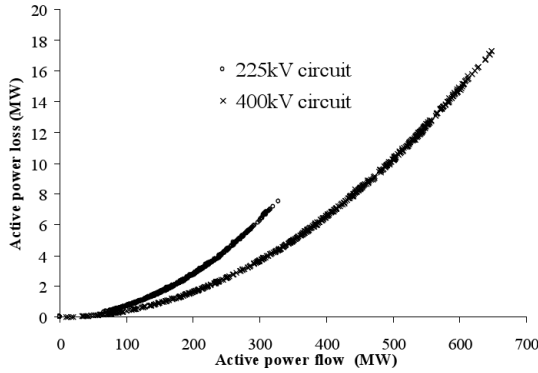


Fig. 1. Estimated active power losses and flows on two particular circuits and for about 600 situations

#### A. Identification of slopes

Slopes are determined from a learning set with the minimization of a cost function that is the average value of the approximation errors in absolute values on all the circuits and situations.

This problem is purely linear and involves a huge set of variables and constraints due to the high number of considered (EHV 400/225 kV) network situations, typically four snapshots per hour for a week time period. Thus a particular attention must be paid to numerical issues arising from such a big problem. In order to work with a minimal number of variables, preliminary studies have shown that only two slopes were necessary to reach the feasibility of problem (1).

$$\min \frac{1}{\kappa \times L} \sum_{\tau=1}^{\kappa} \sum_{l=1}^L \nu_l^{+\tau} + \nu_l^{-\tau} \quad (1)$$

$$\text{s.t.} \begin{cases} \forall \tau = 1.. \kappa, \forall l = 1..L : \\ p_l^{\tau} - \sum_{j=1}^J c_{l,j} \times T_{l,j}^{\tau} = \nu_l^{+\tau} - \nu_l^{-\tau} \\ \forall l = 1..L, \forall j = 1..J - 1 : \\ 0 \leq c_{l,j} \leq c_{l,j+1} \\ \forall \tau \in S : \\ \sum_{l=1}^L \nu_l^{+\tau} = \sum_{l=1}^L \nu_l^{-\tau} \\ \forall \tau = 1.. \kappa, \forall l = 1..L : \\ \nu_l^{+\tau}, \nu_l^{-\tau} \geq 0 \end{cases}$$

where:

- $\kappa$  is the number of cases;
- $S$  is the set of selected snapshots;
- $L$  is the number of circuits in the network;
- $J$  is the number of piecewise-linear approximations;
- $p_l^{\tau}$  is the estimated active loss value for the circuit  $l$  and the situation  $\tau$ ;
- $c_{l,j}$  is the slope of each piecewise-linear approximation on the circuit  $l$ ;
- $T_{l,j}^{\tau}$  is a  $j$ -discretized value of active power flow for the circuit  $l$  and the situation  $\tau$ ;
- $\nu_l^{+\tau}$  and  $\nu_l^{-\tau}$  are slack variables for the absolute value formulation.

The  $j$ -discretized value  $T_{l,j}^{\tau}$  is initially calculated from the estimated active power flow  $T_l^{\tau}$  on the circuit  $l$  and the situation  $\tau$ :

$$T_l^{\tau} = \sum_{j=1}^J T_{l,j}^{\tau} \quad (2)$$

If  $h_l$  denotes the step length associated to the real flow range of the  $l$ -circuit, then all the  $T_{l,j}^{\tau}$  are equal to  $h_l$  except one.

In problem (1), only  $c_{l,j}$ ,  $\nu_l^{+\tau}$  and  $\nu_l^{-\tau}$  are the variables. All other references are data. First constraints give the desired link between losses and flows. Second relations on coefficients  $c_{l,j}$  impose a convexity behavior for the piecewise-linear approximation. The third kind of constraints:  $\sum_{l=1}^L \nu_l^{+\tau} = \sum_{l=1}^L \nu_l^{-\tau}$ , enforce a perfect accuracy on the learning set, i.e. the estimated and approximated global losses are equal. Last constraints deal with the positivity of variables.

Finally, the objective function "approximate losses"  $\ell(T)$  of the DC OPF problem formulated in section II-A can be expressed with the help of the previously calculated coefficients  $c_{l,j}$ , considered as parameters :

$$\ell(T) = \sum_{l=1}^L \sum_{j=1}^J c_{l,j} \times T_{l,j} \quad (3)$$

where  $T_{l,j}$  is now a variable.

#### B. Rejected snapshots

The identification process presented in the section III-A suffers from the difficulty in handling equality constraints. Using multiple piecewise-linear approximations which only have real powers as variables may lead to unfeasible cases. Moreover, for a long time period, nothing guarantees that the

sum of the flows calculated by the state estimator is error-free. Thus, the ability to automatically detect and reject such snapshots is necessary in order to perform a cyclic solution of (1). In order to determine the maximal number of snapshots which respect the constraints:

$$P_\tau = \sum_{l=1}^L \sum_{j=1}^J c_{l,j} \times T_{l,j}^\tau = \sum_{l=1}^L p_l^\tau \quad (4)$$

and exhibit large values of  $P_\tau$ , we perform the following MIP optimization:

$$\min \sum_{\tau=1}^{\kappa} P_\tau r_\tau \quad (5)$$

$$\text{s.t.} \left\{ \begin{array}{l} \forall \tau = 1.. \kappa : \\ r_\tau \in \{0, 1\} \\ \sum_{l=1}^L \sum_{j=1}^J c_{l,j} \times T_{l,j}^\tau \leq P_\tau \times (1 + r_\tau) \\ \sum_{l=1}^L \sum_{j=1}^J c_{l,j} \times T_{l,j}^\tau \geq P_\tau \times (1 - r_\tau) \\ \forall l = 1..L, \forall j = 1..J - 1 : \\ 0 \leq c_{l,j} \leq c_{l,j+1} \end{array} \right.$$

where  $r_\tau$  is a Boolean variable equal to 1 when the situation  $\tau$  is rejected.

Then we define a feasible learning set of selected snapshots used in section III-A as:

$$S = \{\tau \mid r_\tau = 0\} \quad (6)$$

#### IV. MPEC CONSTRAINTS TO MODEL PHASE SHIFTING TRANSFORMERS (PST)

Fig. 2 and 3, describe a technology and type of phase shifter which invalidates the decoupling assumption, and thus requires an AC representation instead of the DC approximation adopted in the previous paragraphs. The phase shift regulation may be on the shunt or the series transformer.

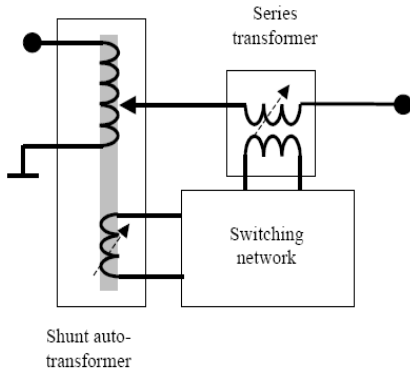


Fig. 2. In-phase regulating auto-transformer

When adjusting transformer tap settings, the current ratio of the in-phase transformer  $r$  and the current phase shift  $\alpha$  become discrete variables,  $X(r, \alpha)$ , the sum of the in-phase transformer and phase shifter reactances, and  $\rho$  vary as a consequence.

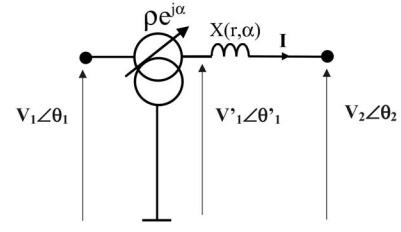


Fig. 3. One phase diagram

$$V_2 + jX(r, \alpha)I = rV_1(1 + e^{j\theta} \frac{\tan \alpha}{\sin \theta - \tan \alpha \cos \theta}) \quad (7)$$

The model (7) captures the more general class of asymmetrical phase shifters : the quadrature boosters are obtained by setting the boost voltage angle parameter  $\theta$  to  $\frac{\pi}{2}$  and keeping  $r$  constant.

For a phase shifting transformer  $d$  ( $d \in P$ ), the associated characteristics, the shifting angle  $\alpha_{t,d}$ , the turns ratio  $\rho_{t,d}$  and the impedance  $Z_{t,d}$  are deduced from the tap position  $t$  ( $t \in A(d)$ ). The following problem (8) determines tap positions which maximize the minimum load increase denoted by  $\Lambda$ . The active nodal injection of each generator bus  $i$  ( $i \in G$ ) is fixed to the estimated value  $P_i^{est}$ , so the problem (8) is equivalent to minimize losses for this choice of load increments.

$$\max \Lambda \quad (8)$$

$$\text{s.t.} \left\{ \begin{array}{l} \forall i \in G \\ P_i^{est} = \phi_i(\theta, V, \alpha, \rho, X) \\ Q_i^{min} \leq \psi_i(\theta, V, \alpha, \rho, X) \leq Q_i^{max} \\ V_i^{est} = V_i \\ \forall i \in N - G \\ \lambda_i P_i^{est} = \phi_i(\theta, V, \alpha, \rho, X) \\ \lambda_i Q_i^{est} = \psi_i(\theta, V, \alpha, \rho, X) \\ \Lambda \leq \lambda_i \\ \text{Voltage Limits :} \\ \forall i \in N - G \quad V_i^{min} \leq V_i \leq V_i^{max} \\ \text{Current Limits :} \\ \forall l \in L \quad I_l(\theta, V, \alpha, \rho, X) \leq I_l^{max} \\ \forall d \in D \\ \sum_{t \in A(d)} x_{t,d} = 1 \\ \alpha_d = \sum_{t \in A(d)} x_{t,d} \alpha_{t,d} \\ \rho_d = \sum_{t \in A(d)} x_{t,d} \rho_{t,d} \\ X_d = \sum_{t \in A(d)} x_{t,d} X_{t,d} \\ \text{MPEC Constraints :} \\ \forall t \in A(d) \quad \min(x_{t,d}, 1 - x_{t,d}) = 0 \end{array} \right.$$

where

- $\theta$  and  $V$  are vectors composed of the voltage angles and magnitudes ;

- $\lambda$  is a vector composed of the nodal load indices ;
- $\phi$  and  $\psi$  are vector valued functions composed of the active and reactive injections
- $I$  is a vector valued function composed of the currents through the lines and transformers;

Denoting by  $\Lambda_0$  the objective value of the problem (8) when the initial topology of the snapshot is used, if a candidate topology, computed through the search described in the previous sections, leads to an objective value of  $\Lambda$  such that  $\Lambda \leq \Lambda_0$ , it means that any proportional load increase causes constraint violations, whatever are the modifications of the phase-shifting transformer tap positions. It is suitable to reject such candidate topology.

In a classical non linear programming approach, discrete variables are treated as continuous ones until they are almost optimized and then roundoff to their nearest discrete values. Here, due to the MPEC constraints, each feasible solution of (8) exhibit the following property : for a phase shifter  $d$ , the variables  $x_{t,d}$  ( $t \in A(d)$ ) are all equal to zero except one, the corresponding index gives the tap position.

The solution of (8) is determined through a relaxation scheme [6] that does not force the strictly feasible regions of the relaxed problems to become empty in the limit. Setting  $x_1 = x_{t,d}$  and  $x_2 = 1 - x_{t,d}$ , a relaxed problem is defined by replacing each MPEC constraint

$$\min(x_{t,d}, 1 - x_{t,d}) = 0 \text{ by } x_1 \geq -\delta_1 \quad x_2 \geq -\delta_2 \quad x_1 x_2 \leq \delta_c$$

Due to the information provided by the multipliers of the above inequalities, there is no need to drive both the relaxation parameters  $\delta_1$ ,  $\delta_2$  and  $\delta_c$  to zero to recover a stationary point of (8). As a result, one can adapt a standard interior-point method [8] without having to modify the search direction, only one interior iteration is performed per relaxed problem. The performance of this relaxation scheme has been proved in practice on a subset of the MacMPEC test problems.

## V. NUMERICAL RESULTS

Three cases are studied, the first on the IEEE 30-bus system, the second and the third on the French EHV transmission network :

- two topologies evaluated by the mean of the PST optimization described in section IV
- a peak load situation where most of the candidate bus merging imply security violations. It corresponds to the smallest set of feasible solutions and leads to a poor decrease of power loss;
- a week time period described by 672 snapshots where the loads are much lower. The objective is to reduce the system energy loss with a small number of topological modifications which meet the security

### A. Two topologies on the IEEE 30-bus system

To check ac network security violations, the problem (8) is solved for the topologies T1 and T2 shown in figure 4.

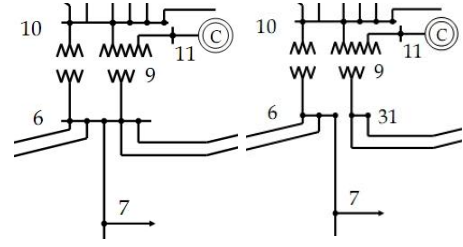


Fig. 4. topologies T1 (initial) and T2 (6 split in two nodes)

The lower voltage magnitude limits at all buses are 0.95 p.u., and the upper limits are 1.1 p.u. for generator buses and 1.06 p.u. for the remaining buses. If an initial branch current is lower than 0.5 p.u., the corresponding branch current limit in (8) is fixed to 0.5 p.u., otherwise an increase of the branch current up to 20% is authorized.

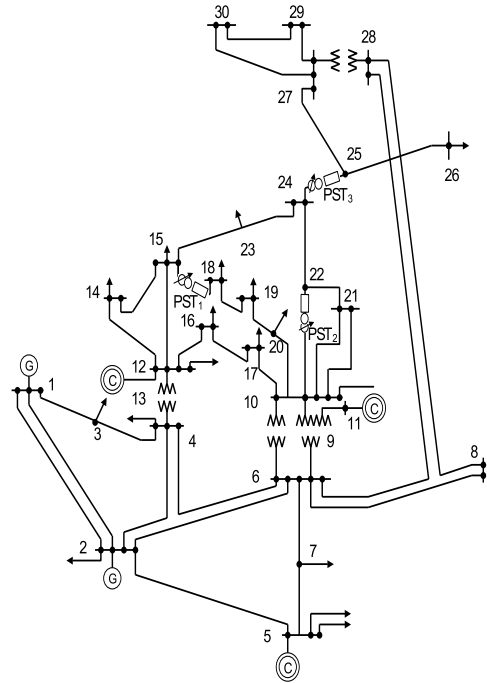


Fig. 5. Modified IEEE 30-bus

Fig. 5 shows that three identical quadrature booster PST transformers have been installed in series with the initial lines on the best locations identified by [9], i.e. the branches 15-18, 10-22 and 24-25. If  $\phi_{max}^{pst}$ ,  $X_{max}^{pst}$  and  $X_0^{pst}$  design respectively the maximal phase shift, the equivalent PST reactances at maximal phase shift and at zero phase shift, the hypothetical parameter values adopted for this test are :

$$\phi_{max}^{pst} = 10^\circ \quad X_{max}^{pst} = 0.15 \text{ p.u.} \quad X_0^{pst} = 0.1 \text{ p.u.} \quad (9)$$

Assuming the reactance of the regulating winding varies as the square of the turns, the current equivalent PST reactance is given by [10]:

$$X_l(\phi_l^{pst}) = X_0^{pst} + (X_{max}^{pst} - X_0^{pst}) \frac{\tan^2 \phi_l^{pst}}{\tan^2 \phi_{max}^{pst}} \quad (10)$$

where  $\phi_l^{pst}$  is the current phase shift of the PST  $l$ . The discrete control device is illustrated by enforcing integer values in

the range  $[-\phi_{max}^{pst}, +\phi_{max}^{pst}]$  for the quantity  $\phi_l$  expressed in degrees.

When the topologies T1 and T2 are successively adopted, the solutions of the problem (8) provide two slightly different objective values  $\Lambda(T1)=1.000$  and  $\Lambda(T2)=0.971$ .

The current and voltage limits have no influence on the value  $\Lambda(T1)$ . On the contrary, the topology T2 is clearly not feasible from the security point of view : while a load shedding has been performed, the best adjustment of the phase shifting transformers leads to a binding current limit constraint on the branch 4-12.

The module  $\rho_l(\phi_l^{pst})$  of the complex turn ratio is represented as function of  $\phi_l^{pst}$  and is given by :

$$\rho_l(\phi_l^{pst}) \cos \phi_l^{pst} = 1.0 \quad (11)$$

From Tables I and II below, we see the coupling effect of equations (10) and (11).

TABLE I  
IMPACT OF PST OPTIMIZATION ON TURN RATIOS

branch	complex turn ratio	Turn ratios (p.u./degrees)	
		topology T1	topology T2
15 – 18	$\rho_1(\phi_1^{pst}) \angle \phi_1^{pst}$	1.0 $\angle$ 0.0	1.0125 $\angle$ - 9
10 – 22	$\rho_2(\phi_2^{pst}) \angle \phi_2^{pst}$	1.0 $\angle$ 0.0	1.0015 $\angle$ + 1
24 – 25	$\rho_3(\phi_3^{pst}) \angle \phi_3^{pst}$	1.0 $\angle$ 0.0	1.0154 $\angle$ + 10

TABLE II  
IMPACT OF PST OPTIMIZATION ON BRANCH REACTANCES

branch	Reactances (p.u.)	
	topology T1	topology T2
15 – 18	0.2185	0.2588
10 – 22	0.1499	0.1504
24 – 25	0.3292	0.3792

### B. Peak Load Situation

The worst-case network for the value of resistive losses is made of 1130 buses and 1872 branches. The values of the objective function enable a classification of the candidate topologies. Fig. 6 illustrates the fact that the value of losses decrease when the maximum number of bus merging increases. However, the gain is seen to reduce quickly as the number of merging exceeds 20.

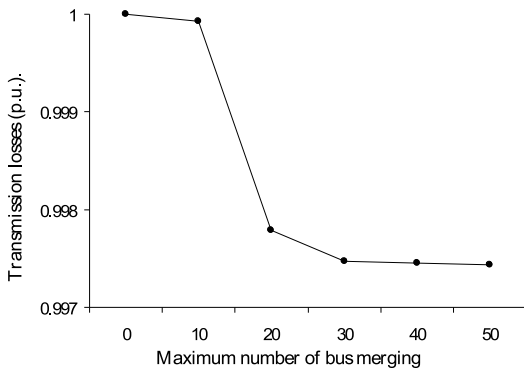


Fig. 6. Loss reduction as a function of number of bus merging

The on-line diagram of tables III and IV show why a specific bus merging at substation *Ulys* has been rejected, see Fig. 7. This new topology would result in:

- current violations when a fault occurs at bus *Ulys* and *Pria* (short-circuit analysis);
- overloads when one of the two lines *Pria-Ulys* is disconnected.

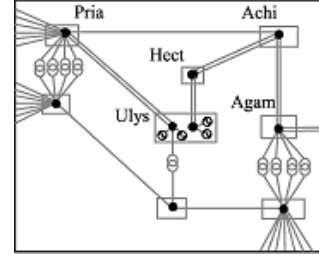


Fig. 7. Sub-network of the french EHV transmission network

TABLE III  
SHORT-CIRCUIT ANALYSIS

Bus	Fault	Positive sequence current (p.u.)	
		Merging Ulys	Original
Ulys	Ulys	1.43j	1.0j & 0.71j
Pria	Pria	1.45j	1.29j
Hect	Hect	1.08j	0.86j

TABLE IV  
CONTINGENCY ANALYSIS

Branch	Outage(s)	Active power flow (p.u.)	
		Merging Ulys	Original
Pria-Ulys(2)	Pria-Ulys(1)	1.17	1.0
Ulys transformer	Pria-Ulys(1)	0.25	0.23
Pria-Achi	Pria-Ulys(1,2)	0.47	1.04

Note that the new configuration of substation *Ulys* perfectly explains the violations: all the four generators are connected to one bus.

### C. Week Time Period

Why are two slopes necessary to achieve the identification process on a week with 672 snapshots? The assessment of piecewise-linear approximation accuracy is based on the optimum solution of (5) which corresponds to:

- none rejected snapshot in the case of two slopes;
- 408 rejected snapshots in the case of one slope.

The last result indicates that power losses cannot be related to active flows with the help of a linear expression. The first result reveals that a convex piecewise-linear approximation is feasible although it requires a periodic identification.

The linear problem (1) is optimized for 672 snapshots with an Interior Point Method initially developed at RTE for non-linear problems [8]. This problem is of a very large size: the number of variables is 2223592 and the number of constraints is 1112468. It is solved on an Intel® Xeon™ 3.6 GHz Linux™ 32 bits platform in 102 iterations and only 33 minutes. Mixed

integer problems presented in subsections III-A and III-B are solved with the commercial optimizer Xpress<sup>MP</sup> [7].

The difficulty mainly relies on the combinatory aspect imposed by the number of authorized network switching in problem (3). As computations are performed off-line, some long optimization times are absolutely not a problem since they remain much lower than a week time period.

Fig. 8, illustrates the gain on energy loss obtained by the mean of one transition of three discrete variables, which implies bus merging in three 225 kV substations.

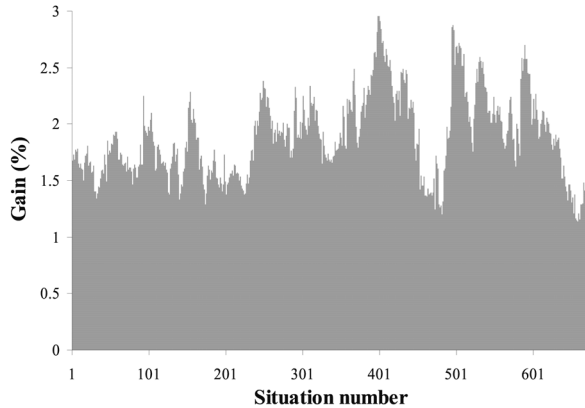


Fig. 8. Gain on losses for the most frequent topological action over a week

The switching strategy is simply determined by the most frequent solution, left-sided on the Fig. 9, of the problem presented in the subsection II-A, processed for every snapshot.

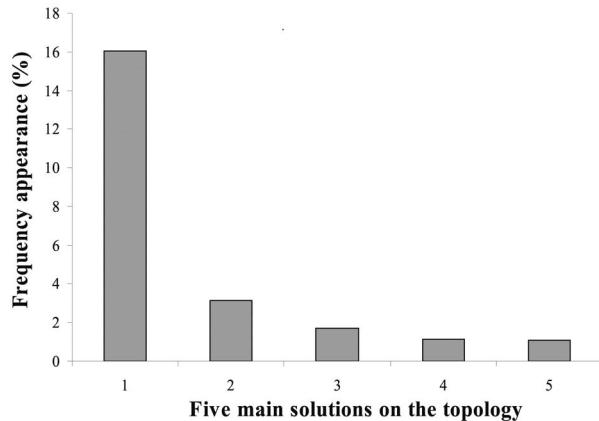


Fig. 9. Best frequency appearance of some topological modifications over a week

Table V shows the optimal mean gain obtained on energy losses over a week for one, two and three possible bus merging among a set of 86 possibilities.

TABLE V  
GAINS ON LOSSES OVER A WEEK FOR OPTIMAL NETWORK CONFIGURATIONS

Number of possible bus merging	Mean gain on loss
1	0.74%
2	1.24%
3	1.63%

## VI. CONCLUSION

Previous work [3] has shown the interest of the network switching to reduce losses. In this paper, a problem of the reduction of energy loss by implementing a weekly cyclic search of topology changes is presented. For each snapshot during the week time period, the calculation of the solution takes into account in a preventive way the static security including short-circuit constraints. Due to the computational burden, the proposed method gives suboptimal but secure solutions.

Numerical results carried out on a real transmission 400/225 kV system are used to investigate if a unique switching strategy holds for a week, i.e. met the security rules and leads to substantial energy loss savings. It appears that a very good candidate is the most frequent set of topological modifications.

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