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Distance Relaying Algorithm Using a DFT-based Modified Phasor Estimation Method

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Abstract-- **In this paper, we propose a distance relaying algorithm using a discrete Fourier transform (DFT)-based modified phasor estimation method to eliminate the adverse influence of exponentially decaying DC offsets. Most distance relays are based on estimating phasors of the voltage and current signals. A DFT is generally used to calculate the phasor of the fundamental frequency component in digital protective relays. However, the output of the DFT contains an error due to exponentially decaying DC offsets. For this reason, distance relays have a tendency to over-reach or under-reach in the presence of DC offset components in a fault current. Therefore, the decaying DC components should be taken into consideration when calculating the phasor of the fundamental frequency component of a relaying signal. The error due to DC offsets in a DFT is calculated and eliminated using the outputs of an even-sample-set DFT and an odd-sample-set DFT, so that the phasor of the fundamental component can be accurately estimated. The performance of the proposed algorithm is evaluated for a-phase to ground faults on a 345 kV, 50 km, simple overhead transmission line. The Electromagnetic Transient Program (EMTP) is used to generate fault signals. The evaluation results indicate that adopting the proposed algorithm in distance relays can effectively suppress the adverse influence of DC offsets.**

Index Terms— **Distance Relay, DC Offset, Phasor Estimation, Discrete Fourier Transform**

I. INTRODUCTION

THE continuous expansion of power systems, in both scale and complexity, has imposed a requirement for fast and accurate fault clearance in order to improve system stability and reliability. If a fault in an important transmission line is not identified and removed as quickly as possible, it might lead to widespread damage to the power system. In order to prevent the damage from spreading to the healthy parts of the power system, protective relays need to detect the faults even within one or two cycles of the power frequency. This makes the task more challenging since less data are provided to extract the desired frequency component.

Over the last two decades, digital distance relays have been applied to power transmission lines. Most distance relays are based on estimating phasors of the voltage and current signals. A discrete Fourier transform (DFT) is generally used to calculate the phasor of the fundamental frequency component in digital protective relays. However the output of the DFT contains an error due to exponentially decaying DC offsets. For this reason, the distance relays have a tendency to over-reach or under-reach in the presence of DC offset components in a fault current. Therefore, the decaying DC components should be taken into consideration in calculating the phasor of the fundamental frequency component of a relaying signal.

Several numerical techniques have been proposed to reduce or remove the adverse effects of a decaying DC component. In [1], a DFT algorithm with a preconditioning digital mimic filter was proposed to suppress the DC offset in a current waveform. The mimic filter can completely remove the decaying DC offset only when the actual and presumed time constants of the DC offset match. Modified Fourier algorithms [2], [3] have been proposed to remove the effect of the DC offset in relaying signals. These methods require one cycle plus two samples to eliminate the effect of the DC offset. Partial sum (PS)-based method using two values that are partially summed with one cycle sample data was proposed in [4]. Although this method is inherently immune to a decaying DC offset and random noise, it produces errors when the input signal contains two decaying DC components with different time constants.

In this paper, we propose a distance relaying algorithm using a DFT-based modified phasor estimation method [5] to eliminate the adverse influence of exponentially decaying DC offsets. The proposed phasor estimation method uses an even-sample-set DFT and an odd-sample-set DFT to reduce the length of the data window to only one cycle. The error due to DC offsets on the DFT is calculated and eliminated using the outputs of the even- and odd-sample-set DFTs so that the phasor of the fundamental component can be accurately estimated. The performance of the distance relay designed by the proposed algorithm is evaluated for a-phase to ground (a-g) faults on a 345 kV, 50 km, simple overhead transmission line. The Electromagnetic Transient Program (EMTP) was

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used to generate the fault signals. To demonstrate the performance of the proposed algorithm, the test results are compared to those of the conventional DFT and the PS-based method [4]. Evaluation result indicates that adopting the proposed algorithm in a distance relay can effectively suppress the adverse influence of DC offsets.

II. PROPOSED DISTANCE RELAYING ALGORITHM

A. New Fourier Algorithm [5]

The fault current flowing through a primary circuit can be generally considered as the combination of an exponentially decaying DC offset and sinusoidal components. If it is assumed that sinusoidal components with frequencies higher than the (N/2–1) order harmonic are eliminated by an anti-aliasing low-pass filter, the discrete current signal measured by a protective relay can be expressed as

$$i\,[n] = \mathrm{A}_0 e^{-n\Delta t/\tau} + \sum_{k=1}^{N/2-1} A_k \cos\left(\frac{2\pi k}{N} n + \varphi_k\right) \tag{1}$$

where, τ and A_0 are the primary time constant and the magnitude of an exponential component, A_k and φ_k are the amplitude and the phase angle of the kth harmonic component, Δt is the sampling interval and N is the number of samples per cycle.

The phasor of the fundamental frequency component is calculated by the DFT as follows.

$$I_{DFT} = \frac{2}{N} \sum_{n=0}^{N-1} i[n] \cdot e^{-j\frac{2\pi}{N}n} = I_{DFT}^{1th} + I_{DFT}^{dc} \tag{2}$$

where $I_{DFT}^{1th} = A_1 e^{j\phi_1}$

$$I_{DFT}^{dc} = \frac{2}{N} A_0 \,(1 - E^N) \Big/ (1 - E e^{-j\frac{2\pi}{N}})$$

$$E = e^{-\Delta t/\tau}$$

Although the DFT is simple and easy to implement, the output of the DFT for the fundamental frequency component, I_{DFT}, contains an error, I_{DFT}^{dc}, due to the exponentially decaying DC offset.

Equation (2) can be decomposed into the even-sample-set DFT, I_{DFT}^{even}, and the odd-sample-set DFT, I_{DFT}^{odd}, as follows.

$$\begin{aligned} I_{DFT} &= \frac{2}{N} \sum_{n=0}^{N-1} i[n] \cdot e^{-j\frac{2\pi}{N}n} \\ &= \frac{2}{N} \sum_{n=0}^{N/2-1} i[2n] \cdot e^{-j\frac{2\pi}{N}2n} + \frac{2}{N} \sum_{n=0}^{N/2-1} i[2n+1] \cdot e^{-j\frac{2\pi}{N}(2n+1)} \\ &= I_{DFT}^{even} + I_{DFT}^{odd} \end{aligned} \tag{3}$$

The even- and odd-sample-set DFTs can be rearranged as follows.

$$\begin{aligned} I_{DFT}^{even} &= \frac{2}{N} \sum_{n=0}^{N/2-1} i[2n] \cdot e^{-j\frac{2\pi}{N}2n} \\ &= \frac{1}{2} A_1 e^{j\phi_1} + \frac{2}{N} A_0 \frac{1 - E^N}{1 - (E e^{-j\frac{2\pi}{N}})^2} \end{aligned} \tag{4}$$

$$\begin{aligned} I_{DFT}^{odd} &= \frac{2}{N} \sum_{n=0}^{N/2-1} i[2n+1] \cdot e^{-j\frac{2\pi}{N}(2n+1)} \\ &= \frac{1}{2} A_1 e^{j\phi_1} + \frac{2}{N} E e^{-j\frac{2\pi}{N}} A_0 \frac{1 - E^N}{1 - (E e^{-j\frac{2\pi}{N}})^2} \end{aligned} \tag{5}$$

The even- and odd-sample-set DFTs have the same fundamental frequency component, $A_1 e^{j\phi_1}/2$. This component can be eliminated by subtracting (5) from (4).

$$I_{DFT}^{even} - I_{DFT}^{odd} = \frac{2}{N} A_0 \,(1 - E^N) \Big/ (1 + E e^{-j\frac{2\pi}{N}}) \tag{6}$$

The DFT output of the fundamental frequency component due to the decaying DC component, I_{DFT}^{dc}, in (2) can be rewritten as follows.

$$I_{DFT}^{dc} = \left(I_{DFT}^{even} - I_{DFT}^{odd}\right) \cdot (1 + E e^{-j\frac{2\pi}{N}}) \Big/ (1 - E e^{-j\frac{2\pi}{N}}) \tag{7}$$

where $E = \dfrac{K_{Im}}{K_{Re} \sin(2\pi/N) - K_{Im} \cos(2\pi/N)}$

$$K_{Re} = Re\left\{I_{DFT}^{even} - I_{DFT}^{odd}\right\}$$

$$K_{Im} = Im\left\{I_{DFT}^{even} - I_{DFT}^{odd}\right\}$$

Therefore, the accurate fundamental frequency component phasor of the input signal, I_{DFT}^{1th}, can be obtained from

$$I_{DFT}^{1th} = I_{DFT} - I_{DFT}^{dc} \tag{8}$$

B. Distance Relaying Algorithm

Fig. 1 shows a flowchart describing distance relaying algorithm adopting a DFT-based modified phasor estimation method. A distance relay is provided with voltage and current signals from one end only of the protected circuit, and is expected to assess the location of a fault on the basis of these locally measured signals. Most distance relays discriminate between load and fault conditions by measuring both the magnitude and angle of the impedance presented to them. This means that the distance relays estimate the phasors of the voltage and current signals to calculate the impedance.

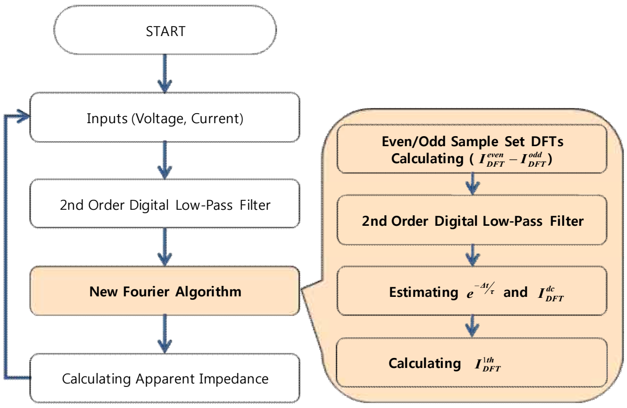


Fig. 1. Flowchart of the proposed algorithm

A DFT is generally used to calculate the phasor of the fundamental frequency component in digital protective relays. However, instead of the conventional DFT, the new Fourier algorithm is used for the distance relaying algorithm. A subtraction process, such as (6), in the new Fourier algorithm normally magnifies high frequency noises in input signals. Although the DFT inherently has very robust characteristics over the high-frequency noises, the subtraction process after the DFT can cause an error. This drawback can be easily suppressed by a simple low-pass filter, like the second one in the right-hand side of the flowchart.

A ground distance relay calculates the apparent impedance using the estimated phasors (V_{app} and I_{app}) as

$$Z_{app} = \frac{V_{app}}{I_{app} + \left(Z_{L0} / Z_{L1} - 1\right) \cdot I_0} \tag{9}$$

where Z_{L1} and Z_{L0} are the positive- and zero-sequence line impedances, respectively, and I_0 is the zero-sequence current from the relaying point into the fault.

III. SIMULATIONS OF THE DISTANCE RELAYING ALGORITHM

A. System Configuration

In this section, the performance evaluation for the proposed distance relaying algorithm is described.

The current and voltage singles are generated by the EMTP. The model system for the simulations is a 345 kV, 50 km overhead transmission with sources at both ends, as shown in Fig. 2. The transmission line parameters used in the simulations are given in Table 1.

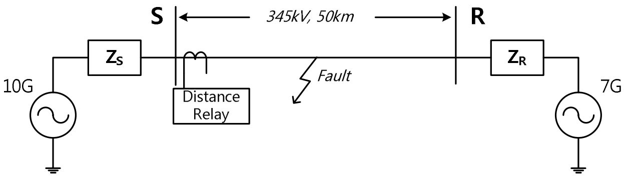


Fig. 2. Single-line diagram of the model system

TABLE I
OVERHEAD TRANSMISSION LINE PARAMETERS

Sequence	Parameter	Value	Units
Positive & Negative	R_1, R_2	0.0345	*Ω/km*
	L_1, L_2	0.9724	*mH/km*
	C_1, C_2	0.0117	*μF/km*
Zero	R_0	0.2511	*Ω/km*
	L_0	2.7058	*mH/km*
	C_0	0.0045	*μF/km*

TABLE II
EQUIVALENT SOURCE PARAMETERS

Sequence	S [GVA]	X/R
Z_{S0}	5.00	5.00
Z_{S1}	10.00	3.00

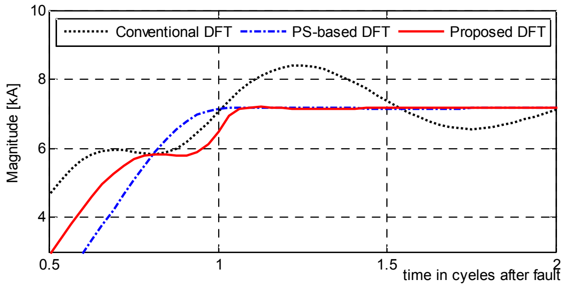
	Case 1		Case 2	
Sequence	S [GVA]	X/R	S [GVA]	X/R
Z_{R0}	3.50	5.00	3.50	7.00
Z_{R1}	7.00	10.00	7.00	14.00

Zone 1 of the distance relay was set to 80% of the transmission line impedance with no intentional time delay. A-phase to ground faults with fault resistances of 0 Ω, 5 Ω, and 10 Ω were applied at a distance of 41 km from the relaying point, *i.e.*, close-in faults from the end of Zone 1. A fault inception angle of 0° was considered for the maximum magnitude of the DC component.

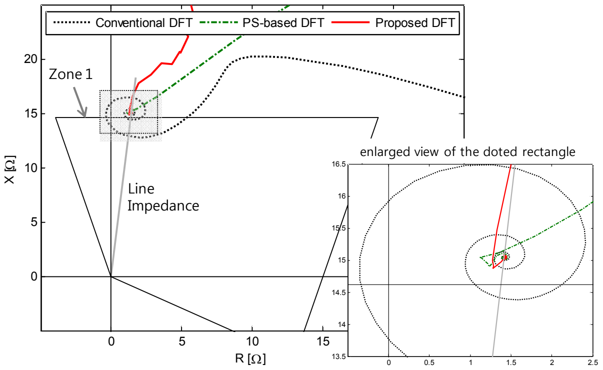
The time constant and amplitude of the decaying DC component varies depending on the system configuration and fault conditions such as the fault location, the fault resistance, and the fault inception angle. The time constant of the decay is generally determined by the fault resistance and the impedance ratio (X/R, the ratio of the inductive reactance to the resistance) of the system. As shown in Table II, different impedance ratios of the remote end source were used for Cases 1 and 2 to consider the effect of the source impedance ratio on the time constant.

B. Case Studies

Figs. 3 to 5 show the magnitude estimation results of fault currents and the measured impedance trajectories designed by the proposed technique for Case 1. For comparison, the results obtained using the conventional DFT and PS-based DFT are also shown in each figure.

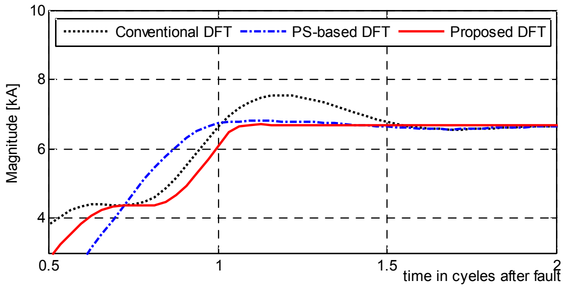


(a) Magnitude estimation results of a fault current

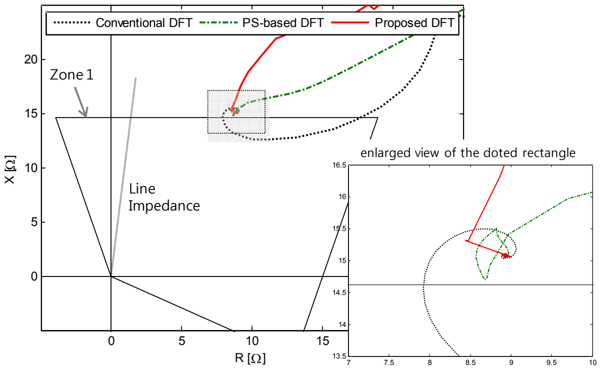


(b) Apparent impedance trajectories

Fig. 3. Test results for Case 1 (fault resistance = 0 Ω)

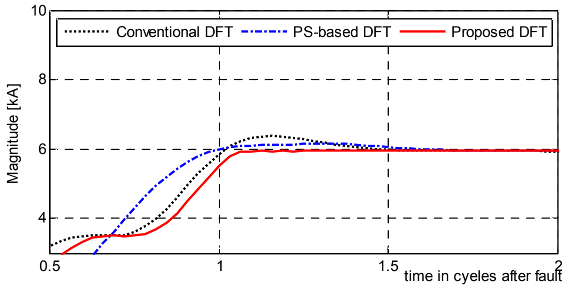


(a) Magnitude estimation results of a fault current

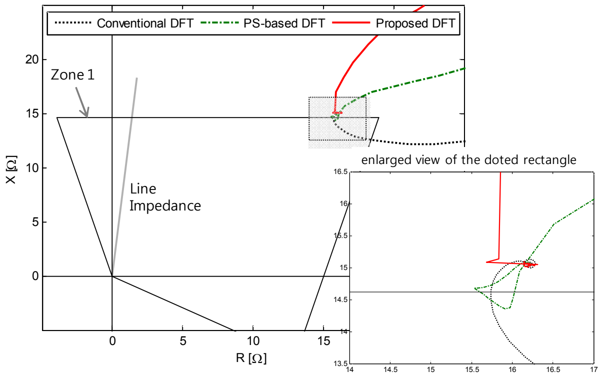


(b) Apparent impedance trajectories

Fig. 4. Test results for Case 1 (fault resistance = 5 Ω)



(a) Magnitude estimation results of a fault current



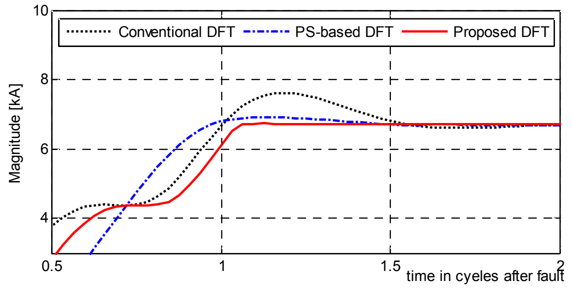
(b) Apparent impedance trajectories

Fig. 5. Test results for Case 1 (fault resistance = 10 Ω)

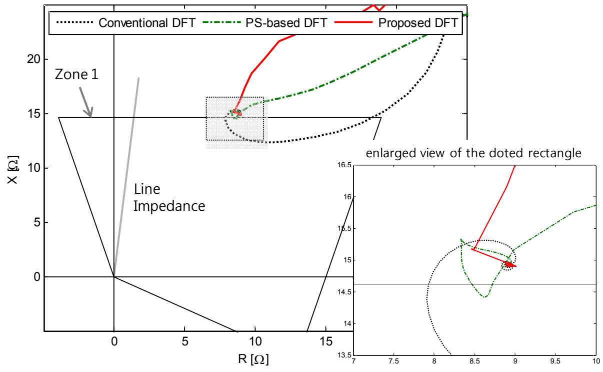
Fig. 3 shows the test results for an ideal a-g fault at 41km from the relaying point. For an ideal fault the fault current flowing through a power line contains only one exponentially decaying DC component which depends on the local source impedance and the line impedance. Compared to the other algorithms, the distance relay designed by the conventional DFT shows not only the largest transient overshoot but also the largest oscillatory response.

Fault resistance is usually small in the case of inter-phase faults. On the other hand, ground faults may introduce high resistance in the fault loop. The most common faults on overhead lines are ground faults that are caused by flashover of an insulator. The fault loop for ground faults includes tower impedance, tower footing resistance, and arc resistance. Tower footing resistance can vary from less than one ohm to several hundred ohms.

For high-resistance ground faults, the time constant of the decay is very small, sometimes less than half of the fundamental frequency cycle. In such cases, decaying DC components do not cause errors that are taken into consideration in calculating the phasor. However, in the case of faults with a fault resistance of less than ten ohms, DC components heavily influence the accuracy and the convergence speed of the estimation of a phasor. Furthermore, the current relaying signal contains two exponentially decaying DC offsets that depend on not only the local source impedance but also on the remote source impedance for a fault with a fault resistance.

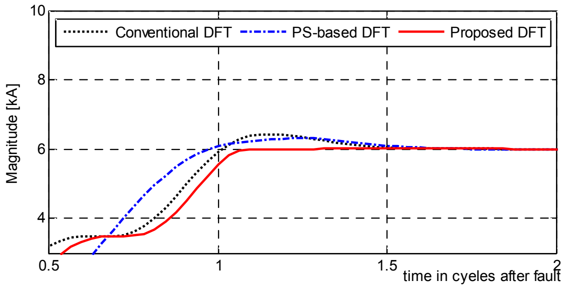


(a) Magnitude estimation results of a fault current

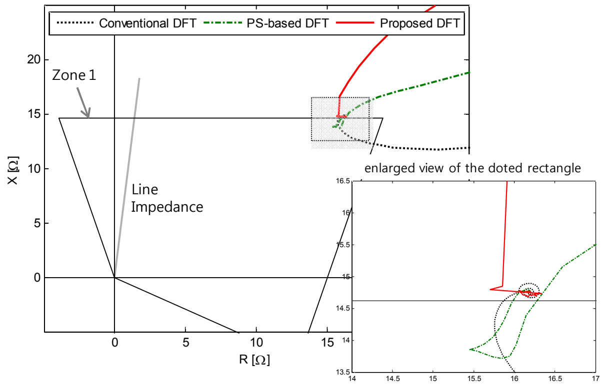


(b) Apparent impedance trajectories

Fig. 6. Test results for Case 2 (fault resistance = 5 Ω)



(a) Magnitude estimation results of a fault current



(b) Apparent impedance trajectories

Fig. 7. Test results for Case 2 (fault resistance = 10 Ω)

Figs. 4 and 5 show test results for the fault with a fault resistance. In this case, as shown in Figs. 4 (a) and 5 (a), there is overshoot in the output of the PS-based method because it cannot completely remove the adverse influence of the DC offsets when the input signal contains two exponentially decaying DC components with different time constants. Thus, the measured impedance using a PS-based method had about a 4–6% transient overreach and showed an oscillatory response for the fault with a fault resistance as shown in Figs. 4(b) and 5(b).

The proposed method completely eliminated the DC offsets because it estimated an exponentially decaying DC offset with an approximately equivalent time constant. Therefore, the measured impedance using the proposed method reached the desired impedance value without an oscillatory response. The transient overreach of the measured impedance using the proposed method was less than 1 %.

Figs. 6 and 7 show the magnitude estimation results of fault currents and the measured impedance trajectories for Case 2. The transient overreach of the apparent impedances using a PS-based method increased compared with Case 1 because the increased impedance ratio of the remote source affected the time constants of the DC components, *i.e.*, one of the time constants that mainly depends on the local source ratio was almost equal to that of Case 1, and the other time constant that greatly depends on the remote source ratio becomes much smaller than that of Case 1.

Sources of supply having different impedance ratios might not only affect the time constants of the DC offsets, but may also change the apparent impedance seen by a distance relay whenever a resistance is present in the fault path. If X/R of the local source is greater than that of the remote source, the fault resistance appears as an impedance with an inductive reactance. If X/R of the local source is less than that of the remote source, the fault resistance appears as an impedance with a capacitive reactance [6], [7]. The more the impedance ratio of the local source differs from that of the remote source, the more the reactance effect may be increased. In the case of the model system used in this study, the fault resistance appears as an impedance with an capacitive reactance. For this reason, the measured impedance trajectory using a PS-based method overreached the boundary of the Zone 1 even when the fault resistance was 5 Ω, as shown in Fig. 6(b). The transient over-reach reduces the sensitivity of the relays and may also increase the operation time because the measured impedance keeps on moving in and out of the relay operation region.

The proposed algorithm was not affected by system and fault conditions. Therefore, the output of the proposed method was stable, and showed superior results. These simulation results demonstrate the superiority of the proposed algorithm in a distance relay design.

IV. CONCLUSIONS

A distance relaying algorithm using a DFT-based modified phasor estimation method to eliminate the adverse influence of exponentially decaying DC offsets was described. The proposed phasor estimation method used an even-sample-set DFT and an odd-sample-set DFT to reduce the length of the data window to only one cycle. The error due to DC offsets on the DFT was calculated and eliminated using the outputs of even- and odd-sample-set DFTs so that the phasor of the fundamental component could be accurately estimated.

A performance evaluation showed that the proposed algorithm was not affected by system and fault conditions and the output was stable without an oscillatory response, unlike some other algorithms. Thus, the evaluation results indicate that adopting the proposed algorithm in a distance relay can effectively suppress the adverse influence of DC offsets in a relaying signal.

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VI. BIOGRAPHIES

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