A novel Z-Matrix Algorithm for Distribution Power Flow Solution

Ting-Chia Ou, *Member, IEEE,* and Whei-Min Lin, *Member, IEEE*

***Abstract*—A direct and rigid algorithm approach for large-scale distribution power flow analysis is proposed in this paper. This algorithm used two primary matrices: B_I and $Z_{V\text{-}BC}$. Two matrices, which are built from the topological characteristics of distribution networks, are used to achieve the power flow solutions. B_I matrix is the bus injection to branch current matrix and the $Z_{V\text{-}BC}$ matrix describes the relationship between the bus voltage mismatches and the branch current. The building algorithm can be accomplished by a simple search technique with two proposed matrices. Four connected cases are considered in this paper. The proposed algorithm is robust and accurate. Test results demonstrate the potential and validity of the proposed algorithm in distribution applications.**

***Index Terms*—**Distribution power flow, distribution automation system, radial network, weakly meshed network. **.**

I. Introduction

THREE phase power flow is a very important tool for the analysis of power system and is used in the operational as well as planning stages in distribution management. Network optimization, state estimation, feeder switching, etc. require a robust and efficient power flow method [1], [2]. Several power flow algorithms specially designed for distribution systems have been proposed in the literature [3]–[15]. Gauss–Seidel and Newton–Raphson (NR) based algorithms are two common techniques used in the industry for power-flow solutions. The Gauss–Seidel method needs more iterations and is known to be slow. The NR algorithm has been used primarily in the Energy Management Systems (EMS), especially the Fast-Decoupled version which tends to exceed any other techniques in performance [3]. Some features cause the traditional power flow methods used in power systems, such as the Gauss-Seidel and Newton-Raphson techniques, to fail to meet the requirements in performance and robustness in distribution system applications including the multiphase and unbalanced network, radial or weakly meshed structure, large number of branches and nodes, high resistance to reactance ratios, wide-ranging resistance and reactance values, etc. In particular, the assumptions necessary for the simplifications used in the standard fast-decoupled Newton-Raphson method [3] are not necessarily in distribution systems. This research tuned to design a computer algorithm and program that meet the requirements of rigorous operational-type power flow and contingency analyses on a large-scale distribution.

One of the most powerful matrices used in the power system analysis is the bus impedance matrix (Z-Matrix). Despite the importance of the Z-Matrix, its usefulness has been constrained by the building process. Z-Matrix is preferred for short circuit analysis with perfectly transposed transmission lines, where short circuit analysis may be performed by using the sequence network with zero, positive and negative sequences. The elements of Z-Matrix related to the monitored contingent lines are often used in combination with pre-contingency load flow to determine the net current change after the switching operation. Traditional building algorithms in [17]-[20] only provide the information between bus voltage and bus current injection. However, applications of Z-Matrix analysis might necessitate relationships among branch current, bus voltage, and bus current injection. A lot of techniques have been proposed to modify the traditional Z-Matrix building algorithms [19]. Among those methods, the Gauss implicit Z-matrix method [9] is the most generally used method, however, this method does not take advantages of the radial and weakly meshed network structure of distribution systems, and requires the solution of a set of equations whose dimension is proportional to the number of buses. Many researches provided some new ideas on how to treat the special topology of distribution systems [12]–[15].

A compensation-based technique was proposed to solve distribution power flow problems in [12]. The iterative-compensation method which uses the bus admittance matrix to simulate the fault conditions was proposed in [8], [11]. Since the forward/backward sweep technique was widely accepted in the solution process of the compensation-based algorithm, new data format and search procedures are essential, which stressed on modeling unbalanced loads and dispersed generators [14]. One of the inconveniences of the compensation-based methods is the need of databases which have to be built and maintained.

The purpose of this paper is to develop an algorithm, which takes advantages of the topological characteristics of distribution systems, and can solve the distribution power flow directly. The new method is systematic, effective, and easily programmable. The time-consuming LU decomposition and forward/backward substitution of the Jacobian matrix or the *Y* admittance matrix, required in the traditional Z-Matrix building algorithms, is not needed in the new development.

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This building algorithm is developed by a simple searching technique with two proposed matrices: $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$, the bus injection to branch current matrix and the branch current to bus voltage mismatch matrix, where four cases are utilized to obtain power flow solutions. The proposed method is robust and very efficient compared to the conventional methods. Test results demonstrate the potential and validity of the proposed algorithm in a large-scale power system.

II. UNBALANCED NETWORK MODEL

A three phase line section model between bus *0* and *k* is shown in Fig. 1. The line parameters can be obtained by the method developed by Carson and Lewis [2].

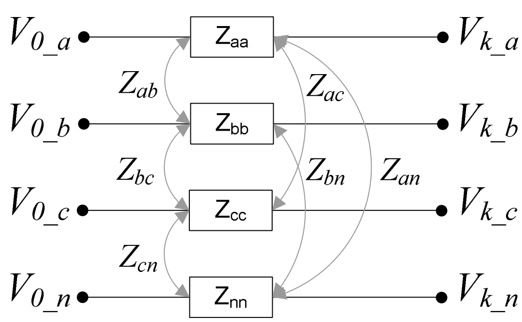

Fig. 1. Three phase line section model

A 4×4 matrix, which takes into account the self and mutual coupling effects of the unbalanced three phase line section, can be described as

$$[Z_{abcn}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{nb} & Z_{nc} & Z_{nn} \end{bmatrix} \quad (1)$$

After Kron's reduction, the matrix dimension will reduce to 3×3, while the effects of the neutral or ground wire are still included in this model and (1) can then be rewritten as

$$[Z_{abc}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \quad (2)$$

The relation between bus voltage mismatches and branch currents in Fig. 1 can be expressed as

$$\begin{bmatrix} V_{0\_a} \\ V_{0\_b} \\ V_{0\_c} \end{bmatrix} - \begin{bmatrix} V_{k\_a} \\ V_{k\_b} \\ V_{k\_c} \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (3)$$

For any phases failed to present, the corresponding row and column in this matrix will contain null-entries. The general form for the voltage mismatches matrix is

$$[\mathbf{\Delta V}_{abc}] = [\mathbf{Z}_{abc}][\mathbf{I}_{abc}] \quad (4)$$

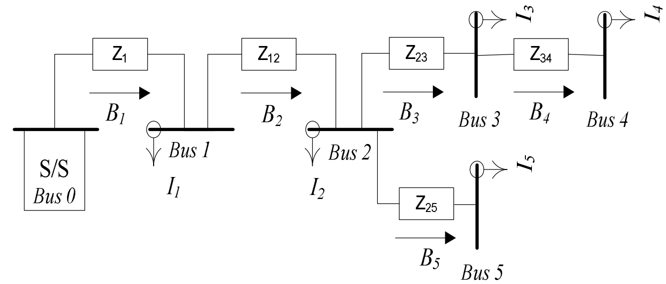

Fig. 2. A 5-bus distribution system

III. $\mathbf{B_I}$ AND $\mathbf{Z_{V\text{-}BC}}$ MATRIX FORMULATION

A 5-bus distribution system shown in Fig. 2 is drawn for example. The power injections can be transformed to equivalent current injection (ECI). For bus *i,* the solution at the k-th iteration for ECI is

$$I_i^k = \frac{P_i - jQ_i}{(V_i^k)^*} \quad (5)$$

where V_i^k and I_i^k are the voltage and the ECI of bus *i* at the *k*-th iteration.

The relationship between the bus current injections and branch currents can be described by (6) for the distribution network. The branch currents B_i can be calculated from bus current injections as

$$\begin{aligned} B_1 &= I_1 + I_2 + I_3 + I_4 + I_5 \\ B_2 &= I_2 + I_3 + I_4 + I_5 \\ B_3 &= I_3 + I_4 \\ B_4 &= I_4 , B_5 = I_5 \end{aligned} \quad (6)$$

Hence, the relationship matrix between the bus current injections and branch currents can be illustrated by

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} \quad (7a)$$

A general form of the matrix can be rewritten as

$$[B] = [\mathbf{B_I}][I] \quad (7b)$$

where $\mathbf{B_I}$ is the bus injection to branch current matrix.

The matrix $\mathbf{B_I}$ is an upper triangular matrix and contains values of 0's and 1's only. The relationship between branch currents and bus voltages of bus 1, 2, and 3 in Fig. 2 can be written as

$$\begin{aligned} V_1 &= V_0 - B_1 Z_1 \\ V_2 &= V_1 - B_2 Z_{12} \\ V_3 &= V_2 - B_3 Z_{23} \end{aligned} \quad (8)$$

where V_i is the voltage of bus *i,* and Z_{ij} is the line impedance between bus *i* and *j*. From (8), V_3 can be rewritten as

$$V_3 = V_0 - B_1 Z_1 - B_2 Z_{12} - B_3 Z_{23} \tag{9}$$

The bus voltages can be described as a function of branch currents, line impedance, and the reference bus voltage. Similar procedures can be performed on other buses by (9); therefore, the relationship matrix between branch currents and bus voltages can be expressed as

$$\begin{bmatrix} V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \end{bmatrix} - \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & Z_{34} & 0 \\ Z_1 & Z_{12} & 0 & 0 & Z_{25} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} \tag{10a}$$

A general form of the ΔV mismatch matrix can be obtained by

$$[\Delta V] = [\mathbf{Z_{V\text{-}BC}}][\mathrm{B}] \tag{10b}$$

where the $\mathbf{Z_{V\text{-}BC}}$ matrix describes the relationship between the bus mismatch voltages and the branch current.

A building algorithm for $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrices can be described as follows.

A. $\mathbf{B_I}$ *matrix*

1) For a distribution system with branch m and bus n, the dimension of the $\mathbf{B_I}$ matrix is $m \times n$.
2) If a new branch current B_j added to bus i and j, copy i-th column of the $\mathbf{B_I}$ matrix to the j-th column of the bus and fill a "1" to the position of the j-th row and j-th column.
3) Repeat procedure (2) until all branch current are included in the $\mathbf{B_I}$ matrix.

B. $\mathbf{Z_{V\text{-}BC}}$ *matrix*

From KVL, the $\mathbf{Z_{V\text{-}BC}}$ matrix building algorithm can be created as follows.

4) For a distribution system with branch m and bus n, the dimension of the $\mathbf{Z_{V\text{-}BC}}$ matrix is $n \times m$.
5) If the branch impedance Zij is added between bus i and j, copy i-th row of the $\mathbf{Z_{V\text{-}BC}}$ matrix to the j-th row and fill the impedance "Z_{ij}" to the position of the j-th row and j-th column.
6) Repeat procedures (5) until all branch impedance are included in the $\mathbf{Z_{V\text{-}BC}}$ matrix.

The proposed algorithm can easily be extended to a multiphase system or bus. If the line section between bus i and bus j is a three phase line section, the corresponding branch current B_i will be a 3×1 vector and the "1" in the $\mathbf{B_I}$ matrix will be a 3×3 identity matrix. Similarly, if Z_{ij} in the $\mathbf{Z_{V\text{-}BC}}$ matrix is a 3×3 impedance matrix and the building algorithms of the $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrices are similar. In addition, the building algorithms can be developed on the original network database, where data processing time can be reduced and the proposed method can be easily integrated with the existent distribution power flow.

C. A **Z** *based power flow solution*

$\mathbf{B_I}$ matrix describes the relationship between bus current injections and branch currents. The $\mathbf{Z_{V\text{-}BC}}$ matrix represents the relationship between branch currents and bus voltages. The corresponding variations of branch currents, generated by the bus current injections, can be computed by $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrices directly. Combining (7b) and (10b), the relationship between bus current injections and bus mismatch voltages can be rewritten as

$$[\Delta V] = [\mathbf{Z_{V\text{-}BC}}][\mathbf{B_I}][I] = [\mathrm{Z_{DPF}}][I] \tag{11}$$

And the solution for distribution power flow can be achieved by solving (5) and (12) iteratively

$$\left[\Delta V^{k+1}\right] = [\mathrm{Z_{DPF}}]\left[I^k\right] \tag{12}$$

In the notation to be used in our analysis existing buses will be identified by numbers or the letters i, j, and k. The original bus $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrix is identified as $\mathbf{B_{I,origin}}$ and $\mathrm{Z_{origin}}$. At bus k the original voltage will be denoted by V_k, the voltage mismatches after modifying $\mathbf{Z_{V\text{-}BC}}$ matrix will be ΔV_k. Listed below cases are considered in this section.

IV. NETWORK MODELING SCENARIOS

A distribution feeder is usually radial with load buses spread along line sections and various phases. It is ordinary to model the feeder with the (P, Q) bus for load and a swing bus at the substation. To take advantage of the topological characteristics for distribution systems, equivalent current injection model has been a convenient scheme to solve the radial network [4]. Two derivative matrices, the bus injection to branch current matrix and the branch current to bus voltage mismatches matrix, were built based on ECI in the proposed method [4]–[7].

There are several types of modifications in which a branch having impedance Z_{new} is added to a network at bus i, the original voltage will be denoted by V_i, and ΔV_i will denote the voltage change at that bus. A 5-bus distribution system shown in Fig. 2 is drawn for illustration and four cases are considered in this section.

Case 1. Adding Z_{new} *from a new bus to the reference bus*

A new bus is connected to the reference bus (*bus 0*) through Z_{new}. The original reference bus voltage remains the same when a current I_{new} is injected at the new bus, as shown in Fig. 3.

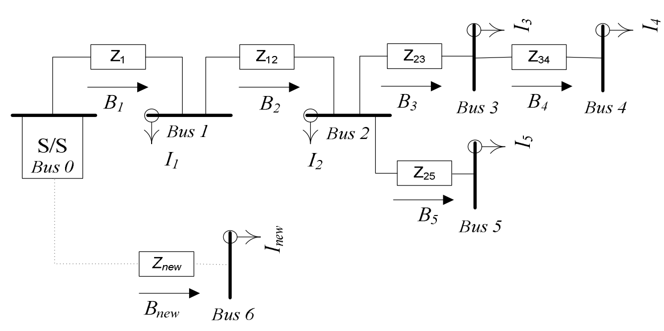

Fig. 3. *Adding* Z_{new} *from a new bus to the reference bus*

The addition of a new bus connected to the reference bus*0* through Z_{new}, the $\mathbf{B_I}$ matrix will be

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_{new} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_{new} \end{bmatrix} \tag{13a}$$

The general form for the modified $\mathbf{B_I}$ matrix is

$$\begin{bmatrix} B \\ B_{new} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ I_{new} \end{bmatrix} \tag{13b}$$

ΔV mismatches matrix can be written as

$$\begin{bmatrix} V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \end{bmatrix} - \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_{new} \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & Z_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{new} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_{new} \end{bmatrix} \tag{13c}$$

The general form for (13c) is

$$\begin{bmatrix} \Delta V \\ \Delta V_{new} \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 \\ 0 & Z_{new} \end{bmatrix} \begin{bmatrix} B \\ B_{new} \end{bmatrix} \tag{13d}$$

Combining (13b) and (13d), the new ΔV mismatches matrix can be rewritten as

$$\begin{bmatrix} \Delta V \\ \Delta V_{new} \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 \\ 0 & Z_{new} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ I_{new} \end{bmatrix} \tag{13e}$$

Case 2. Adding Z_{new} from a new bus to an existing bus k

A new bus connected to an existing bus through Z_{new} with I_{new} injected at the new bus will have the current injecting into existing bus *k* be the sum of I_k plus I_{new}, as shown in Fig. 4.

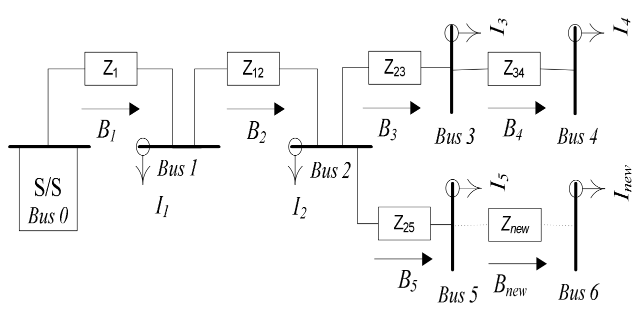

Fig. 4. *Adding Z_{new} from a new bus to an existing bus k*

The addition of a new bus connected through branch impedance Z_{new} to an existing bus *k* .The current I_{new} flows into the network at bus *k*, and the $\mathbf{B_I}$ matrix will be

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_{new} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_{new} \end{bmatrix} \tag{14a}$$

The general form for the modified $\mathbf{B_I}$ matrix is

$$\begin{bmatrix} B \\ B_{new} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & col.(k) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ I_{new} \end{bmatrix} \tag{14b}$$

The new column is to copy the elements of the *k*-th column of $\mathbf{B}_{\mathbf{I},\ origin}$ to the new column. Fill "1" to the position of the new row and the new column. ΔV mismatches matrix is

$$\begin{bmatrix} V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \end{bmatrix} - \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_{new} \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & Z_{25} & 0 \\ Z_1 & Z_{12} & 0 & 0 & Z_{25} & Z_{new} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_{new} \end{bmatrix} \tag{14c}$$

The general form for (14c) is

$$\begin{bmatrix} \Delta V \\ \Delta V_{new} \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 \\ row\_k & Z_{new} \end{bmatrix} \begin{bmatrix} B \\ B_{new} \end{bmatrix} \tag{14d}$$

Combining (14b) and (14d), the new ΔV mismatches matrix can be rewritten as

$$\begin{bmatrix} \Delta V \\ \Delta V_{new} \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 \\ row\_k & Z_{new} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & col.(k) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ I_{new} \end{bmatrix} \tag{14e}$$

Case 3. Adding Z_{new} between two existing buses i and j

Adding a new branch with impedance Z_{new} between buses *i* and *j* in Fig. 5, shows that these buses extract current from the original network. The new branch current B_{new} flowing from bus *i* to bus *j*, causes voltage changes at bus *j* and bus *i*.

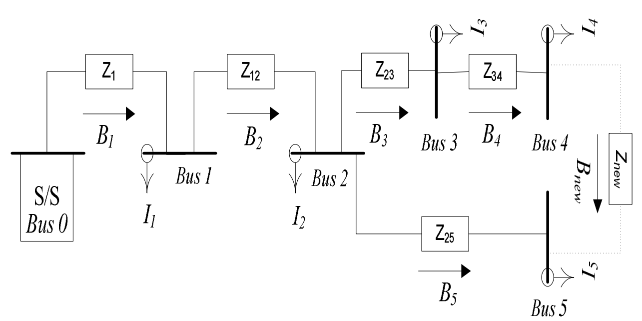

Fig. 5. *Adding Z_{new} between two existing buses i and j*

Adding branch impedance Z_{new} between buses *i* and *j*, Fig. 5 shows a new loop. Taking the new branch current into account, the current injections of bus 4 and bus 5 will be

$$I_4' = I_4 + B_{new}$$
$$I_5' = I_5 - B_{new} \tag{15a}$$

The $\mathbf{B_I}$ matrix will be

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 + B_{new} \\ I_5 - B_{new} \end{bmatrix} \tag{15b}$$

Equation (15b) can be rewritten as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} B_{new} \\ -B_{new} \end{bmatrix} \tag{15c}$$

And the modified $\mathbf{B_I}$ matrix can be obtained as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ B_{new} \end{bmatrix} \tag{15d}$$

The general form for the modified $\mathbf{B_I}$ matrix is

$$\begin{bmatrix} B \\ B_{new} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & col.(i-j) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ B_{new} \end{bmatrix} \tag{15e}$$

The new column is column i minus column j of $\mathbf{B}_{\mathbf{I},\ origin}$. The new row filled with "0" in $\mathbf{B}_{\mathbf{I},\ origin.}$ Finally, fill "1" to the position of the off-diagonal.

Considering *case3* shown in Fig. 5, KVL for this loop can be written as

$$Z_{23}B_3 + Z_{34}B_4 + Z_{new}B_{new} - Z_{25}B_5 = 0 \tag{16a}$$

Combining (16a) and (10a), the new ΔV mismatch matrix can be written as

$$\begin{bmatrix} V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \\ 0 \end{bmatrix} - \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & Z_{25} & 0 \\ 0 & 0 & Z_{23} & Z_{34} & -Z_{25} & Z_{new} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_{new} \end{bmatrix} \tag{16b}$$

The general form for (16b) and the modified $\mathbf{Z_{V\text{-}BC}}$ matrix is

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 \\ row.(i-j) & Z_{new} \end{bmatrix} \begin{bmatrix} B \\ B_{new} \end{bmatrix} \tag{16c}$$

Combining (15e) and (16c), the new ΔV mismatch matrix can be rewritten as

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 \\ row.(i-j) & Z_{new} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & col.(i-j) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ B_{new} \end{bmatrix} \tag{16d}$$

The new row is *row i* minus row j of $\mathbf{Z}_{\text{origin}}$. Finally, fill Z_{new} to the position of the off-diagonal.

The building algorithm of Step 2) and 5) for the $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrices can be modified as in (15) and (16).

Case 4. Adding a new bus through Z_{new} between two existing buses i and j

The addition of a new bus k connected through Z_{new} from existing bus i *to* bus j in Fig. 6, shows the new branch current B_{new} flowing from bus i to bus j through the new bus (I_{new}=0). When the new bus k is injected with I_{new}, the change in voltage at each bus is caused by the branch current $-B_{new}$ at bus i, the branch current $-B_{kj}$ at the new bus, and B_{kj} at bus j.

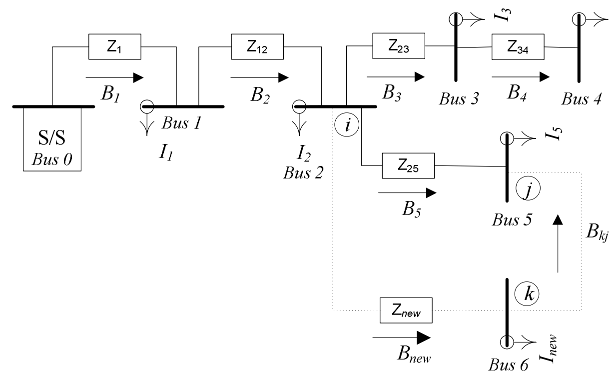

Fig. 6. *Adding a new bus through Z_{new} between two existing buses i and j*

To add a new bus k through Z_{new} between two existing buses i and j in Z_{origin}, in Fig. 6, which shows that the branch current B_{kj} is flowing from bus k to bus j. The change in voltage at each bus is caused by the injection I_{new} at bus k and branch current B_{kj}. The modified $\mathbf{B_I}$ matrix will be

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_{new} \\ B_{kj} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_{new} \\ B_{kj} \end{bmatrix} \tag{17a}$$

The general form for (17a) with the modified $\mathbf{B_I}$ matrix is

$$\begin{bmatrix} B \\ B_{new} \\ B_{kj} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & col.(i) & col.(k-j) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ I_{new} \\ B_{kj} \end{bmatrix} \tag{17b}$$

The first new column is to copy elements of the i-th column of $\mathbf{B}_{\mathbf{I},\ origin}$ to the new column. And fill "1" to the position of the first new row and column. The second new column is created by column k minus column j of $\mathbf{B}_{\mathbf{I},\ origin}$. The new rows fill "0" of $\mathbf{B}_{\mathbf{I},\ origin.}$ The off-diagonal elements of the new row and column are "1"'s.

The bus voltage changes are established by the new row, which are the elements of row k of Z_{origin} after subtracting row j from row k of Z_{origin} and multiplying the result by branch current. Considering *case4* shown in Fig. 6, KVL for this loop can be written as

$$Z_{new}B_{new} - B_5 Z_{25} = 0 \tag{18a}$$

Combining (18a) and (10a), the new ΔV mismatch matrix can be written as

$$\begin{bmatrix} V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \\ V_0 \\ 0 \end{bmatrix} - \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_{new} \\ 0 \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & 0 & 0 & 0 & 0 \\ Z_1 & Z_{12} & Z_{23} & Z_{34} & 0 & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & Z_{25} & 0 & 0 \\ Z_1 & Z_{12} & 0 & 0 & 0 & Z_{new} & 0 \\ 0 & 0 & 0 & 0 & Z_{25} & -Z_{new} & 0 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_{new} \\ B_{kj} \end{bmatrix} \tag{18b}$$

The general form for the modified $\mathbf{Z_{V\text{-}BC}}$ matrix is

$$\begin{bmatrix} \Delta V \\ \Delta V_{new} \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 & 0 \\ row(i) & Z_{new} & 0 \\ & row(k-j) & 0 \end{bmatrix} \begin{bmatrix} B \\ B_{new} \\ B_{kj} \end{bmatrix} \tag{18c}$$

The first new row is *rowing* i of $\mathbf{Z}_{\text{origin}}$. Finally, fill Z_{new} to the position of the off-diagonal. The second new row is row k minus row j of Z_{origin}. The off-diagonal element of the second new row and column is "0".

Combining (17b) and (18c), the new ΔV mismatches matrix can be rewritten as

$$\begin{bmatrix} \Delta V \\ \Delta V_{new} \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{origin} & 0 & 0 \\ row(i) & Z_{new} & 0 \\ row(k-j) & & 0 \end{bmatrix} \begin{bmatrix} \mathbf{B}_{\mathbf{I},origin} & col.(i) & col.(k-j) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I \\ I_{new} \\ B_{kj} \end{bmatrix} \tag{18d}$$

The building algorithm of Step 2) and 5) for the $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrix can be modified as in (17) and (18).

From (13) to (18), it can be seen that the building algorithms of $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrices are similar, the nonzero terms of these two matrices are transposed and can be built by the same subroutine in a program. For example, the voltages mismatches of (16d) can be rewritten as

$$\begin{bmatrix} \Delta V \\ 0 \end{bmatrix} = [\mathbf{Z_{V\text{-}BC}}][\mathbf{B_I}] \begin{bmatrix} I \\ B_{new} \end{bmatrix} = \begin{bmatrix} A & M^T \\ M & N \end{bmatrix} \begin{bmatrix} I \\ B_{new} \end{bmatrix} \tag{19}$$

Applying Kron's Reduction to (19), the modified algorithm for weakly meshed networks can be obtained by

$$\Delta V = \left[A - M^T N^{-1} M \right] [I] \tag{20}$$

Note that except for some modifications needed to be done for the $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrices. Therefore, when the topology of a distribution power system is weakly meshed, the proposed method can obtain better performance for the power flow solution.

V. SIMULATION RESULTS

The proposed three phase power flow algorithm was implemented using MATLAB and tested on a Windows-XP-based AMD-Athlon-2500+ PC. Three methods are used for tests and the convergence tolerance is set to 0.001 p.u.

Method 1: The Gauss implicit Z-matrix method [9].

Method 2: The traditional Newton-Raphson method.

Method 3: The proposed algorithm

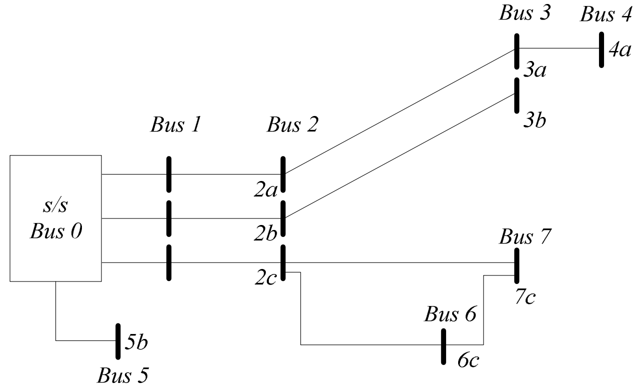

Fig.7. Eight-bus distribution system

A. Accuracy Comparison

An eight-bus system, including the three phase, double-phase, and single-phase line sections and buses is shown in Fig. 7 for testing and comparisons. The final converged voltage solutions of method 3 are very close to other existent methods shown in Table I.

TABLE I
FINAL CONVERGED VOLTAGE SOLUTIONS

Bus number	Method 1		Method2		Method3		Phase
	Voltage (Pu)	Angle (Rad)	Voltage (Pu)	Angle (Rad)	Voltage (Pu)	Angle (Rad)	
0	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	A
0	1.0000	-2.4960	1.0000	-2.4960	1.0000	-2.4960	B
0	1.0000	2.4960	1.0000	2.4960	1.0000	2.4960	C
1	0.9864	0.0054	0.9861	0.0054	0.9863	0.0054	A
1	0.9876	-2.4887	0.9878	-2.4887	0.9877	-2.4887	B
1	0.9833	2.4812	0.9835	2.4812	0.9834	2.4812	C
2	0.9627	0.0046	0.9629	0.0046	0.9628	0.0046	A
2	0.9721	-2.4864	0.9719	-2.4864	0.9719	-2.4864	B
2	0.9636	2.4798	0.9632	2.4798	0.9633	2.4798	C
3	0.9582	0.0039	0.9583	0.0039	0.9583	0.0039	A
3	0.9684	-2.4832	0.9686	-2.4832	0.9686	-2.4832	B
4	0.9566	0.0032	0.9563	0.0032	0.9563	0.0032	A
5	0.9883	-2.4916	0.9884	-2.4916	0.9884	-2.4916	B
6	0.9625	2.4786	0.9626	2.4786	0.9626	2.4786	C
7	0.9612	2.4778	0.9614	2.4778	0.9615	2.4778	C

B. Performance Test

A main feeder trunk with 3×90-phase buses was obtained from the Taiwan Power Company (TPC) for the performance test. The single and double-phase laterals have been lumped to form the unbalanced loads for testing purposes. The substation is modeled as the slack bus. This trunk is duplicated for the various sizes for tests as shown in Table II [6].

TABLE II
TEST FEEDER

Feeder No.	No. of Nodes	Length (km)

1	45	1.5
2	90	2.5
3	135	3.2
4	180	4.0
5	270	7.4

Fig.8 illustrates the number of the normalized execution time (NET) for the three methods. The performance of method 3 is set to 1.0 for Feeder1. We can find that method 3 is more efficient and fast especially when the network size increases. For a 270-node system, method 3 is almost 22 times faster than method 1.

Some branches will be connected to form various numbers of loops to the test feeder to make the system "meshed" for tests as shown in Table III. The result shows that the number of iterations (IT) of the proposed method is stable, but the normalized execution time will increase when the numbers of meshed loops increase, since the meshed networks add the nonzero terms to the $\mathbf{B_I}$ and $\mathbf{Z_{V\text{-}BC}}$ matrices.

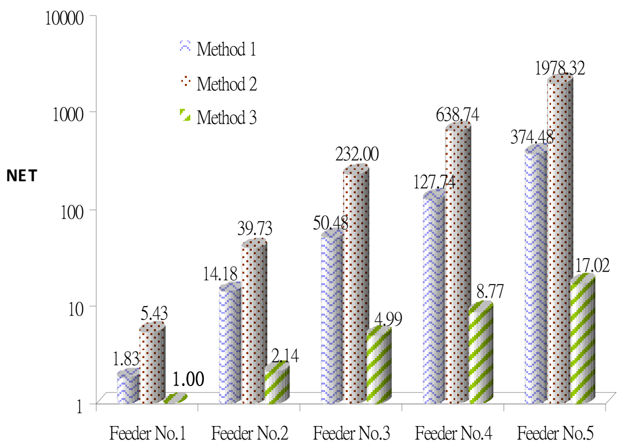

Fig.8. Number of the normalized execution time

TABLE III
NUMBER OF IT AND NET

Feeder No.	No. of Loops	NET	IT
1	1	1.12	3
2	2	5.27	3
3	4	17.32	3
4	5	37.68	4
5	6	115.60	4

C. Robustness Test

The ill-condition problems of the Jacobian matrix or Y admittance matrix make the power flow program diverge. It usually occurs when the system contains some very short/long lines. Two cases are shown for the IEEE 30-bus system, which is known to be a relatively weaker system for test [21], [22].

A heavy load test at bus No. 30 for IEEE 30-bus is shown in Table IV. The load is adjusted by multiplying a factor K_L that ranges from 0 to 3. The voltage sag occurs (0.94885 pu) when K_L is 2.15, and power-flow solution needs more iterations to converge. Increasing K_L, the system further weakened, and the proposed method is very robust.

TABLE IV
NUMBER OF ITERATION FOR HEAVY LOAD TEST

K_L	Method 1	Method 2	Method 3
0.5	3	3	3
1	3	3	3
1.5	4	3	3
2	5	4	4
2.5	5	4	4
3	5	4	4

VI. DISCUSSION AND CONCLUSION

A direct approach for calculating distributed power flow solutions was proposed in this paper. Two matrices, which are developed from the topological characteristics of distribution systems, are used to solve load flow problem. The $\mathbf{B_I}$ matrix represents the relationship between bus current injections and branch currents, and the $\mathbf{Z_{V\text{-}BC}}$ matrix represents the relationship between branch currents and bus voltages. These two matrices are combined to form a direct approach for solving power flow problems with four cases classified according to connected structure. The time-consuming LU decomposition and forward/backward substitution of the Jacobian matrix or the *Y* admittance matrix, required in the traditional Z-Matrix building algorithms, is not needed in the new development. Test results demonstrate that the proposed algorithm is suitable for large-scale distribution systems. The ill-conditioned problem that usually occurs in the traditional solution techniques can be avoided here. Extensive comparisons of the proposed method with other load flow methods and the integration of other equipment models such as transformer, capacitors, and voltage regulators into the proposed method will be discussed in future works.

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VIII. Biographies

Ting-Chia Ou (M’05) was born in 1971. He received his BS-EE from the National Taiwan University of Science and Technology, Taipei, Taiwan, in 1996 and MS-EE from the National Sun Yat-Sen University, Kaohsiung, Taiwan, in 2002. He is currently pursuing his Ph.D. degree at National Sun Yat-Sen University. He was an Electrical Engineer at Taiwan Railway Administration, Kaohsiung, Taiwan, in 1998. He joined Institute of Nuclear Energy Research in 2008, Tao Yuan, Taiwan, and worked with nuclear power system research. His research interests include Distribution system, Power system operation, Hybrid generation system, and Smart grid. Mr. Ou is a Member of *IEEE*.

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