Paper accepted for presentation at 2009 IEEE Bucharest Power Tech Conference, June 28th - July 2nd, Bucharest, Romania

A METHOD OF TRANSIENT ELECTROMECHANICAL PROCESSES MODELING IN POWER SYSTEMS

Nikolay F. Djagarov
Department of Electrical Engineering, Varna Technical University, Bulgaria
jagarov@ms.ieee.bg

Abstract – A non-iterative method for transient electromechanical processes calculation in power system with arbitrary topology network and *RL* equivalent scheme is described. The electrical machines models (synchronous generator, induction motor) and static *RL*-loads are written in Cauchy form in respect of currents and angular velocity. With exclusion of the current derivatives from network equations we obtained algebraic equations system for a node voltages calculation. The automatic method for the system model build is proposed. The new methods are illustrated by simulation of transients in 15-node power system.

Keywords: non-iterative method for transient electromechanical processes calculation, automatic method for the system model build

1 INTRODUCTION

The transient electromechanical processes in power systems are modeled for both stability and quality regulation investigations. In this the basic difficulties are connected with non-linearity of the investigated systems, with non-stationary of transients and with the solving problem dimension. Independently that the computational technology progresses, the limits of the speed and memory still exists. The automation of all investigation process is a problem of contemporary interest. The automation of mathematical models build is the more sophisticated part of this process. The efficiency of mathematical model is evaluated by means of its dimension and requisite of time and memory for its investigation.

In transient stability programs, the electric transients of the stators windings of all machines and of the tie lines interconnecting of the power system components are neglected [1,2,3]. Generally, this approximation permits a much larger step size when integrating differential equations of the power system components. However, the response predicted by reduced-order simulation is only an approximation of the actual system response.

The popular program EMTP [4] is proposed to investigate electromagnetic transients in multinode networks. The trapezoidal rule is used to convert the differential equations of the network components into algebraic equations involving voltages, currents and past values. These algebraic equations use a nodal approach. The solution of transient process is then obtained using triangular factorization. In [5,6] is proposed additional capabilities for simulation of rotating machinery in EMTP, using two interface methods, compensation and prediction. The implementation of electrical machines lead to clumsiness and inefficiency (it is used fixed time-step size, but the processes have a components of various time constants, necessity of iteration due to variable rotor speed, involve the inversion of matrices at each time step, numerical oscillations).

Although it is possible to simulate power systems in detail (with stator and network transients included). Typically, the shunt capacitance of the tie lines interconnecting the various component is small and it is may be neglected, particularly in stiffly connected power systems.

In [7] is proposed a procedure for establishing a state space model of power system. This procedure utilizes Kirchoff's current law to establish the stator currents of a so-called root machine at each bus. The state model of the root machine includes the stator current and derivatives of stator currents as inputs. The stator voltages represent the outputs of the root model. All other machines connected to that bus are referred to as nonroot machines. The derivatives of stator currents, which are needed as inputs to the root machine are established by summing the derivatives of stator currents of all nonroot machines and tie lines connected to the root machine (bus). The basic drawback of this procedure is its complexity.

In [8,9] is presented a non-iterative algorithm for calculation a transients in autonomous power system with radial network, including/non-including of stator and tie-lines transients. The general algorithm is derived by a root resistance for 1^{-th} node and excluding the derivatives of stator windings from network equations. In [10] the non-iterative algorithm including stator and tie lines transients is developed and added for an arbitrary power system. The object of this paper is further development and addition of non-iterative investigation method.

2 ELEMENT MODELS OF POWER SYSTEM

The basic problems of the used methods are the automatic build of models, the simulation of various disturbances and numerical investigation of processes.

The electrical machines models (synchronous generator, induction motor) and static *RL*-load are written in Cauchy form in respect of the currents and angular velocity.

978-1-4244-2235-7/09/$25.00 ©2009 IEEE

It is accepted that the generator equations are written in *d,q,0* frame that is connected with its rotor (and is rotated with angular velocity ω_k) but motor and static load equations are written in synchronous frame $d,q,0^{syn}$ (is rotated with synchronous velocity ω_s).

Synchronous generator

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{I}_s\\ \boldsymbol{I}_r\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}_{ss} & \boldsymbol{A}_{sr}\\ \boldsymbol{A}_{rs} & \boldsymbol{A}_{rr}\end{bmatrix}.\begin{bmatrix}\boldsymbol{I}_s\\ \boldsymbol{I}_r\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_{ss} & \boldsymbol{B}_{sr}\\ \boldsymbol{B}_{rs} & \boldsymbol{B}_{rr}\end{bmatrix}.\begin{bmatrix}\boldsymbol{U}_s\\ u_f\end{bmatrix}=$$
$$=\begin{bmatrix}\boldsymbol{H}_s\\ \boldsymbol{H}_r\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_{ss} & \boldsymbol{B}_{sr}\\ \boldsymbol{B}_{rs} & \boldsymbol{B}_{rr}\end{bmatrix}.\begin{bmatrix}\boldsymbol{U}_s\\ u_f\end{bmatrix}; \qquad (1)$$
$$\frac{d}{dt}\omega_k=\frac{1}{\tau_m}\left(T_{pm}+T_e\right);$$

where: $\boldsymbol{A}$ and $\boldsymbol{B}$ are parameter-depended matrices defined in Appendix; through subscribes *s* and *r* note stator and rotor variables and parameters respectively; $\boldsymbol{I}_s=\left[i_d, i_q, i_0\right]^t$; $\boldsymbol{I}_r=\left[i_f, i_g, i_h\right]^t$; $\boldsymbol{U}_s=\left[u_d, u_q, u_0\right]^t$; $T_e=x_{ad}\cdot(i_d+i_f+i_g)\cdot i_q-x_{aq}\cdot(i_q+i_h)\cdot i_d$ is the electromagnetic torque; T_{pm} is the prime mover torque; τ_m is the rotor mechanical time constant.

For the stator windings we may be written:

$$\frac{d}{dt}\boldsymbol{I}_s=\left(\boldsymbol{H}_s+\boldsymbol{B}_{sr}\cdot u_f\right)+\boldsymbol{B}_{ss}\cdot\boldsymbol{U}_s=\boldsymbol{H}_{ss}+\boldsymbol{B}_{ss}\cdot\boldsymbol{U}_s \qquad (2)$$

The stator equations are transformed to synchronous reference frame by premultiplying with transformation matrix:

$$\boldsymbol{K}_{sk}\frac{d}{dt}\boldsymbol{K}_{sk}^{-1}\cdot\boldsymbol{K}_{sk}\cdot\boldsymbol{I}_s=\boldsymbol{K}_{sk}\cdot\boldsymbol{H}_{ss}+\boldsymbol{K}_{sk}\cdot\boldsymbol{B}_{ss}\cdot\boldsymbol{K}_{sk}^{-1}\cdot\boldsymbol{K}_{sk}\cdot\boldsymbol{U}_s=$$
$$=\frac{d}{dt}\boldsymbol{I}_s^s+\boldsymbol{K}_{sk}\left(\boldsymbol{K}_{sk}^{-1}\right)^{\prime}\cdot\boldsymbol{I}_s=\boldsymbol{H}_{ss}^s+\boldsymbol{B}_{ss}^s\cdot\boldsymbol{U}_s^s; \qquad (3)$$

where:

$$\boldsymbol{K}_{sk}=\begin{bmatrix}\cos\delta_{sk} & \sin\delta_{sk} & 0\\ -\sin\delta_{sk} & \cos\delta_{sk} & 0\\ 0 & 0 & 1\end{bmatrix} \qquad (4)$$

is the transformation matrix; $\delta_{sk}=\int\left(\omega_s-\omega_k\right)dt$ is the displacement angle of the rotor relative to the synchronous reference frame; $\boldsymbol{B}_{ss}^s=\boldsymbol{K}_{sk}\cdot\boldsymbol{B}_{ss}\cdot\boldsymbol{K}_{sk}^{-1}$; $\boldsymbol{K}_{sk}^{-1}=\boldsymbol{K}_{sk}^{t}$; $\boldsymbol{H}_{ss}^s=\boldsymbol{K}_{sk}\cdot\boldsymbol{H}_{ss}$;

$$\left(\boldsymbol{K}_{sk}^{-1}\right)^{\prime}=\frac{d}{dt}\boldsymbol{K}_{sk}^{-1}=-\begin{bmatrix}\sin\delta_{sk} & \cos\delta_{sk} & 0\\ -\cos\delta_{sk} & \sin\delta_{sk} & 0\\ 0 & 0 & 0\end{bmatrix}.\left(\omega_s-\omega_k\right);$$
$$\boldsymbol{K}_{sk}\left(\boldsymbol{K}_{sk}^{-1}\right)^{\prime}=\left(\omega_s-\omega_k\right).\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}.$$

Let rewrite (3) in Cauchy form:

$$\frac{d}{dt}\boldsymbol{I}_s^s=\boldsymbol{H}_{ss}^s-\boldsymbol{K}_{sk}\left(\boldsymbol{K}_{sk}^{-1}\right)^{\prime}\cdot\boldsymbol{I}_s+\boldsymbol{B}_{ss}^s\cdot\boldsymbol{U}_s^s=$$
$$=\boldsymbol{H}_s^s+\boldsymbol{B}_{ss}^s\cdot\boldsymbol{U}_s^s; \qquad (5)$$

where $\boldsymbol{H}_s^s=\boldsymbol{H}_{ss}^s-\boldsymbol{K}_{sk}\left(\boldsymbol{K}_{sk}^{-1}\right)^{\prime}\cdot\boldsymbol{I}_s$.

Induction motor

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{I}_s\\ \boldsymbol{I}_r\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}_{ss} & \boldsymbol{A}_{sr}\\ \boldsymbol{A}_{rs} & \boldsymbol{A}_{rr}\end{bmatrix}.\begin{bmatrix}\boldsymbol{I}_s\\ \boldsymbol{I}_r\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_s\\ \boldsymbol{B}_r\end{bmatrix}.\boldsymbol{U}_s=$$
$$=\begin{bmatrix}\boldsymbol{H}_s\\ \boldsymbol{H}_r\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_s\\ \boldsymbol{B}_r\end{bmatrix}.\boldsymbol{U}_s; \qquad (6)$$
$$\frac{d}{dt}\omega_r=\frac{1}{\tau_m}\left(T_e-T_l\right);$$

where: $\boldsymbol{A}$ and $\boldsymbol{B}$ are parameter-depended matrices defined in Appendix; through subscribes *s* and *r* note stator and rotor variables and parameters respectively; $\boldsymbol{I}_s=\left[i_d, i_q, i_0\right]^t$; $\boldsymbol{I}_r=\left[i_{rd}, i_{rq}, i_{r0}\right]^t$; $\boldsymbol{U}_s=\left[u_d, u_q, u_0\right]^t$; $T_e=x_{ad}\cdot(i_{rd}\cdot i_q-i_{rq}\cdot i_d)$ is the electromagnetic torque; T_l is the load torque which is generally mechanical input to the machine; τ_m is the rotor mechanical time constant.

For the stator windings we may be written:

$$\frac{d}{dt}\boldsymbol{I}_s=\boldsymbol{H}_s+\boldsymbol{B}_s\cdot\boldsymbol{U}_s \qquad (7)$$

Static R-L load

$$\frac{d}{dt}\boldsymbol{I}=\boldsymbol{A}.\boldsymbol{I}+\boldsymbol{B}.\boldsymbol{U}=\boldsymbol{H}+\boldsymbol{B}.\boldsymbol{U}; \qquad (8)$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are parameter-depended matrices defined in Appendix.

Line-link

The tie lines of power system sort out to a line-links and line-trees by them topological affiliations. The equations of the line-link are written in Cauchy form (see fig.1).

$$\frac{d}{dt}\boldsymbol{I}_j=\boldsymbol{A}.\boldsymbol{I}_j+\boldsymbol{B}.\left(\boldsymbol{U}_j-\boldsymbol{U}_i\right)=$$
$$=\left(\boldsymbol{A}.\boldsymbol{I}_j-\boldsymbol{B}.\boldsymbol{U}_i\right)+\boldsymbol{B}.\boldsymbol{U}_j=\boldsymbol{H}_j+\boldsymbol{B}.\boldsymbol{U}_j \qquad (9)$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are parameter-depended matrices defined in Appendix.

Line-tree

$$\boldsymbol{U}_i-\boldsymbol{U}_j=-\boldsymbol{Z}_{ij}\cdot\boldsymbol{I}_{ij}-\boldsymbol{L}_{ij}\frac{d}{dt}\boldsymbol{I}_{ij}; \qquad (10)$$

where $\boldsymbol{Z}$ and $\boldsymbol{L}$ are parameter-depended matrices defined in Appendix.

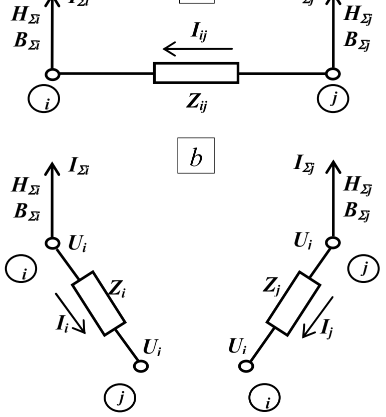

Figure 1

3 SYSTEM DESCRIPTION AND MODELS

The combine of the separate element models to common system model is a key problem in the building of power system aggregate (general) model by someone electrical method. The use of Kirchoff's current law is a preference because it is allowed easily to simulate a different disturbances and calculate unknown node voltages that are in right side of element models (1), (6), (8).

First, the network lines sort out to line-tree and line-link. The line-link models (9) assign to the element models and with help of line-tree models (10) the system general model it is built.

For each node that consist a electrical machines, a static loads and a line-links is calculated summary node current:

$$\boldsymbol{I}_{\Sigma k} = \sum_{j=1}^{p} m_j \cdot \boldsymbol{I}_j ; \qquad (11)$$

where: $m_j = \dfrac{S_j}{S_{G\Sigma}}$ is a scale coefficient; S_j is an element apparent power; $S_{G\Sigma} = \sum_{j=1}^{m} S_{Gj}$ is a total generator apparent power; $\boldsymbol{I}_{\Sigma k} = \left[i_{d\Sigma}, i_{q\Sigma}, i_{0\Sigma} \right]^t$ is summary node current its components are sum of the elements current, which are incident to node; p is number of the elements incident to k-node.

The Kirchoff's current law for node may be expressed in following form:

$$\boldsymbol{I}_{\Sigma 1} + \boldsymbol{I}_{\Sigma 2} + \ldots + \boldsymbol{I}_{\Sigma k} + \ldots + \boldsymbol{I}_{\Sigma n} = 0 ; \qquad (12)$$

where n is a number of power system nodes.

This equation in differential form:

$$\frac{d}{dt}\boldsymbol{I}_{\Sigma 1} + \frac{d}{dt}\boldsymbol{I}_{\Sigma 2} + \ldots + \frac{d}{dt}\boldsymbol{I}_{\Sigma k} + \ldots + \frac{d}{dt}\boldsymbol{I}_{\Sigma n} = 0 . \qquad (13)$$

Replacing the differentials by its right side (5), (7), (8) and (9) we yield:

$$\begin{aligned} &\boldsymbol{H}_{\Sigma 1} + \boldsymbol{H}_{\Sigma 2} + \ldots + \boldsymbol{H}_{\Sigma k} + \ldots + \boldsymbol{H}_{\Sigma n} + \\ &+ \boldsymbol{B}_{\Sigma 1} \cdot \boldsymbol{U}_1 + \boldsymbol{B}_{\Sigma 2} \cdot \boldsymbol{U}_2 + \ldots + \boldsymbol{B}_{\Sigma k} \cdot \boldsymbol{U}_k + \ldots + \boldsymbol{B}_{\Sigma n} \cdot \boldsymbol{U}_n = 0 \end{aligned} \qquad (14)$$

where: $\boldsymbol{B}_{\Sigma k} = \sum_{j=1}^{p} m_j \cdot \boldsymbol{B}_j = diag\left[b_{d\Sigma}, b_{q\Sigma}, b_{0\Sigma} \right]_k$;

$\boldsymbol{H}_{\Sigma k} = \sum_{j=1}^{m} m_j \cdot \boldsymbol{H}_j = \left[h_{d\Sigma}, h\, i_{q\Sigma}, h_{0\Sigma} \right]_k^t$; m is number of elements incident to k-node.

Replacing the line-tree currents $\boldsymbol{I}_{ij}$ and its differentials $\frac{d}{dt}\boldsymbol{I}_{ij}$ in equation (10) by the summary node currents $\boldsymbol{I}_{\Sigma j}$ and its differentials $\frac{d}{dt}\boldsymbol{I}_{\Sigma j}$, replacing differentials by its right side and adding yield line-tree equations to (14) we obtain:

$$\left(\boldsymbol{N}_1 + \boldsymbol{L} . \boldsymbol{N}_2 . \boldsymbol{B} \right) \boldsymbol{U} = -\left[\boldsymbol{L} . \boldsymbol{N}_2 . \boldsymbol{H} + \boldsymbol{Z} . \boldsymbol{N}_2 . \boldsymbol{I} \right] = \boldsymbol{A} . \boldsymbol{U} = \boldsymbol{Y} ; \qquad (15)$$

where:

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{U}_1 \\ \boldsymbol{U}_2 \\ \vdots \\ \boldsymbol{U}_k \\ \vdots \\ \boldsymbol{U}_n \end{bmatrix}, \quad \boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}_{\Sigma 1} \\ \boldsymbol{I}_{\Sigma 2} \\ \vdots \\ \boldsymbol{I}_{\Sigma k} \\ \vdots \\ \boldsymbol{I}_{\Sigma n} \end{bmatrix}, \quad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{\Sigma 1} \\ \boldsymbol{H}_{\Sigma 2} \\ \vdots \\ \boldsymbol{H}_{\Sigma k} \\ \vdots \\ \boldsymbol{H}_{\Sigma n} \end{bmatrix}$$

are node voltages, summary node currents and summary node vectors $\boldsymbol{H}$ respectively; $\boldsymbol{L} = diag\left\lfloor \boldsymbol{L}_{ij} \right\rfloor$; $\boldsymbol{Z} = diag\left\lfloor \boldsymbol{Z}_{ij} \right\rfloor$; $\boldsymbol{B} = diag\left\lfloor \boldsymbol{B}_{\Sigma j} \right\rfloor$ are diagonal matrices of inductions, impedances and summary node matrices $\boldsymbol{B}_{\Sigma j}$; $\boldsymbol{N}_1$ и $\boldsymbol{N}_2$ are network matrices which describe topological feature of the net.

Excluding current derivatives from network equations we obtained algebraic equation system, which allows us to calculate the unknown node voltages non-iteratively. This is the basic advantage of the proposed method. Second benefit is that the building of general power system model by the equations system (15) may be automatized as the system (15) matrices are diagonal (except $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$) and the vectors are created easily for each node.

Only network matrices $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ have a singular form. For its construction is used the correspond pattern matrices $\boldsymbol{N}_{1p}$ and $\boldsymbol{N}_{2p}$. Each unit element of pattern matrices corresponds to a unity matrix with dimension 3x3 of matrices $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$. Below its construction will be present on example of power system shown on fig.2. Obvious, matrix $\boldsymbol{N}_{1p}$ is a tree graph of network incidence matrix, which for the considered example would have following form:

$\boldsymbol{N}_{1p} =$

	1	1								1	1	1		1
	1													
		1	1		1	1	1							
			1	1										
				1										
					1									
						1								
							1	1						
								1	1					
									1					
										1				
											1			
												1		
													1	
														1

The matrix $\boldsymbol{N}_{2p}$ presents the line-tree currents replacement by the summary node currents. Each row corresponds to one line-tree current and each column corresponds to one summary node current. It is used also matrix $\boldsymbol{N}_{1p}$. Starting from lowest row each row multiplies

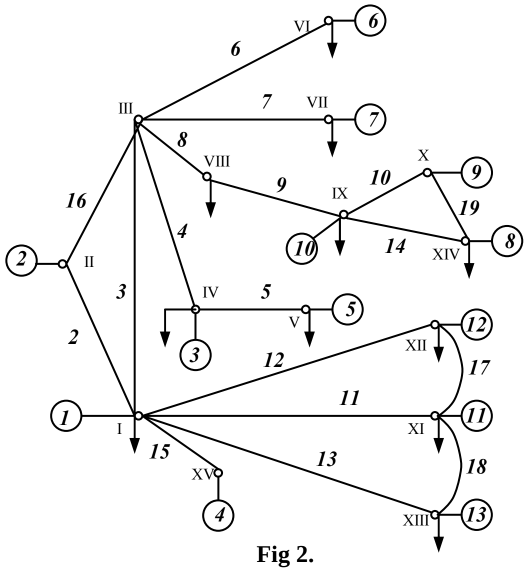

Fig 2.

$N_{2p} =$

	1	1	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$	$\tilde{1}$	1	1	1	$\tilde{1}$	1
	1													
		1	1	$\tilde{1}$	1	1	1	$\tilde{1}$	$\tilde{1}$				$\tilde{1}$	
			1	1										
				1										
					1									
						1								
							1	1	$\tilde{1}$				$\tilde{1}$	
								1	1				1	
									1					
										1				
											1			
												1		
													1	
														1

by each previous row and in dependence of the obtained results (zero or non-zero) it is added or it's not added logically to current row:

$$
\begin{aligned}
&\boldsymbol{N}_{2p} = \boldsymbol{N}_{1p}; \\
&\boldsymbol{N}_{2p,k} \cap \boldsymbol{N}_{2p,k-1} = q_{k-1}; \\
&if \quad q_{k-1} \neq 0, \quad then \quad \boldsymbol{N}_{2p,k-1} = \boldsymbol{N}_{2p,k} \cup \boldsymbol{N}_{2p,k-1}; \\
&if \quad q_{k-1} = 0, \quad then \quad \boldsymbol{N}_{2p,k-1} = \boldsymbol{N}_{2p,k-1};
\end{aligned}
\tag{16}
$$

where $k = \overline{n,1}$ and n is the number of system node.

For considered example the pattern matrix $\boldsymbol{N}_{2p}$ will have form shown above.

4 DISTURBANCES SIMULATION

The proposed method of a transient calculation allows simulate arbitrary disturbances in power system that are caused by changes in the regulators, parameters, numbers of elements and network topology. The element (generator, motor, static load) turning on/off is performed by respective model adding/excluding in the corresponding node. Simultaneously the total sums of the vectors $\boldsymbol{H}_{\Sigma k}$ and $\boldsymbol{I}_{\Sigma k}$, and a matrix $\boldsymbol{B}_{\Sigma k}$ for corresponding node are changed. If a generator is turned on or off it is necessary to recalculate a total generator apparent power $S_{G\Sigma} = \sum_{j=1}^{m} S_{Gj}$ and a scale coefficient $m_j = \dfrac{S_j}{S_{G\Sigma}}$.

Highest performs are necessary to be carried out at the topology network change. In that case it is necessity to build a new pattern matrices $\boldsymbol{N}_{1p}$ and $\boldsymbol{N}_{2p}$, and a new equations system (15). The small changes of the network, for example turning on or off of a particular line, can be simulated by a parameters change (matrices $\boldsymbol{Z}_{ij}$ и $\boldsymbol{L}_{ij}$) of the switched line.

The fault at the given node and its turn off by protection gear is simulated parametrically by sudden change of *RL*-load parameters.

On figures 3-7 are shown part of results obtained simulating fault in node 12 at time 1500 [rad] with fault canceling at time 1550 [rad]. Shown curves are: $\delta_{3\text{-}4}$ (delta 3-4); voltage in node 12 (U12); $\delta_{12\text{-}2}$ (delta 12-2); current of link-line 16 (ILL16); current of generator 12 (IG12). On figures 7-8 are shown part of results obtained simulating change of reference voltage of AVR of Synchronous Generator 8 in node 14 from 1 [p.u.] to 1.08 [p.u.] at time 1500 [rad] and return to initial state at time 1550 [rad]. Shown curves are: angular velocity of generator 12 (wk12); current of the static load 2 (ISL2).

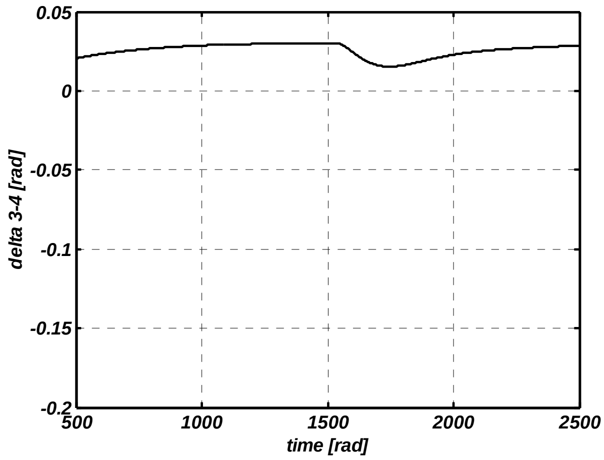

Figure 3

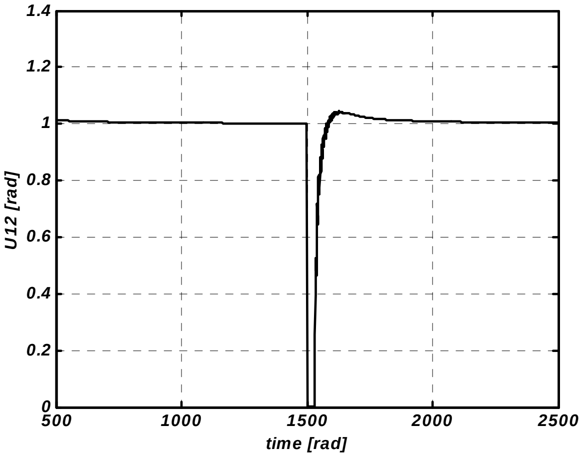

Figure 4

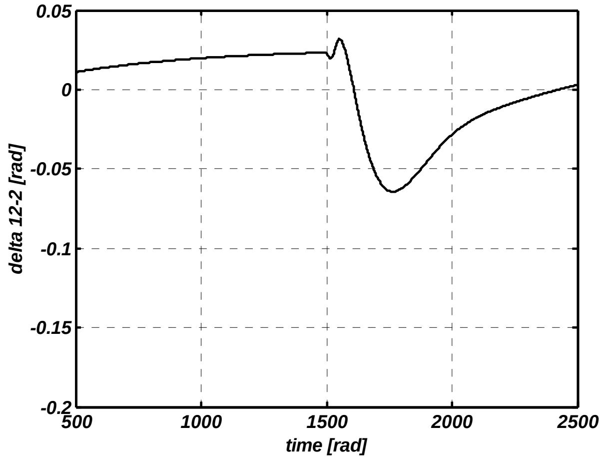

Figure 5

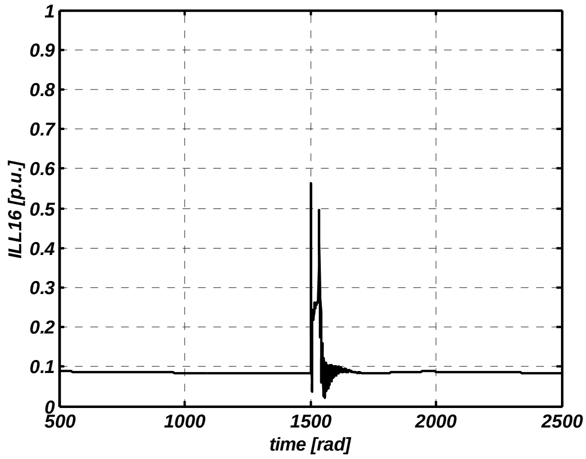

Figure 6

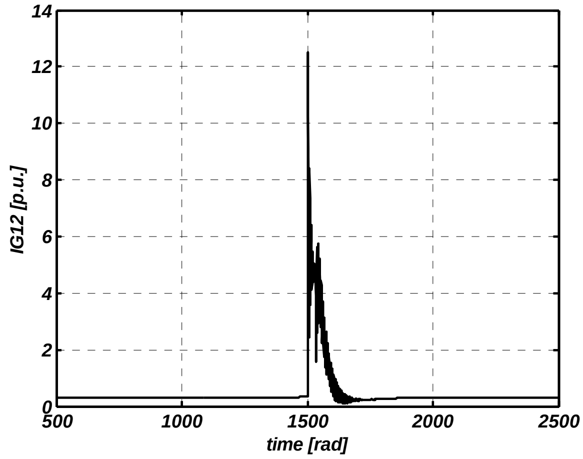

Figure 7

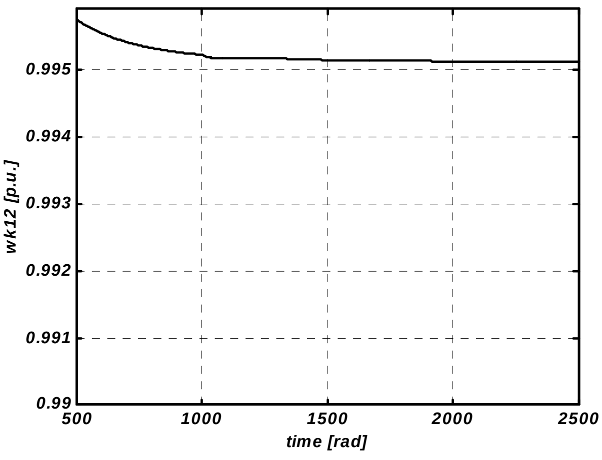

Figure 8

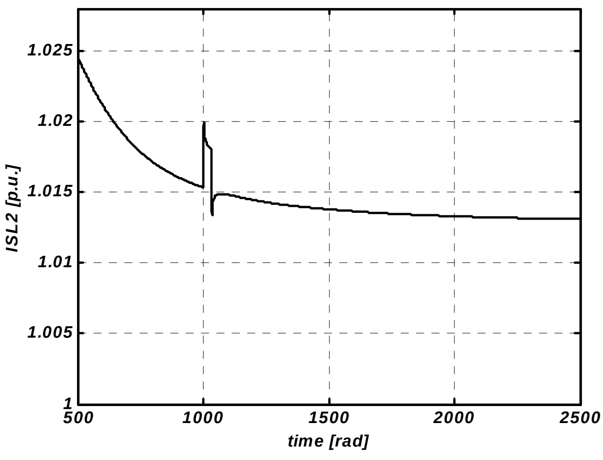

Figure 9

5 CONCLUSION

Proposed is a new method for automatic compose of the models, which describe transient electromechanical processes in power system with arbitrary number and type of elements and arbitrary topology of network. Mathematical models obtained using this method are efficient, since they use minimal computational resources (memory and computing time). Method can be easily implemented using state of the art program instruments. Companion paper [11] shows current accomplishment in MATLAB environment. Obtained experimental results demonstrates correctness and efficient of method.

REFERENCES

[1] J.M.Undrill,"Structure in the Computation of

Power System Nonlinear Dynamic Response”, IEEE Trans. on Power Apparatus and Systems, Vol. PAS-88, pp.1-6, Jan. 1969.

[2] W.Janischewskyj and P.Kundur, “Simulation of the Non-Linear Dynamic Response of Interconnected Synchronous Machines”, *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-91, N5, pp.2064-2077, Sept./Oct. 1972.

[3] P.C.Krause, F.Nozari, T.L.Skvarenina and D.W. Olive, “The Theory of Neglecting Stator Transients”, *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-98, N1, pp.141-148, Jan./Feb. 1979.

[4] Computer Analysis of Electric Power System Transients. Edited by J.A. Martinez-Velasco, Published by IEEE Inc, 1997, 619 p.

[5] H.K.Lauw and W.S.Meyer, “Universal Machine Modeling for the Representation of Rotating Machinery in an Electromagnetic Transients Program”, *IEEE Trans. on Apparatus and Systems*, Vol. PAS-101, N6, pp.1342-1352, June, 1982.

[6] H.K.Lauw, “Interfacing for Universal Multi-Machine System Modeling in an Electromagnetic Transients Program”, *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-104, N9, pp.2367-2373, Sept., 1985.

[7] J.S.Mayer and O.Wasynczuk, “An Efficient Method of Simulating Stiffly Connected Power Systems with Stator and Network Transients Included”, *IEEE Trans. on Power Systems*, Vol.6, N3, pp.922-929, Aug, 1991.

[8] V.M.Sendurev, “An Algorithm of Electromechanical Transient Processes in Autonomous Power System by Full and Reduced Park-Gorev’s Equations”, *Electricity*, (Moscow) N5, pp.16-20, May, 1978.

[9] K.K.Ketner, I.A.Kozlova and V.M.Sendurev, “An Algorithm of Transient Processes Calculation in Autonomous Power Systems”. Zinatne, Riga, 1981, 166 p.

[10] N.F.Djagarov, “A Calculation of Transient Processes in Power Systems with a Complex Network”, *Electricity*, (Moscow) N1, pp.9-16, Jan. 1990.

[11] N.F.Djagarov and K.V.Malikov “Creating an electromechanical transients modeling program in MATLAB and SIMULINK”, same issue

APPENDIX

System model parameters

The electrical machines models are written in per unit system of equals mutual inductance and electromotive force (e.m.f.) MF. All values are given in p.u. with a base power equal to the machine rating. Below non-zero elements of matrices ***A, B, Z*** and ***L*** are expressed.

Synchronous generator

$$d_d = x_d \cdot x_f \cdot x_g - x_{ad}^2 \cdot (x_d + x_f + x_g - 2\, x_{ad});$$

$$q_d = x_q \cdot x_h - x_{aq}^2; \quad b_{44} = \frac{-a_{44}}{r_f}; \quad b_{54} = \frac{-x_l \cdot x_{ad}}{d_d};$$

$$b_{11} = \frac{x_g \cdot x_f - x_{ad}^2}{d_d}; \quad b_{22} = \frac{x_h}{q_d}; \quad b_{14} = b_{14} = -\frac{x_{gl} \cdot x_{ad}}{d_d};$$

$$b_{51} = -\frac{x_{fl} \cdot x_{ad}}{d_d}; \quad b_{62} = -\frac{x_{aq}}{q_d}; \quad b_{33} = \frac{1}{x_0};$$

$$a_{11} = -r_s \cdot b_{11}; \quad a_{12} = x_q \cdot b_{11} \cdot \omega_k; \quad a_{14} = -r_f \cdot b_{41};$$

$$a_{15} = -r_g \cdot b_{51}; \quad a_{16} = x_{aq} \cdot b_{11} \cdot \omega_k; \quad a_{21} = -x_d \cdot b_{22} \cdot \omega_k;$$

$$a_{22} = -r_s \cdot b_{22}; \quad a_{24} = a_{25} = -x_{ad} \cdot b_{22} \cdot \omega_k; \quad a_{26} = -r_h \cdot b_{62};$$

$$a_{41} = -r_s \cdot b_{41}; \quad a_{42} = x_q \cdot b_{41} \cdot \omega_k; \quad a_{44} = \frac{\left(x_{ad}^2 - x_d \cdot x_g\right) \cdot r_f}{d_d};$$

$$a_{45} = \frac{x_l \cdot x_{ad} \cdot x_g}{d_d}; \quad a_{46} = x_{aq} \cdot b_{41} \cdot \omega_k; \quad a_{33} = -\frac{r_s}{x_0};$$

$$a_{51} = -r_s \cdot b_{51}; \quad a_{52} = x_q \cdot b_{51} \cdot \omega_k; \quad a_{56} = x_{aq} \cdot b_{51} \cdot \omega_k;$$

$$a_{54} = \frac{x_s \cdot x_{ad} \cdot r_f}{d_d}; \quad a_{55} = \frac{\left(x_{ad}^2 - x_d \cdot x_f\right) \cdot r_g}{d_d}; \quad a_{66} = \frac{x_q \cdot r_h}{q_d};$$

$$a_{61} = -x_d \cdot b_{62} \cdot \omega_k; \quad a_{62} = -r_s \cdot b_{62}; \quad a_{64} = a_{65} = -x_{ad} \cdot b_{62} \cdot \omega_k \cdot$$

Induction motor

$$x_d' = x_s - \frac{x_{ad}^2}{x_r}; \quad b_{11} = b_{22} = \frac{1}{x_d'}; \quad b_{41} = b_{52} = \frac{-x_{ad}}{x_r \cdot x_d'};$$

$$a_{11} = a_{22} = -r_s \cdot b_{11}; \quad a_{12} = -a_{21} = \left(\omega_s + \frac{x_{ad}^2 \cdot \omega_r}{x_r \cdot x_d'} \right);$$

$$a_{14} = -a_{25} = r_r \cdot b_{41}; \quad a_{15} = -a_{24} = x_{ad} \cdot b_{11} \cdot \omega_r;$$

$$a_{41} = a_{52} = -r_s \cdot b_{41}; \quad a_{42} = -a_{51} = x_s \cdot b_{41} \cdot \omega_r;$$

$$a_{44} = a_{55} = \frac{-x_s \cdot r_r}{x_r \cdot x_d'}; \quad a_{45} = -a_{54} = \left(\omega_s - \frac{x_s \cdot \omega_r}{x_d'} \right)$$

$$b_{33} = \frac{1}{x_0}; \quad a_{33} = -\frac{r_s}{x_0}.$$

Static R-L load

$$a_{11} = a_{22} = -\frac{r_l}{l_{l1}}; \qquad a_{33} = -\frac{r_l}{l_{l0}}; \qquad a_{12} = -a_{21} = \omega_s;$$

$$b_{11} = b_{22} = \frac{1}{l_{l1}}; \; b_{33} = \frac{1}{l_{l0}}.$$

Link line

$$a_{11} = a_{22} = -\frac{r_{ll}}{l_{ll1}}; \qquad a_{33} = -\frac{r_{ll}}{l_{ll0}}; \qquad a_{12} = -a_{21} = \omega_s;$$

$$b_{11} = b_{22} = \frac{1}{l_{ll1}}; \; b_{33} = \frac{1}{l_{ll0}}.$$

Tree line

$$z_{11} = z_{22} = z_{33} = r_{lt}; \; z_{12} = -z_{21} = -l_{lt} \cdot \omega_s;$$

$$l_{11} = l_{22} = l_{lt1}; \; l_{33} = l_{lt0}.$$